

Wakes and Wind Farms

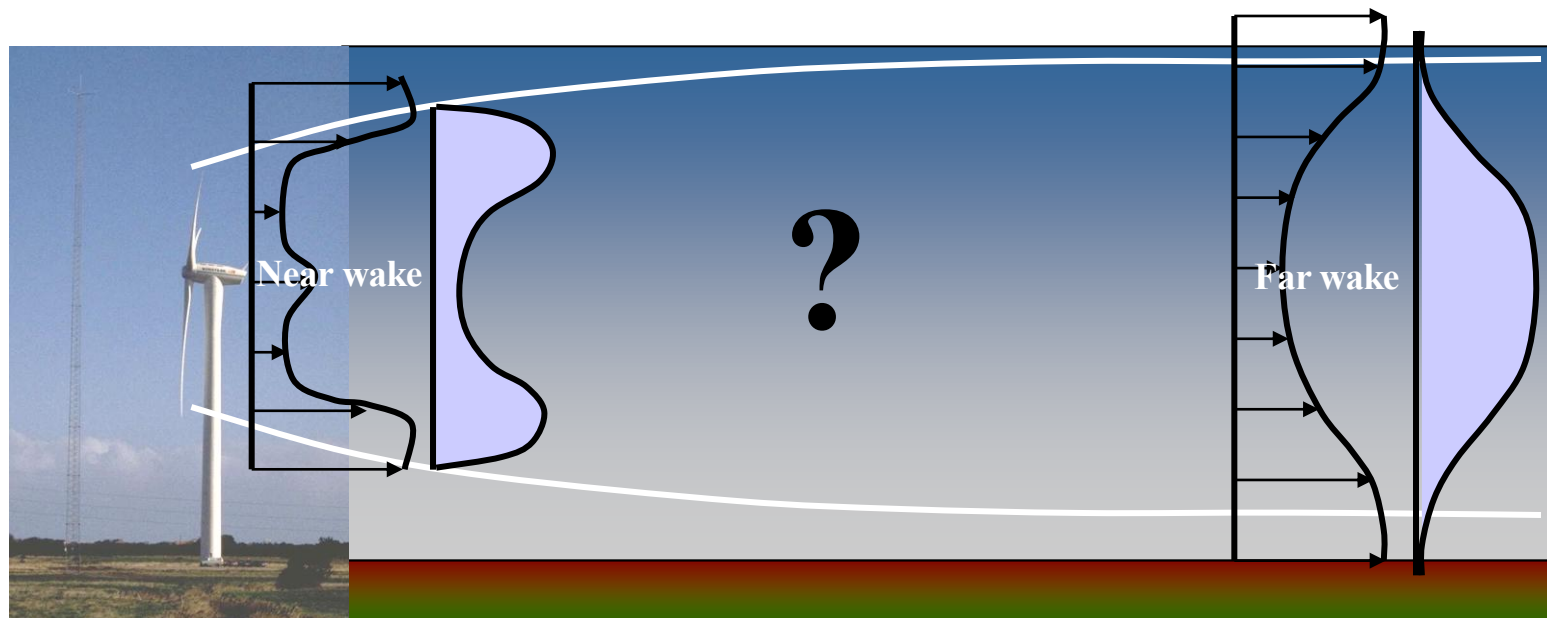
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Valery Okulov, Hamid Sarlak, Kurt S. Hansen***

Department of Mechanical Engineering (MEK)
Technical University of Denmark

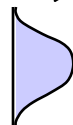
**Collaborators: Stefan Ivanell, Dan Henningson and other members of the
Nordic Consortium on Optimization and Control of Wind Farms**

Wake Aerodynamics

Wake development:



: Axial velocity



: Turbulence intensity

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Wake Aerodynamics

Basic questions:

- **How important is the dynamics of the vortex system**
- **How does the strength of the vortices depend on the blade load**
- **How does roll-up take place**
- **Can we determine the conditions for stability**
- **What is the relationship between vortex dynamics and meandering**
- **How does the added turbulence intensity relate to the loading**
- **Dynamics of the wake interaction between more turbines**
- **How do we optimize wind turbines in large parks**
- **Can we determine the mutual influence between wind farms**

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PhD-project: Søren Juhl Andersen

Simulation and prediction of wakes and wake interaction in wind farms

Year 1:

Literature review on wake models.

Courses

Further development of an existing CFD code to conduct a thorough parametric study of the flow field in the wake of a single wind turbine. The parametric study aims at investigating the following:

- **The influence of shear in the inflow(i.e. a gradient in the atmospheric boundary layer)**
- **Vortex collapsing**
- **Transition between near- and far-wake**
- **Relation between induced turbulence and thrust**
- **How to correctly add ambient and induced turbulence**

Year 2:

Develop guidelines based on the parametric study of a single wind turbine

Continued parametric study on the flow field within wind turbine farms investigating the following influences:

- **The spacing between individual wind turbines.**
- **Stable or unstable atmospheric boundary layer, including vertical mixing.**
- **Wake meandering**

Year 3:

Analysis and interpretation of parametric studies leading to a procedure to optimize wind farms.

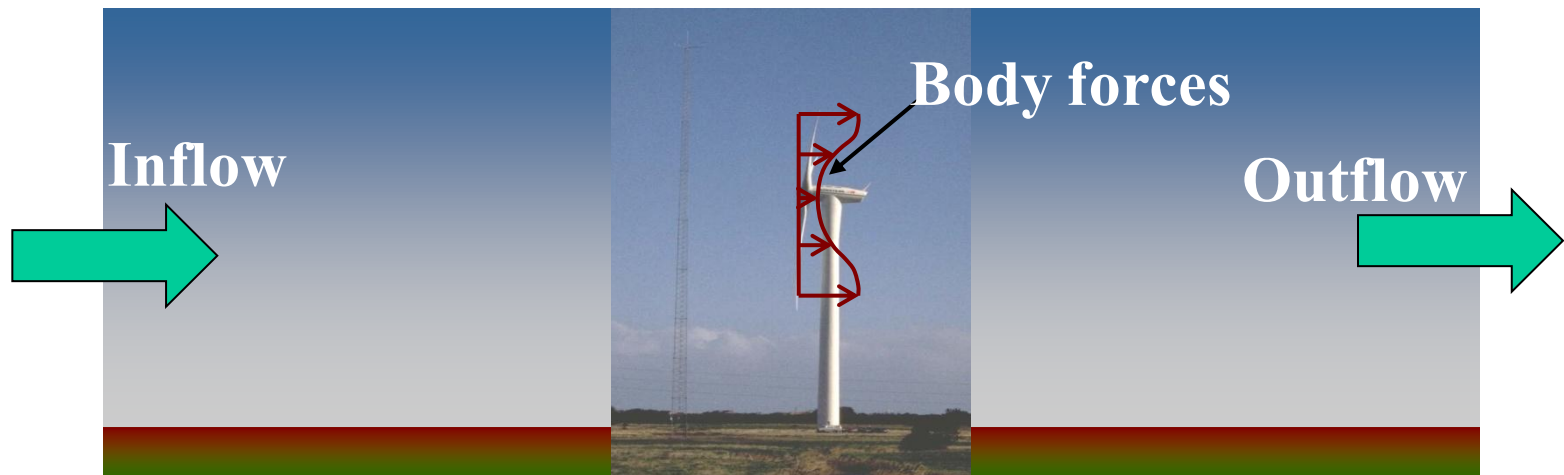
PhD Project: Hamid Sarlak

“Simulation and prediction of wakes in offshore wind farms subject to turbulent and stratified atmospheric boundary layers”

- **Literature survey on wake models based on CFD. Further development of an existing CFD code to include modelling of inflow effects on scales relevant for offshore wind farms**
- **Modelling of turbulent and thermal atmospheric boundary layer with existing CFD code. Development of engineering predictive tool. Comparison to experiments with particular emphasis on densely located wind turbines, such as the Lillgrund wind farm.**
- **Interaction between wind farms.**
- **Numerical experiments and parametrical studies. Guidelines/recommendations.**

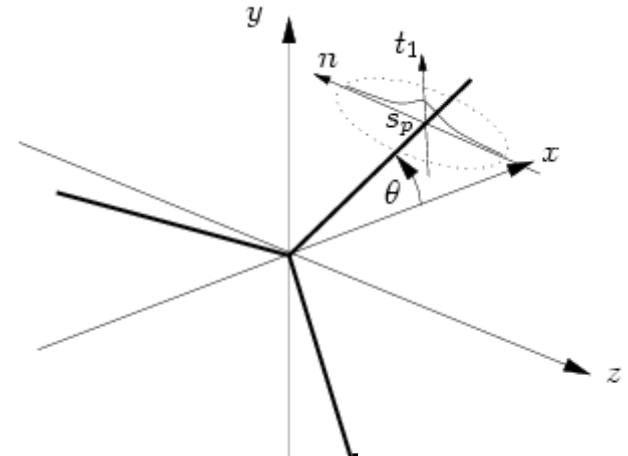
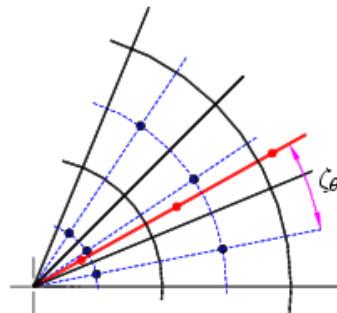
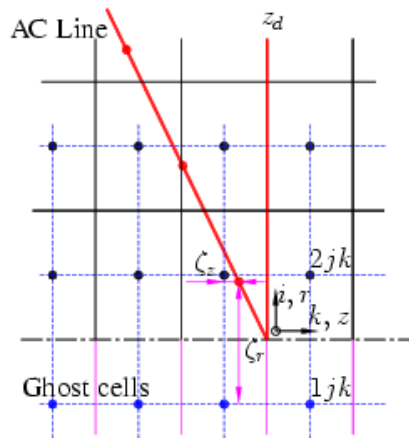
The actuator line technique

- Basic idea:
- Replace rotor blades by body forces
 - Determine body forces from aerofoil data
 - Simulate flow domain using DNS or LES



The Actuator Line Technique

- Regular 3D Grid
- $\mathbf{V}_{r\theta z}$ – Linear interpolation
- No tip correction is applied



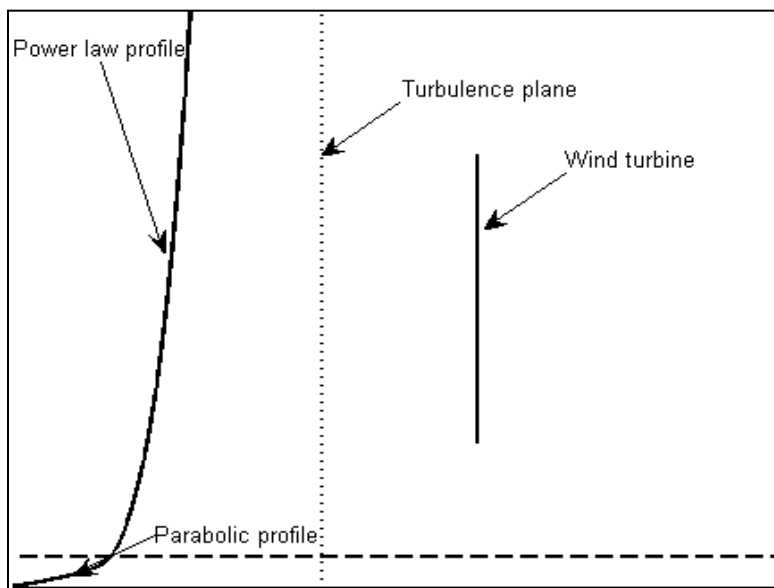
Forces smearing

$$\mathbf{f}(\mathbf{x}) = \sum_{i=1}^B \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathbf{F}^i(s) \eta(p^i) dndt_1$$

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla p + \mu \nabla^2 \mathbf{V} + \mathbf{f}_{body}$$

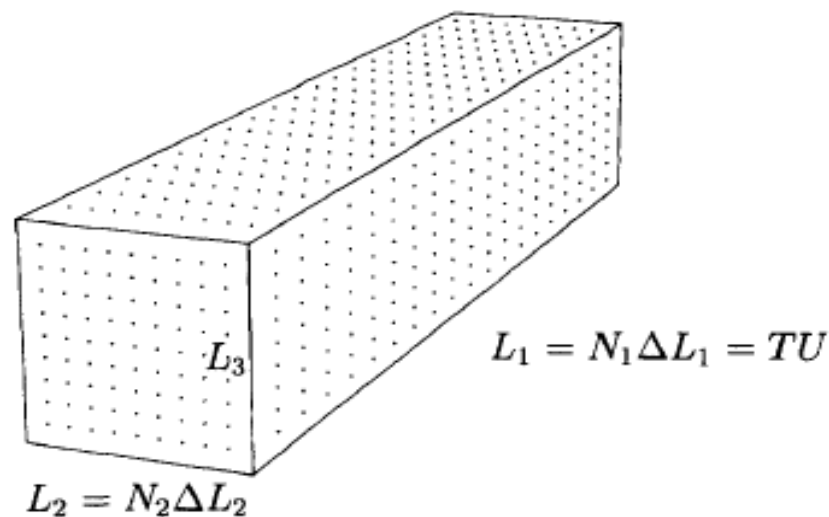
Wind Shear and Turbulence

$$w(y) = \begin{cases} w_0 \cdot (c_2 y^2 + c_1 y) & y \leq \Delta \\ w_0 \cdot \left(\frac{y}{h_{hub}} \right)^\alpha & y > \Delta \end{cases}$$



$$\mathbf{f} = \dot{m}\mathbf{u} + \rho\varepsilon \frac{\partial \mathbf{u}}{\partial t}$$

$$\mathbf{f}_\varepsilon = \mathbf{f} \otimes \eta_\varepsilon, \quad \eta_\varepsilon(d) = \frac{1}{\varepsilon\sqrt{\pi}} \exp\left(-\left(\frac{d}{\varepsilon}\right)^2\right)$$



Power law wind shear profile

Model of wind turbulence

Wind Turbine Wake Aerodynamics

Horns Rev offshore wind farm:



Simulation of the Horns Rev Wind Farms

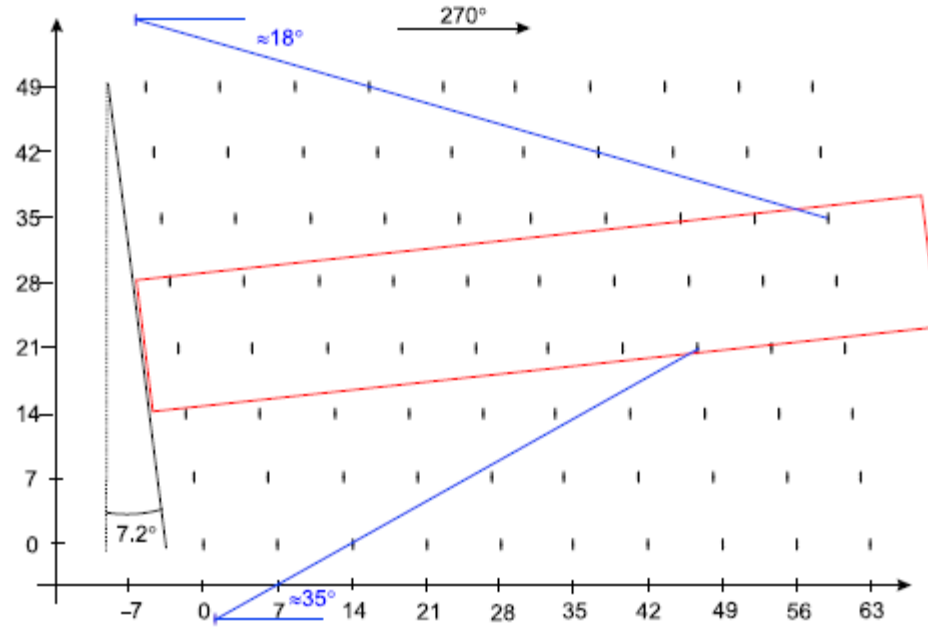


FIGURE 1. Layout of Horns Rev Wind Farm

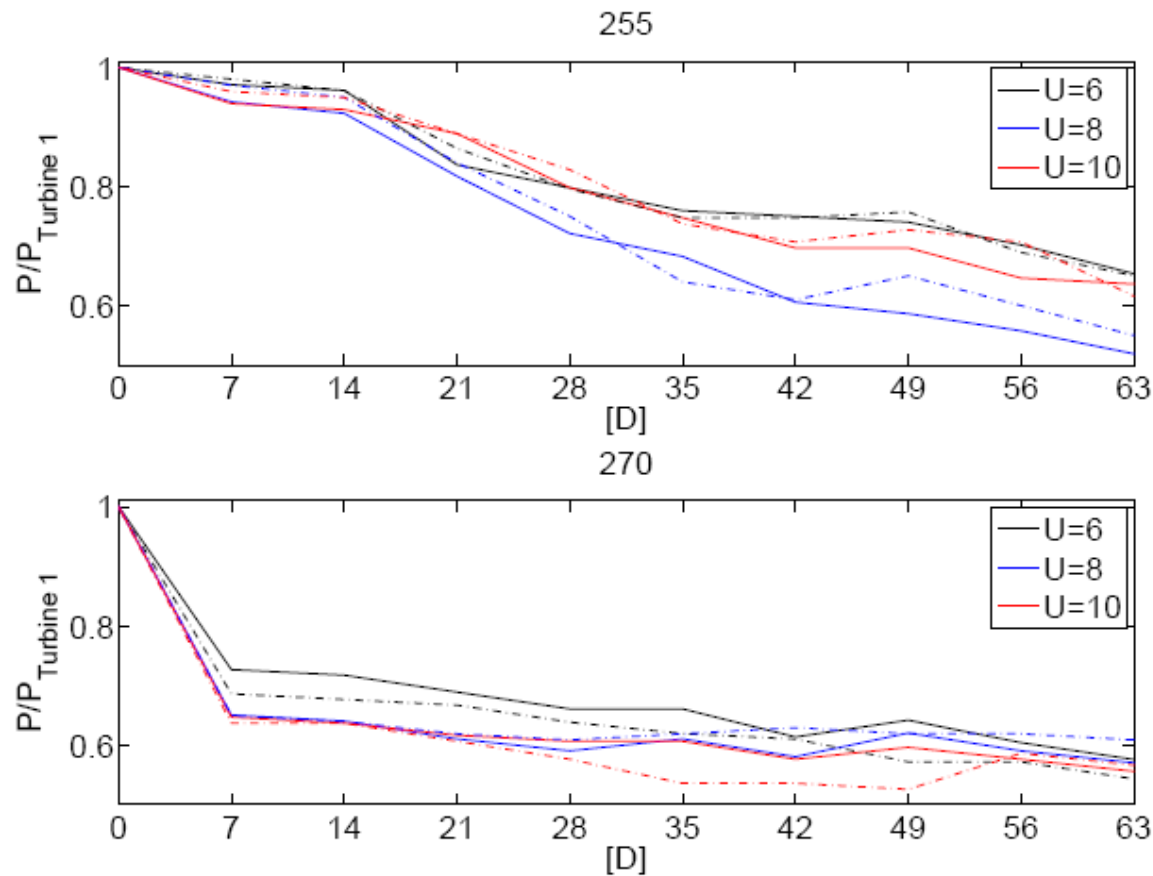


FIGURE 5. Experimental data from Horns Rev. The figures show the production at two inflow angles, 255 and 270 degrees. The solid lines represents the results for column number 4, and the dotted lines the results for column number 5. The result is depicted for three different wind speeds.

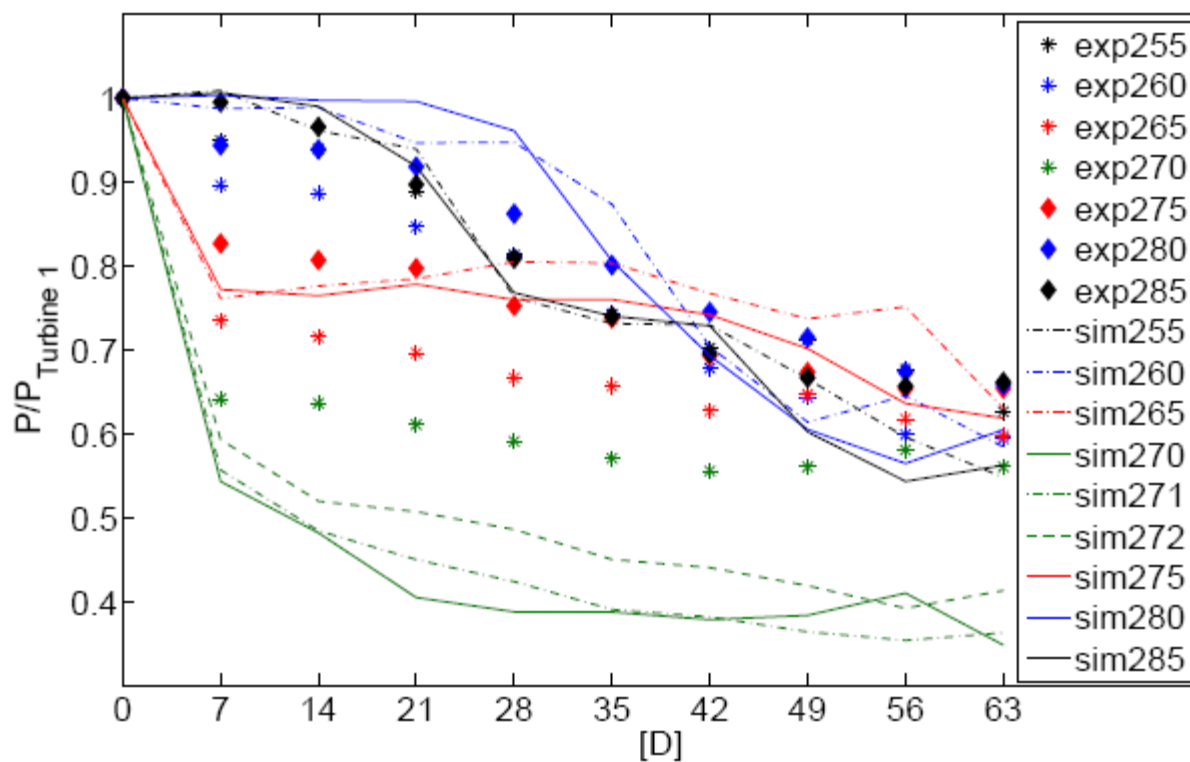
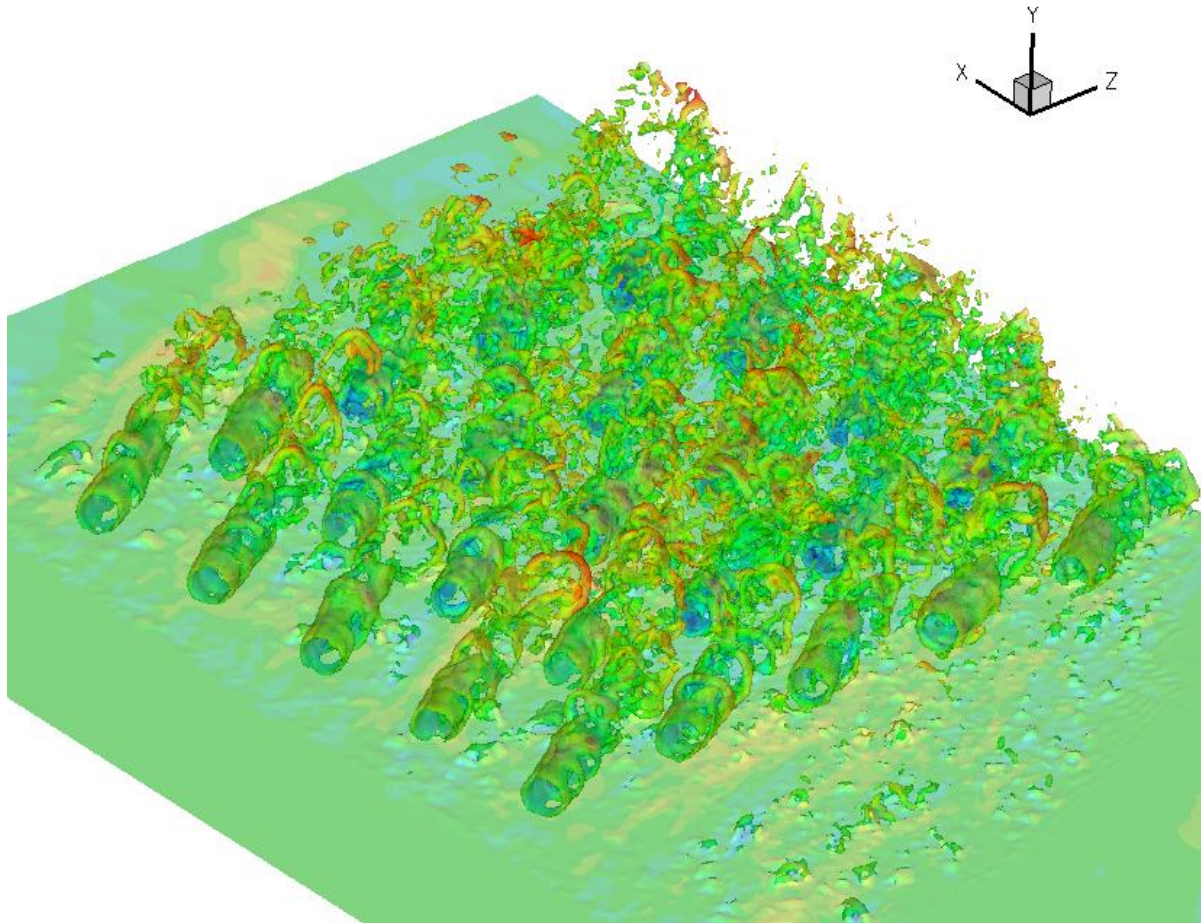


FIGURE 13. Simulation results compared with measurements. Results from both simulations and measurements are shown for inflow angles between 255 and 285 degrees, i.e., ± 15 degrees from the westerly direction.

Modelling of Turbulent and Atmospheric Turbulence



**Vorticity shed from 5x5 turbines in a farm
computed by actuator disk method**

Further validation: Lillgrund wind farm

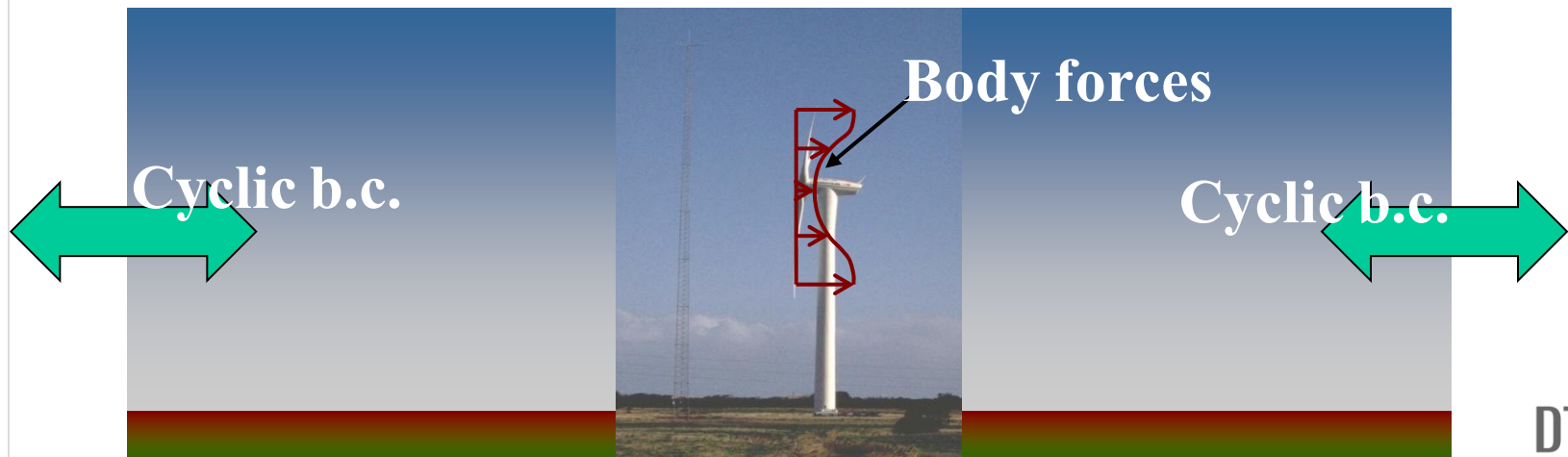
Data required:

- Blade type
- Rotor geometry
- Tip speed
- Performance



Simulation of turbulence inside wind farm

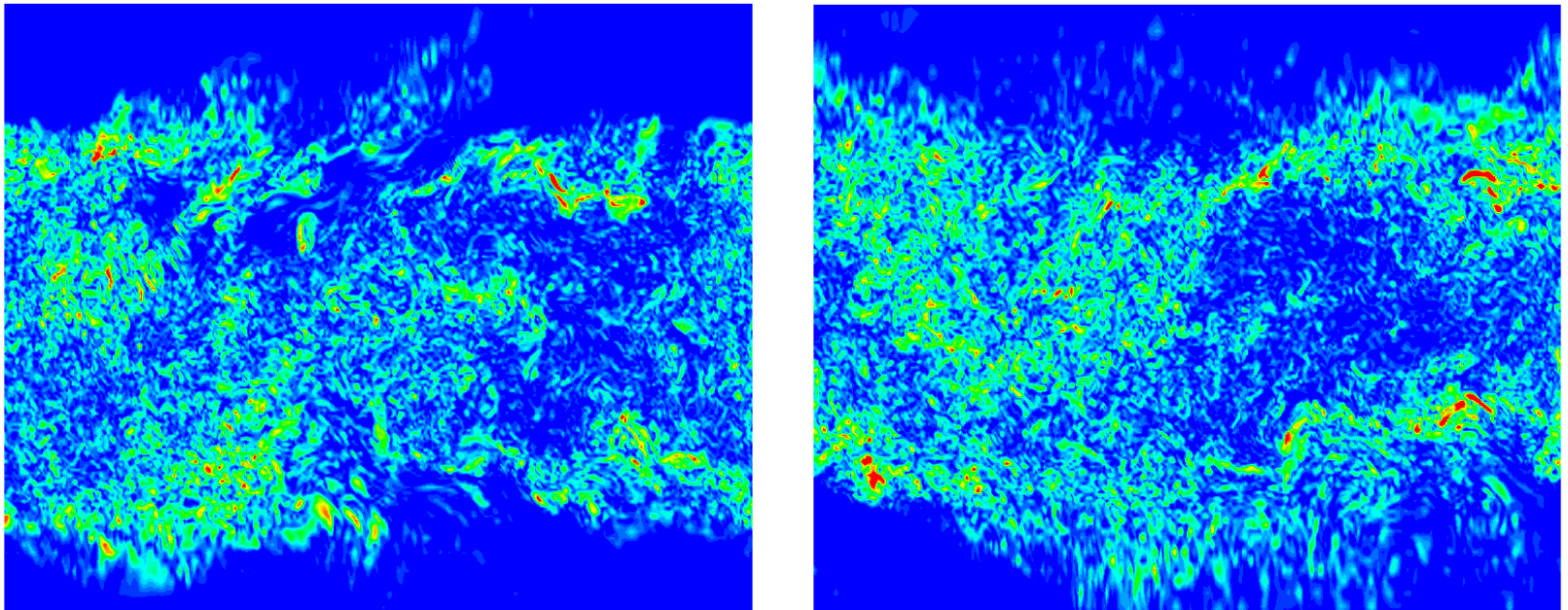
- Basic idea:
- Replace rotor blades by body forces
 - Determine body forces from aerofoil data
 - Simulate an 'infinite' row of turbines using cyclic boundary conditions



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Simulation of turbulence inside wind farm

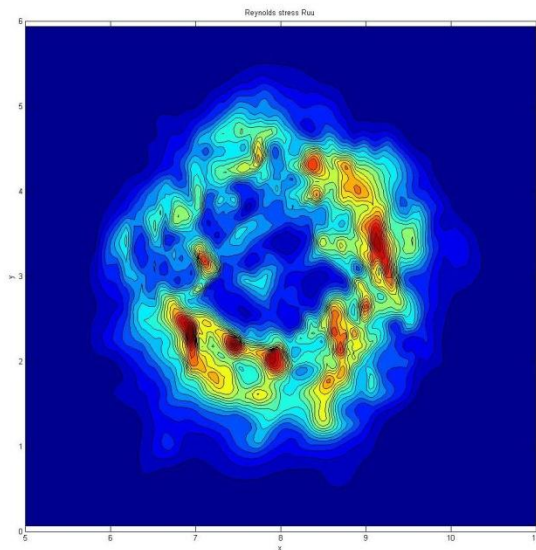
Cross sectional turbulent flow fields:



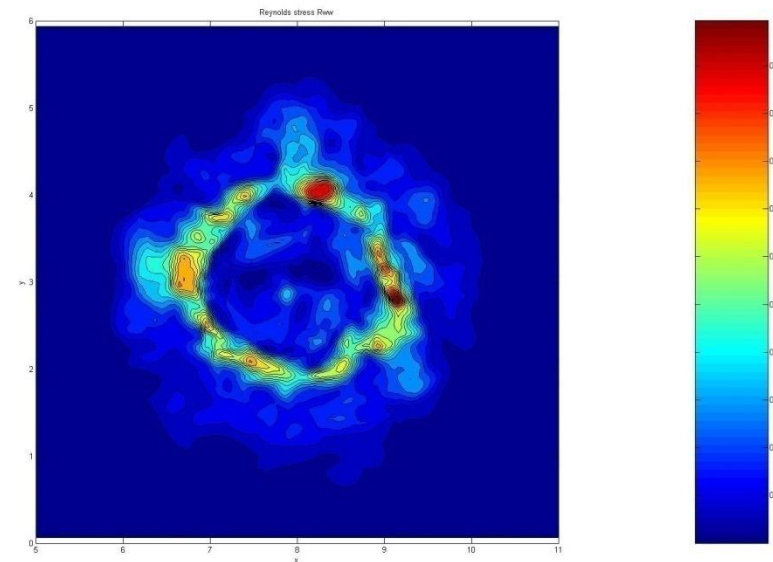
Iso-vorticity contours in the final stage

Simulation of turbulence inside wind farm

Reynolds stresses:



$$\overline{u'u'}$$



$$\overline{w'w'}$$

Decomposition and reconstruction of wind field

Reconstruction of velocity field:

$$V^{(k)}(\vec{x}, t) = \overline{V^{(k)}}(\vec{x}) + \sum_{i=1}^N a_i(t) \phi_i^{(k)}(\vec{x})$$

Amplitude function for mode 'i' at time t: $a_i = a_i(t)$

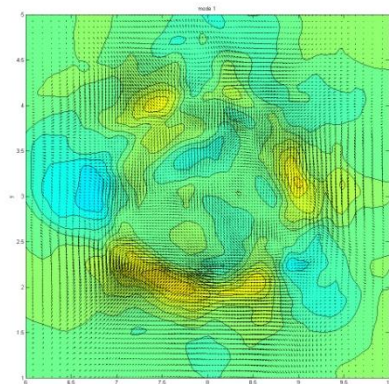
Spatial mode 'i': $\phi_i^{(k)}(\vec{x})$

Galerkin projection on the Navier-Stokes equations:

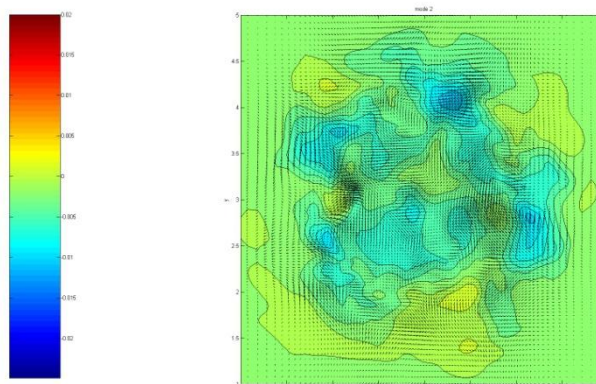
$$\frac{\partial a_i}{\partial t} = \alpha_i + \beta_{ij} a_j + \gamma_{ijk} a_j a_k$$

Reconstruction of turbulence inside wind farm

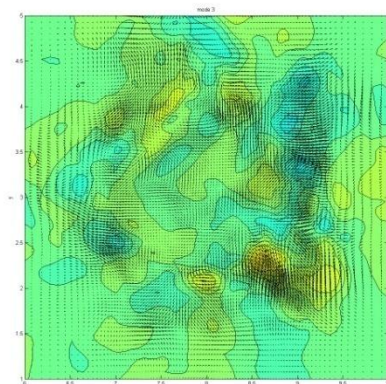
Proper Orthogonal Decomposition:
$$V^{(k)}(\vec{x}, t) = \overline{V^{(k)}}(\vec{x}) + \sum_{i=1}^N a_i(t) \phi_i^{(k)}(\vec{x})$$



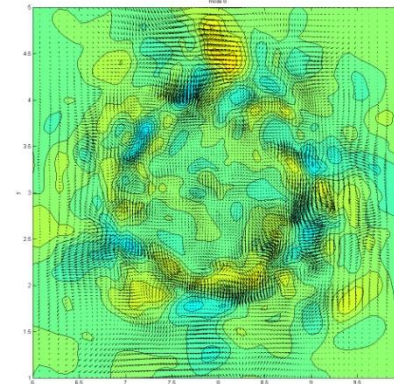
POD Mode 1



POD Mode 3

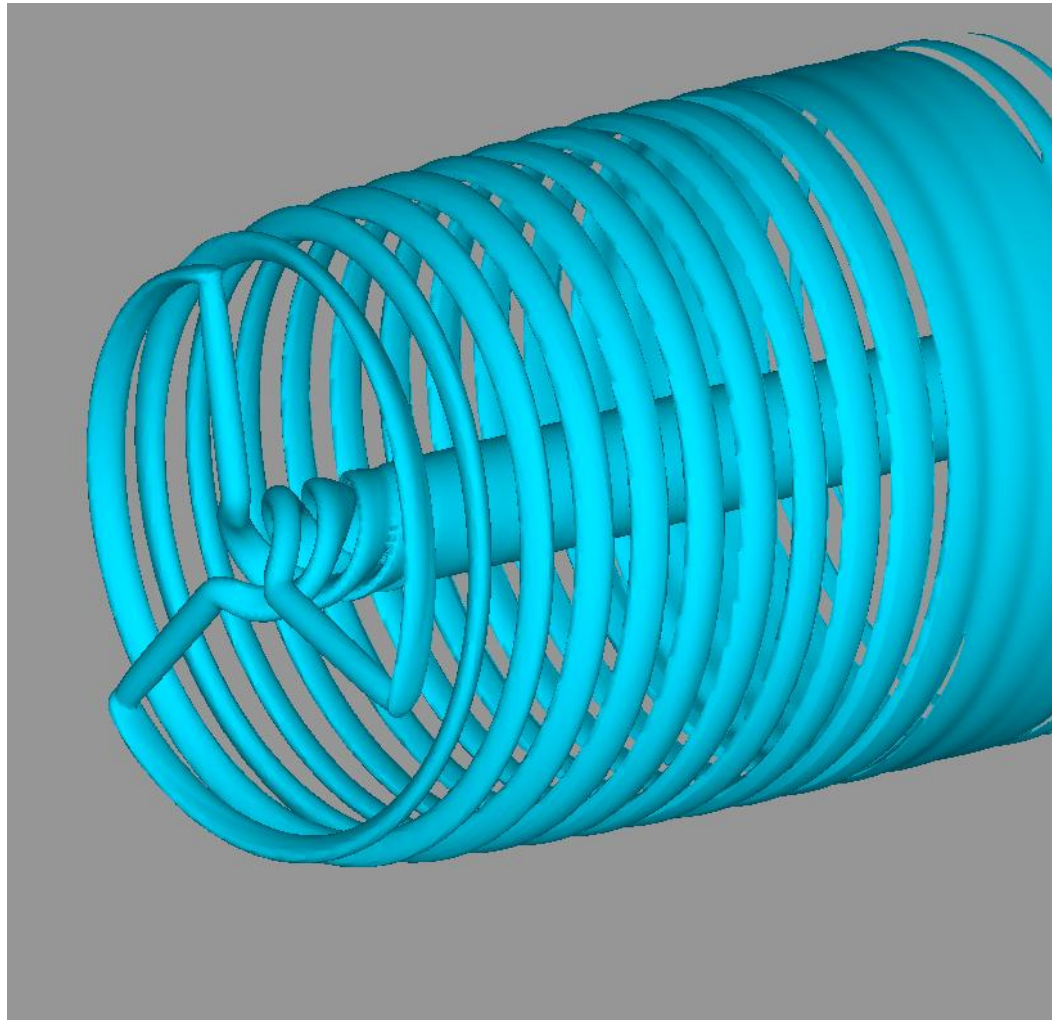


POD Mode 2

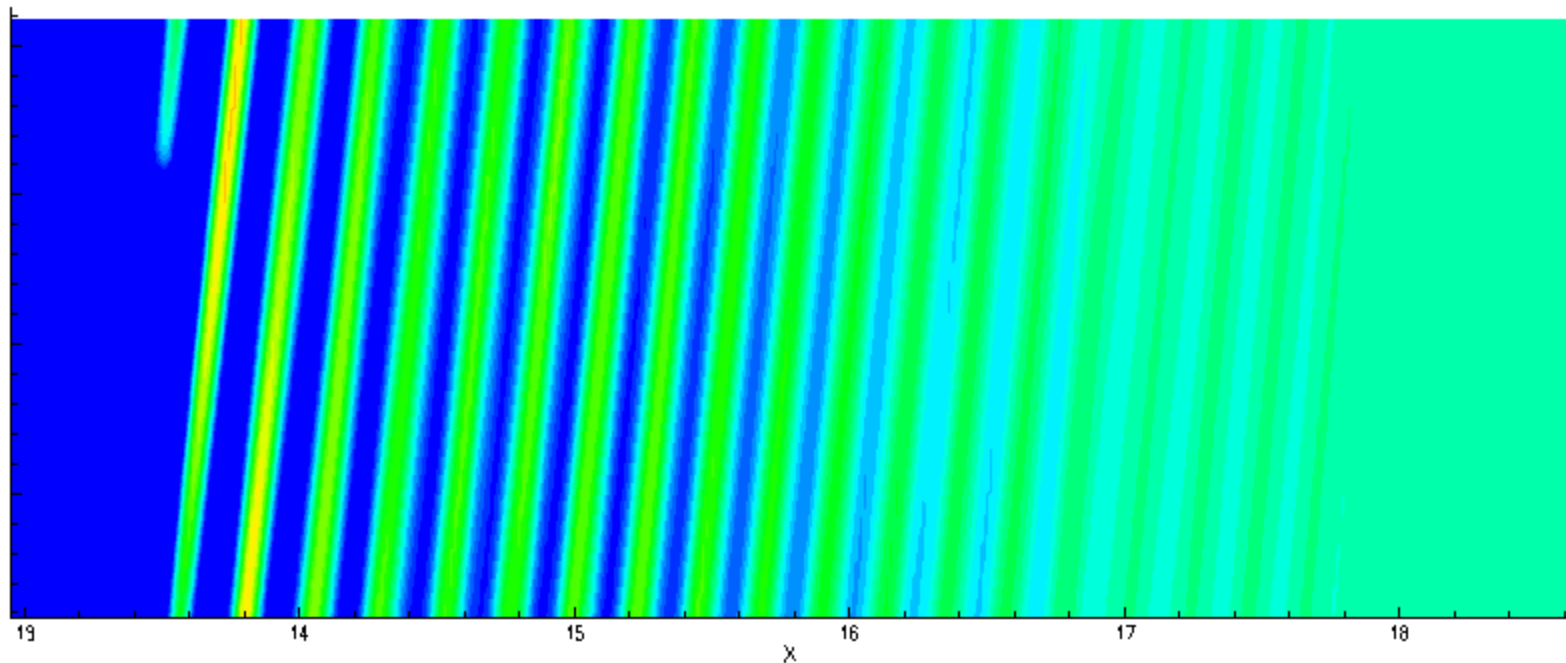


POD Mode 8

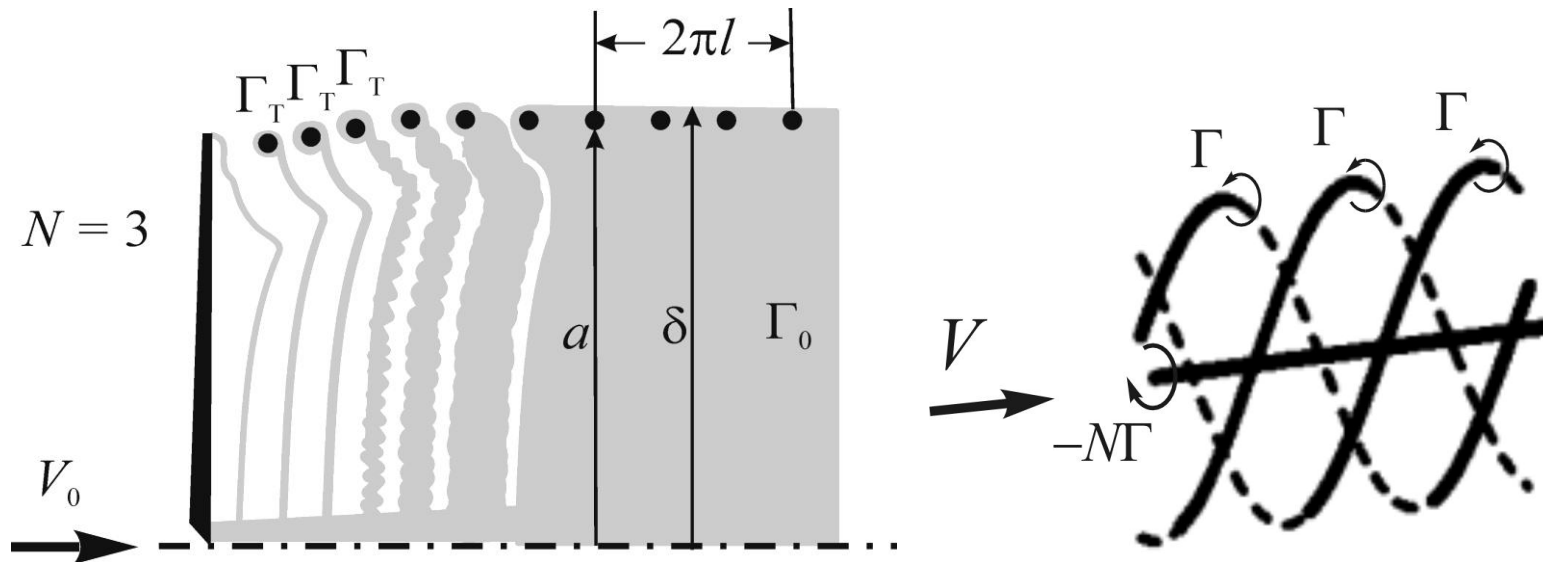
Stability analysis of vortex structures in the wake of a rotor



Stability of vortex structures in the wake of a rotor



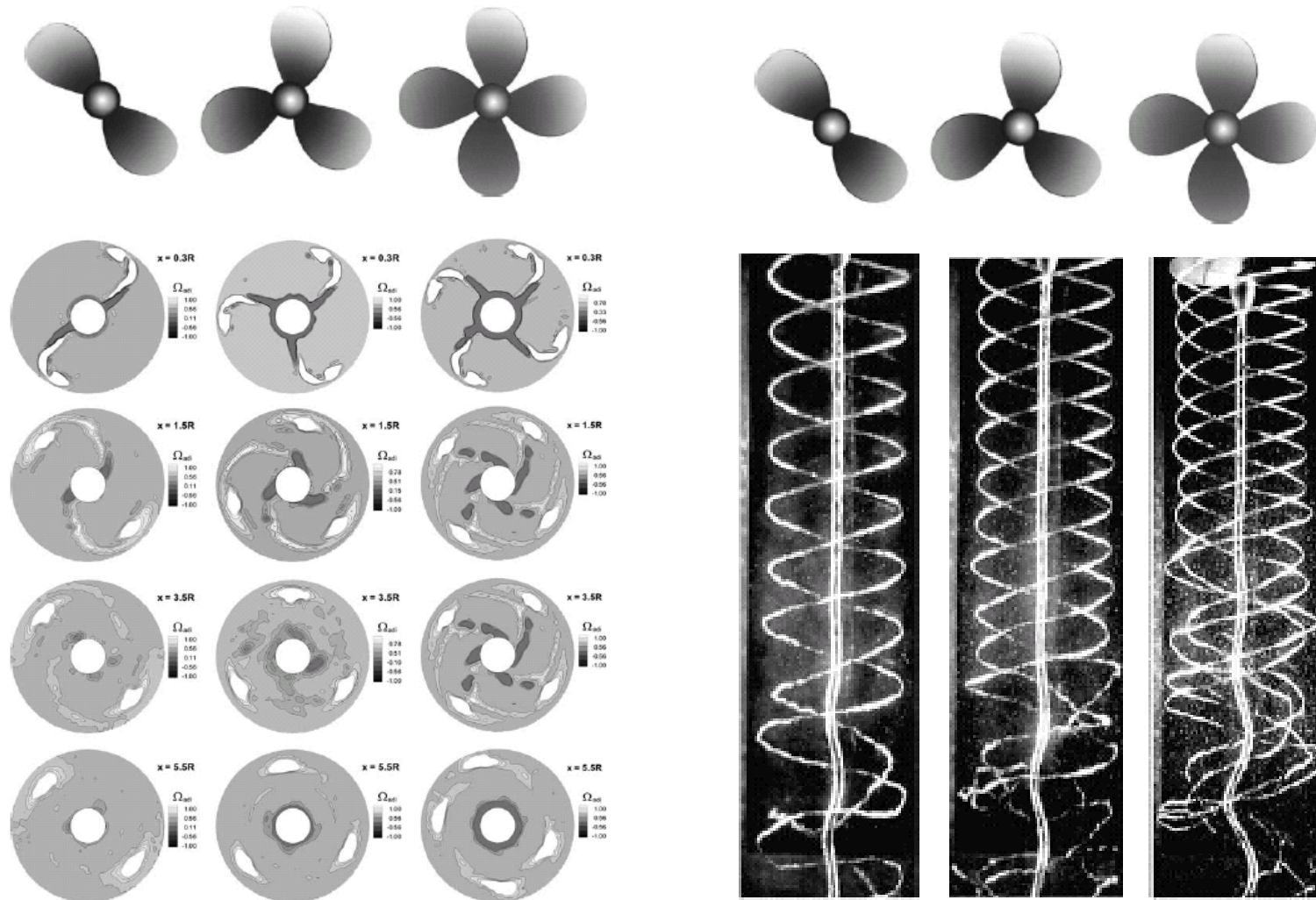
Stability analysis for tip vortices when $N\Gamma = -\Gamma_0$



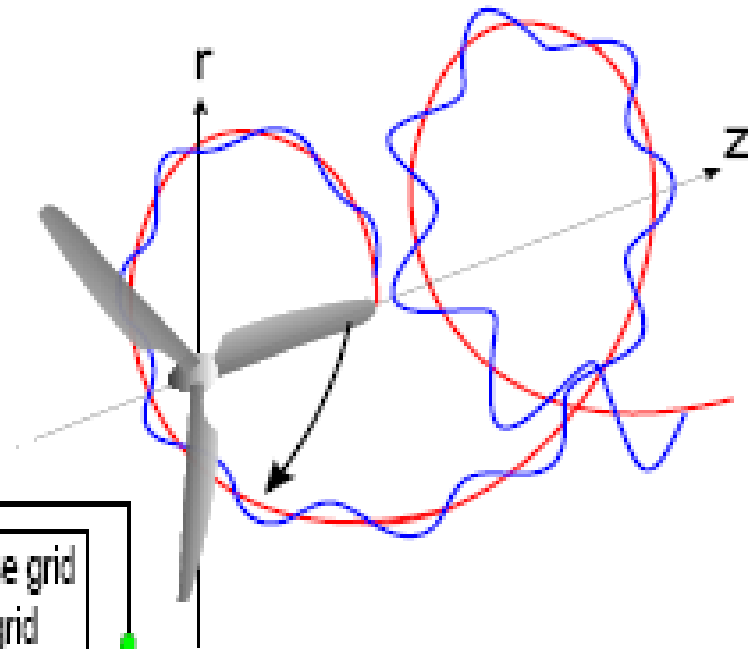
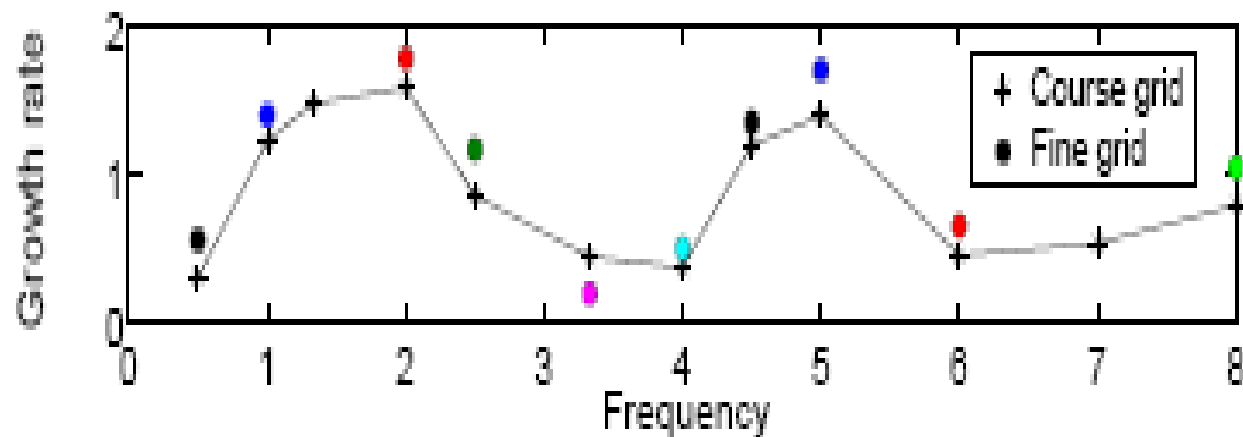
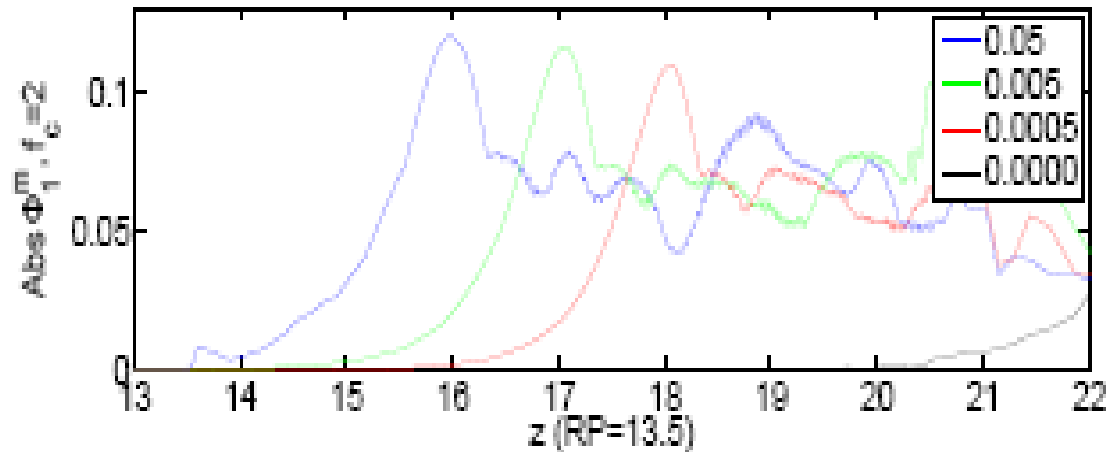
A wake consisting of tip vortices and a root vortex is always unconditional unstable

Test of Joukowski's model on instability

PIV measurements in wake cross-sections behind ship screw (Felli et al; 2008)



Stability analysis of vortex structures



Wind Turbine Wakes and Wind Farms

Summary

- A numerical wake model based on the actuator line technique and body forces has been developed
- Computational results have been compared to experimental data. The agreement is generally very good.
- It has been possible to carry out initial computations of a wind farm under simplified conditions
- To take into account realistic wind conditions, including the atmospheric boundary layer, annual temperature variations, humidity, etc, demands access to very large computing facilities, but is realistic using existing computing tools