Length scales and the spectral tensor

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Motivation

- Connect turbulence length scale with mixing length scale
- Explore the three parameters of the Mann model as functions of stability and wind speed
- Expand the spectral tensor model to cover other stabilities

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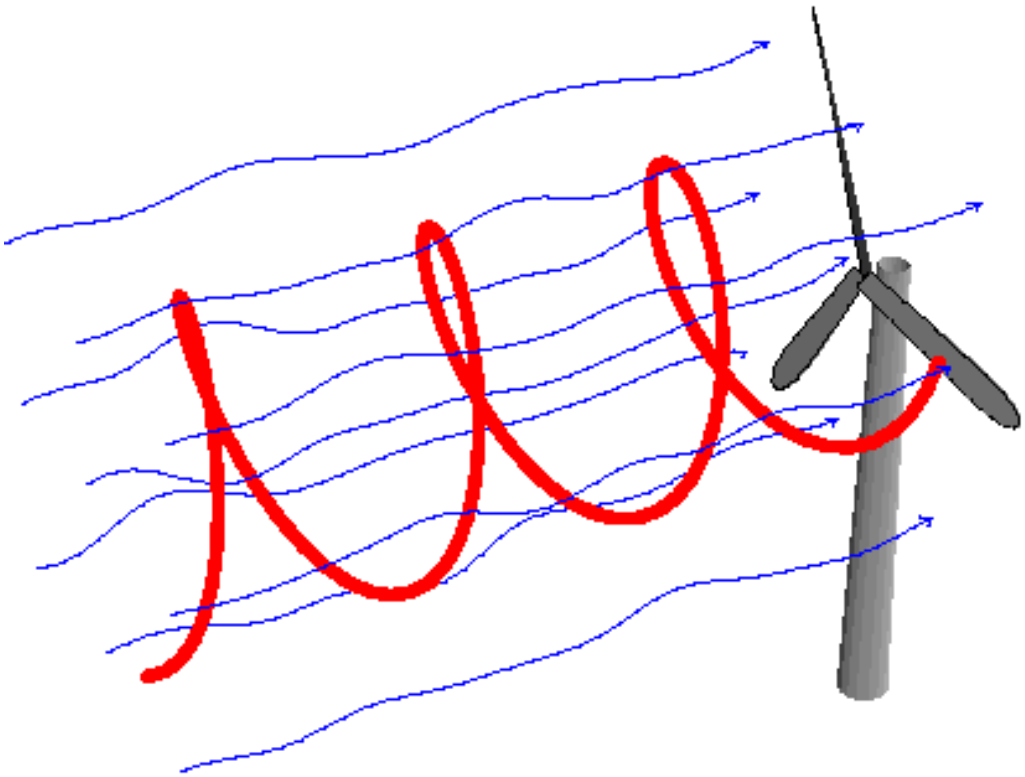
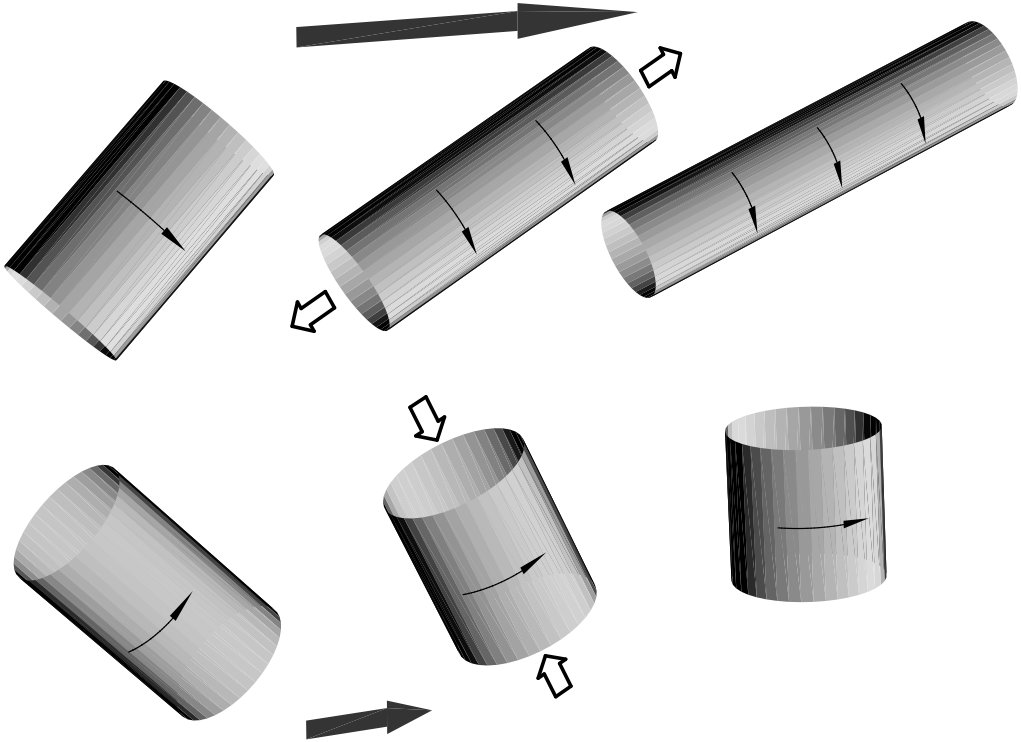
Turbulence for wind turbine load modeling



The purpose is to describe spatial and temporal fluctuations with relevance for wind turbine load calculations and how instruments may sense these fluctuations remotely.

Rapid distortion theory

Rapid distortion theory (RDT) was originally formulated to calculate turbulence in a wind tunnel contraction.
It was later used to model the response of turbulence to shear.



Length scale of the wind profile

$$l = \frac{u_*}{\partial U / \partial z}. \tag{1}$$

Surface layer

- Neutral conditions:

$$(l_{SL})_N = \kappa z. \tag{2}$$

- Diabatic conditions:

$$l_{SL} = (l_{SL})_N {\phi_m}^{-1} = \kappa z {\phi_m}^{-1}, \tag{3}$$

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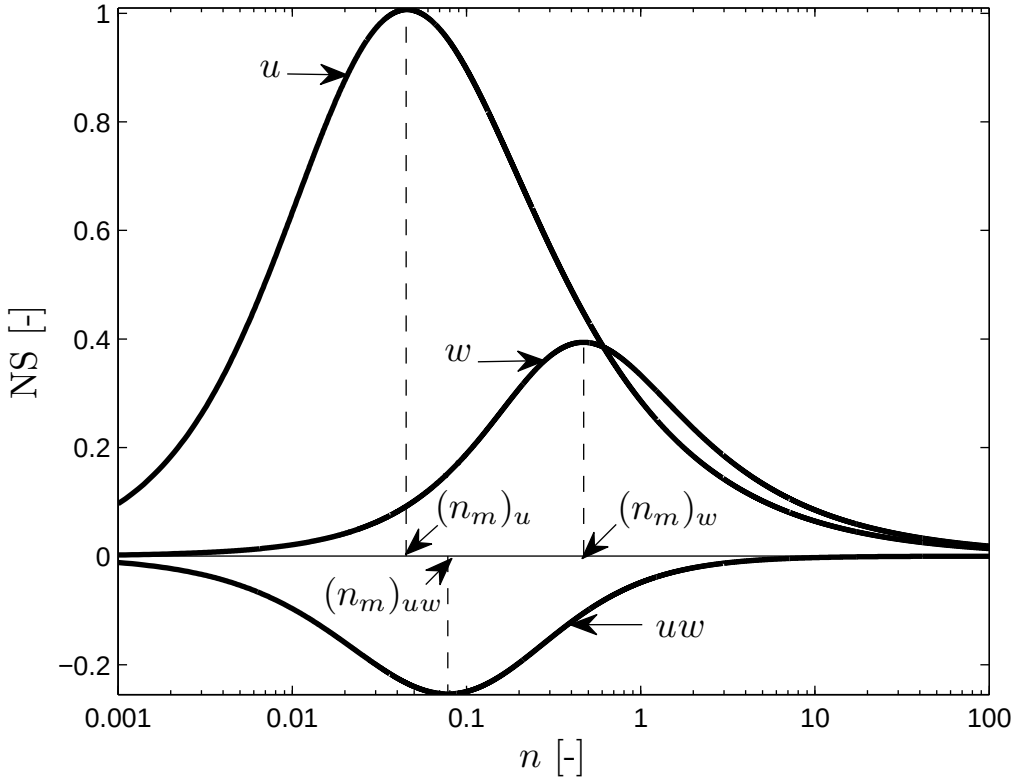
Length scale of the turbulence

Surface layer and neutral conditions



Kaimal et al. (1972):

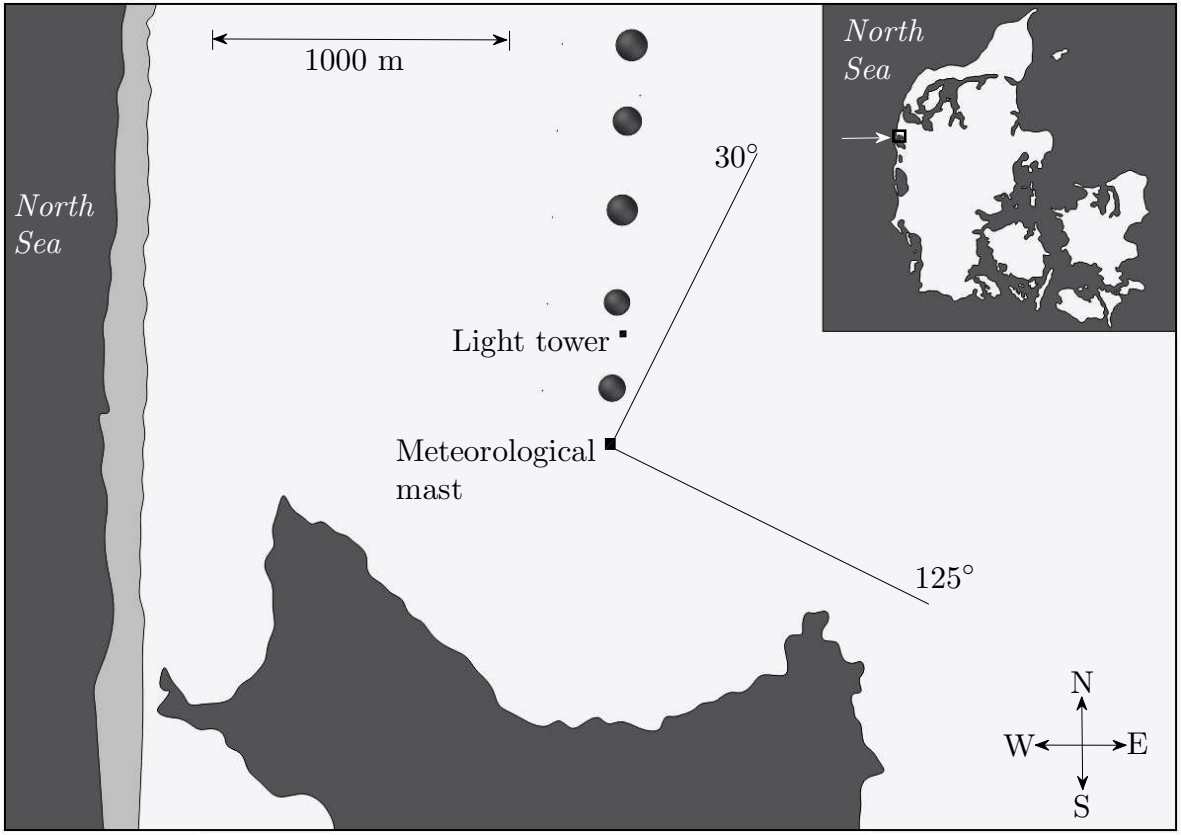
$$\frac{fS_u(f)}{{u_{*o}}^2} = \frac{102n}{(1+33n)^{5/3}}$$

$$\frac{fS_w(f)}{{u_{*o}}^2} = \frac{2.1n}{1+5.3n^{5/3}}$$

$$\frac{-fC_{uw}(f)}{{u_{*o}}^2} = \frac{12n}{(1+9.6n)^{7/3}}$$

where $n = fz/U$.

Site and measurements



- 20 Hz Sonics at 10, 20, 40, 60, 80. 100 and 160 m
- 10-min time series collected for ∼1 year

Results: diabatic observations

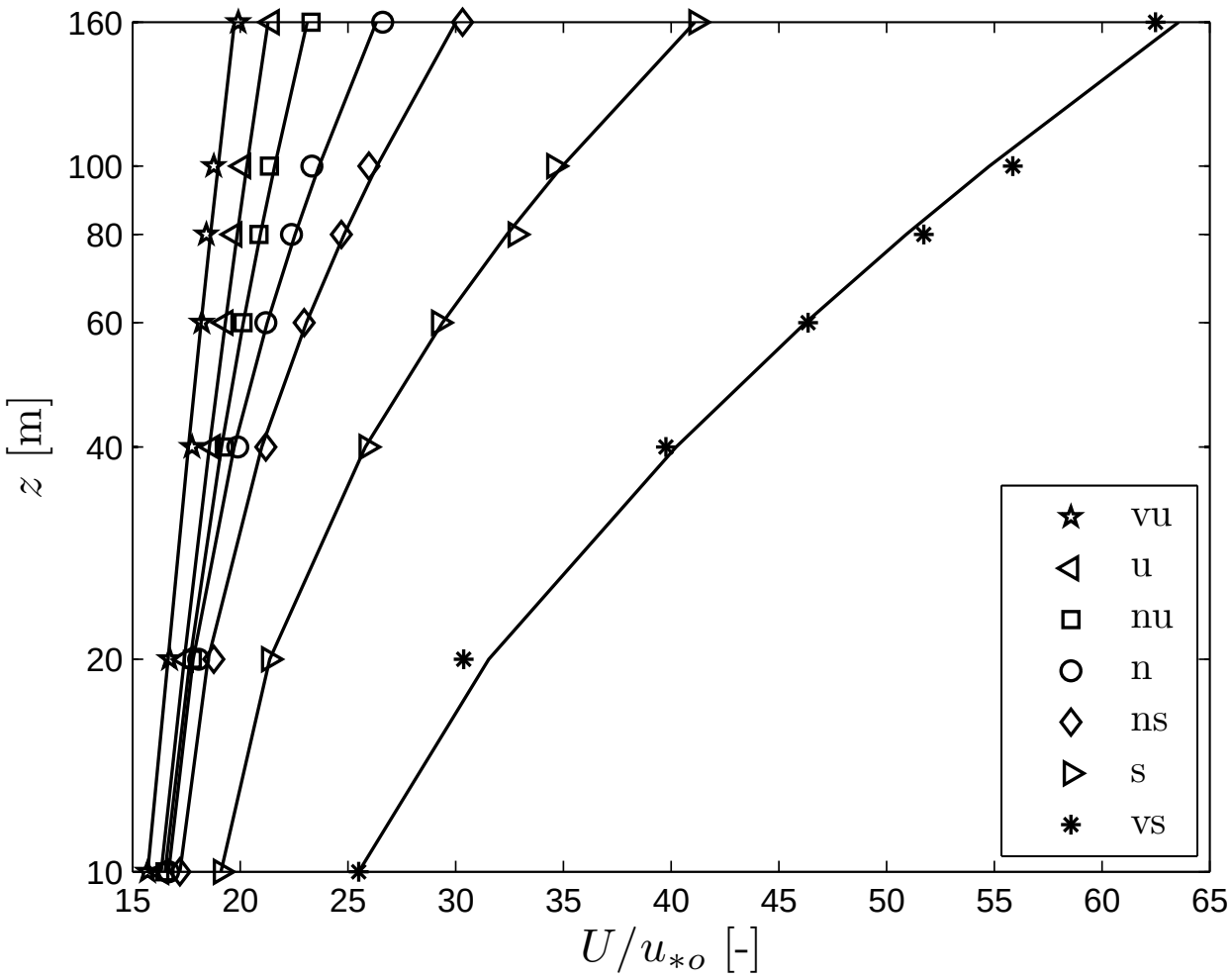
Obukhov length interval [m]	Atmospheric stability class	L [m]	u_{*o} [m s^{-1}]	z_o [m]	z_i [m]	No. of 10-min data
$-100 \leq L \leq -50$	Very unstable (vu)	-74	0.35	0.013	600	397
$-200 \leq L \leq -100$	Unstable (u)	-142	0.41	0.012	600	459
$-500 \leq L \leq -200$	Near unstable (nu)	-314	0.40	0.012	550	292
$\lvert L \rvert \geq 500$	Neutral (n)	5336	0.39	0.013	488	617
$200 \leq L \leq 500$	Near stable (ns)	318	0.36	0.012	451	439
$50 \leq L \leq 200$	Stable (s)	104	0.26	0.008	257	1144
$10 \leq L \leq 50$	Very stable (vs)	28	0.16	0.002	135	704

- $z_i = C \frac{u_{*o}}{|f_c|}$ for neutral and stable conditions
- z_o from

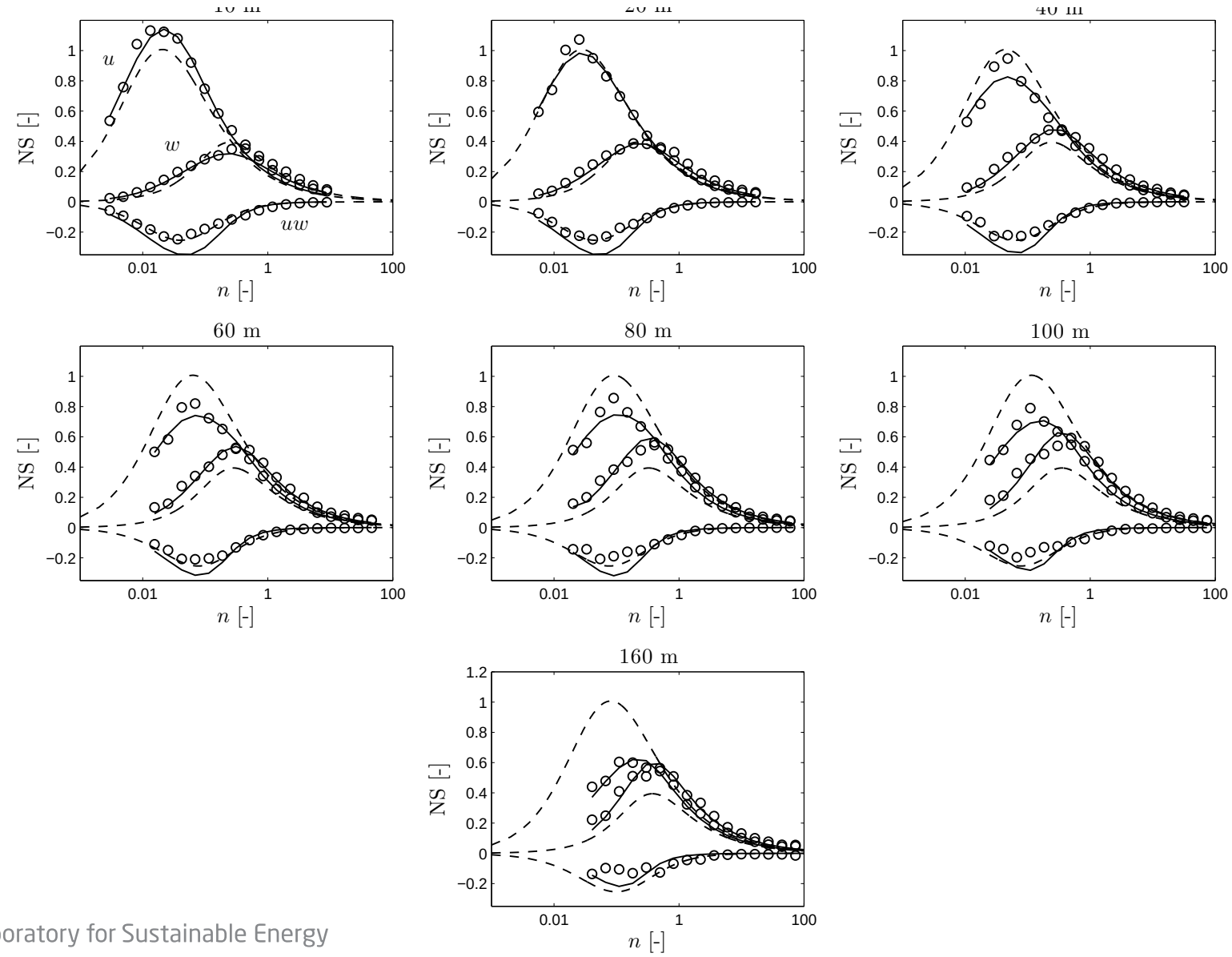
$$U = \frac{u_{*o}}{\kappa} \left[\ln \left(\frac{z}{z_o} \right) - \psi_m \right] \tag{4}$$

Results: diabatic observations

- $\partial U/\partial z$ from the fitting of Högström (1988) is used for l



Very unstable spectra



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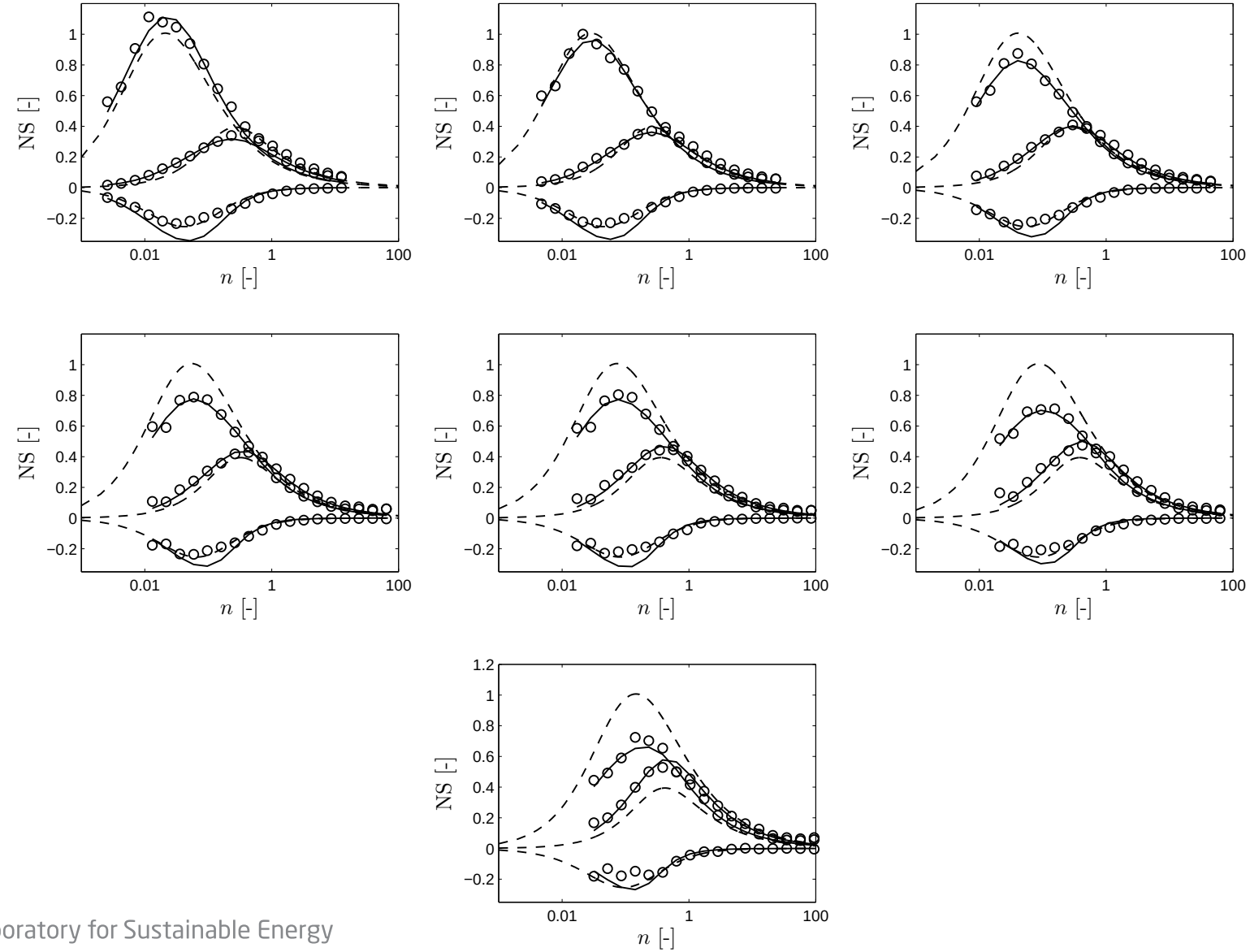
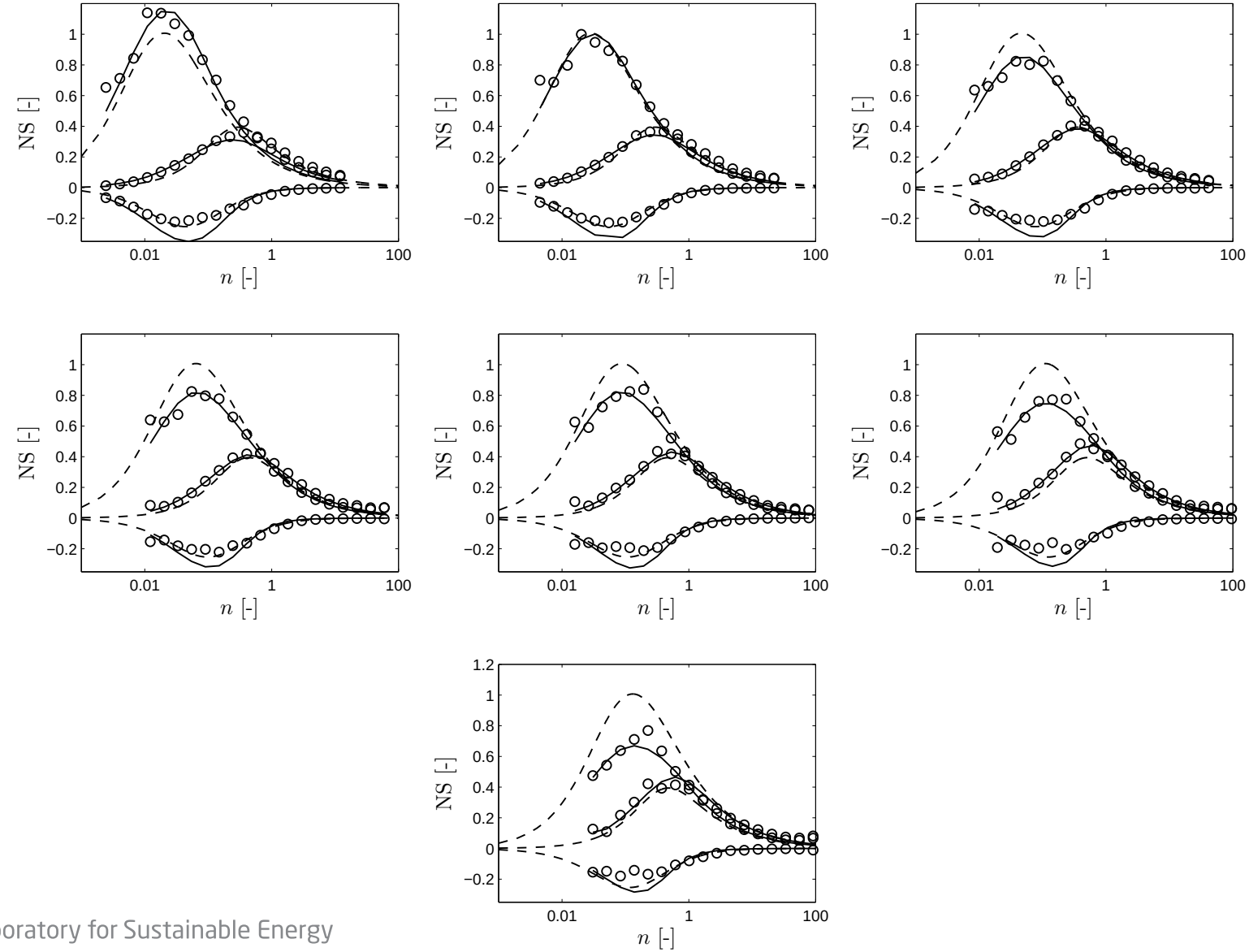
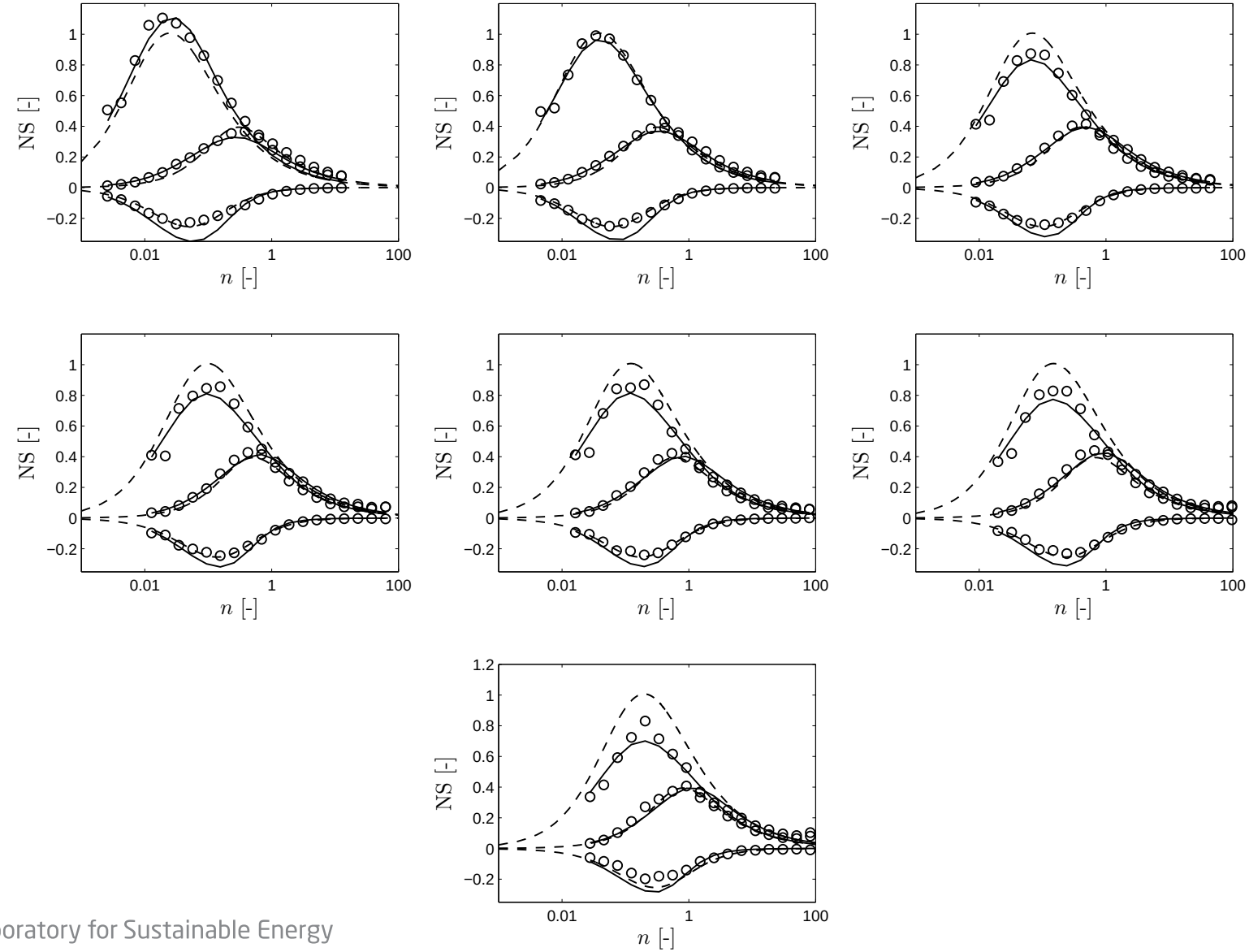


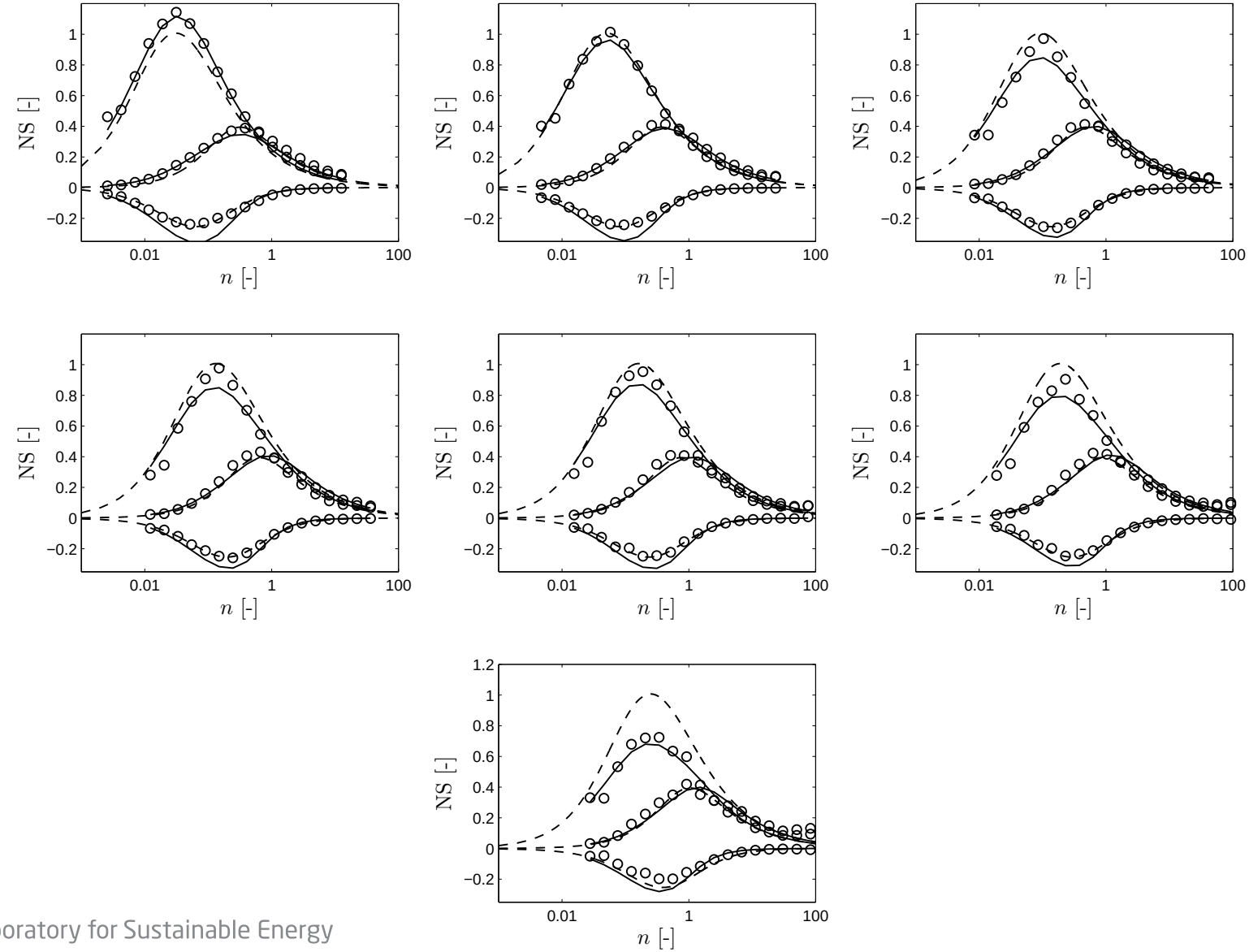
Near unstable spectra



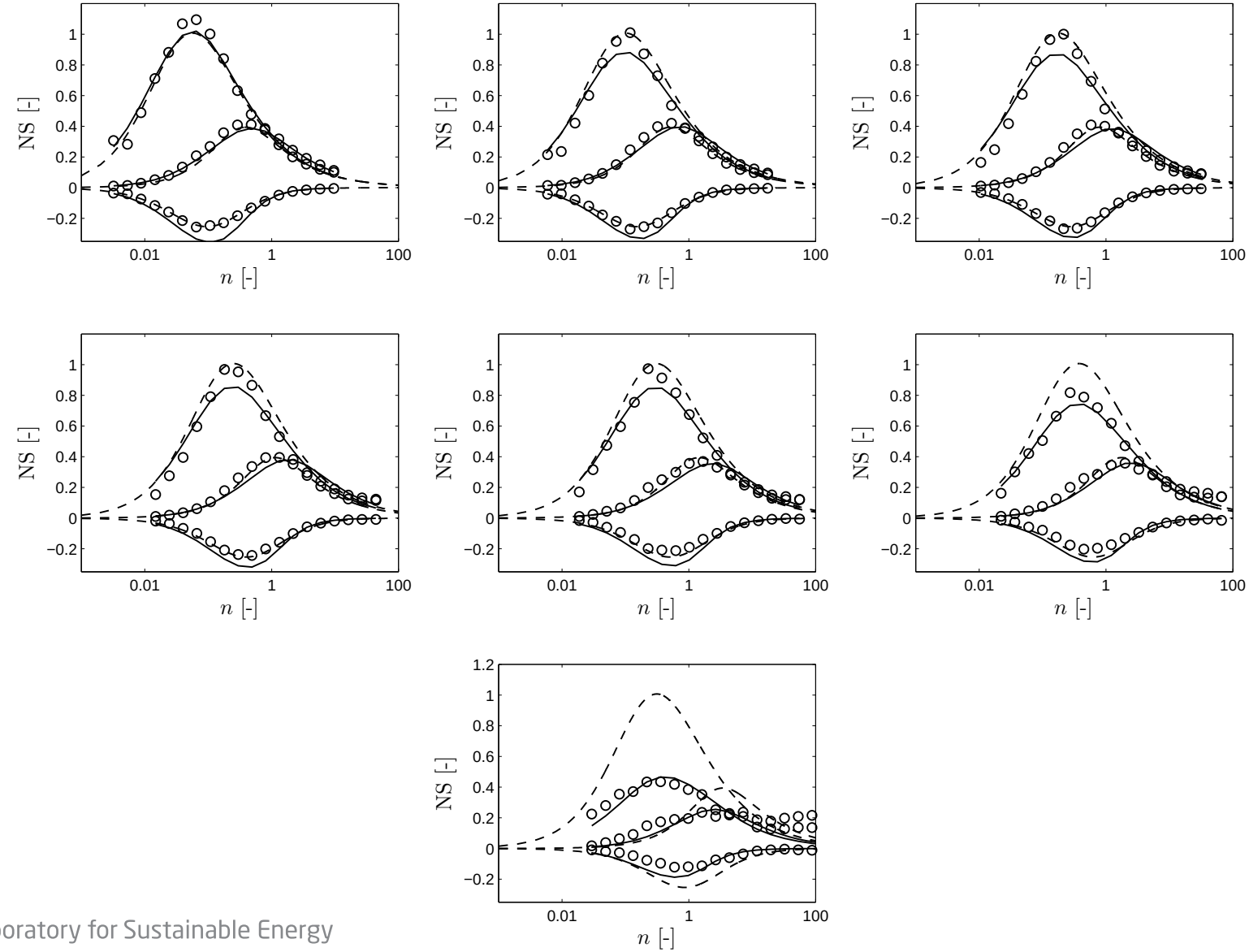
Neutral spectra



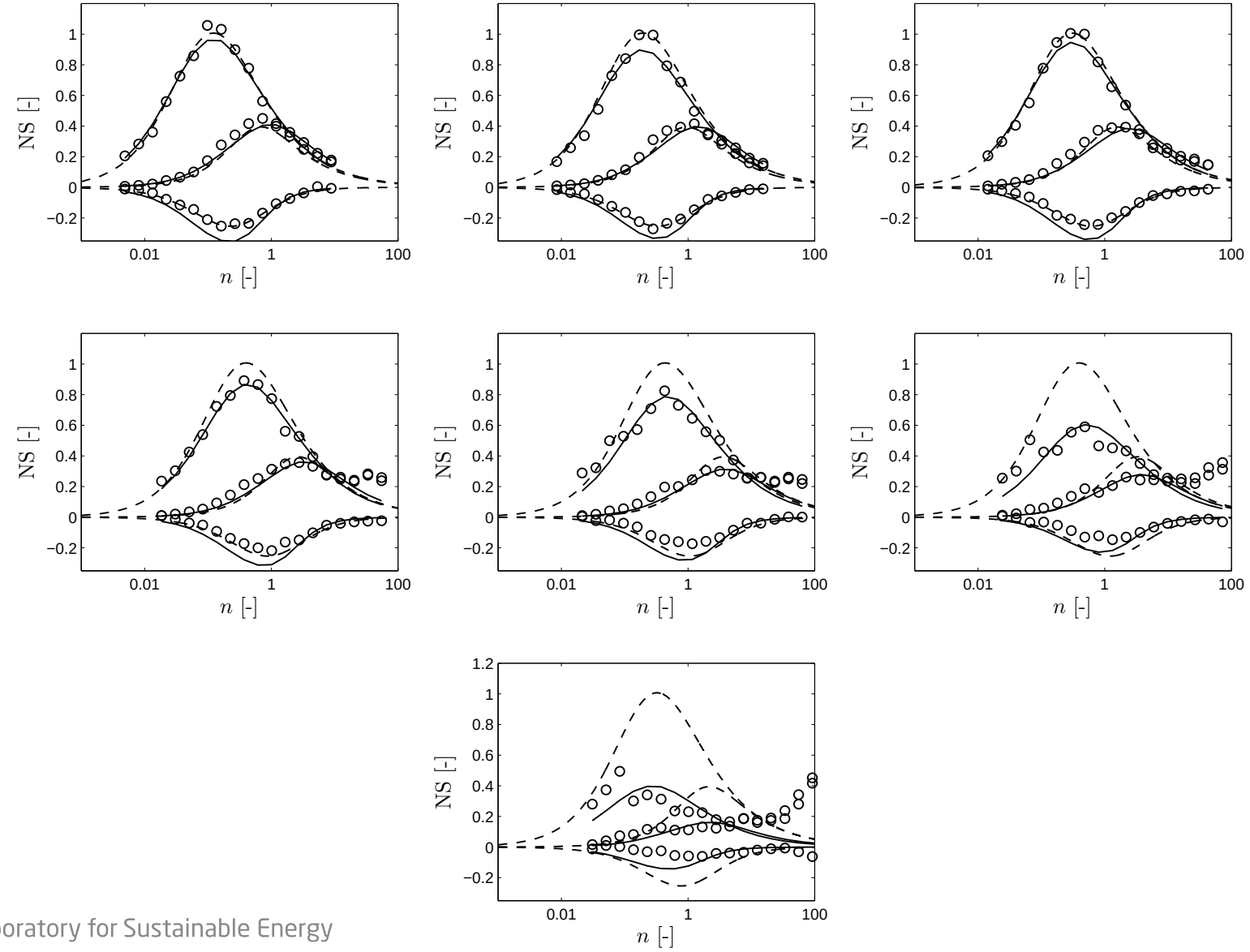
Near stable spectra



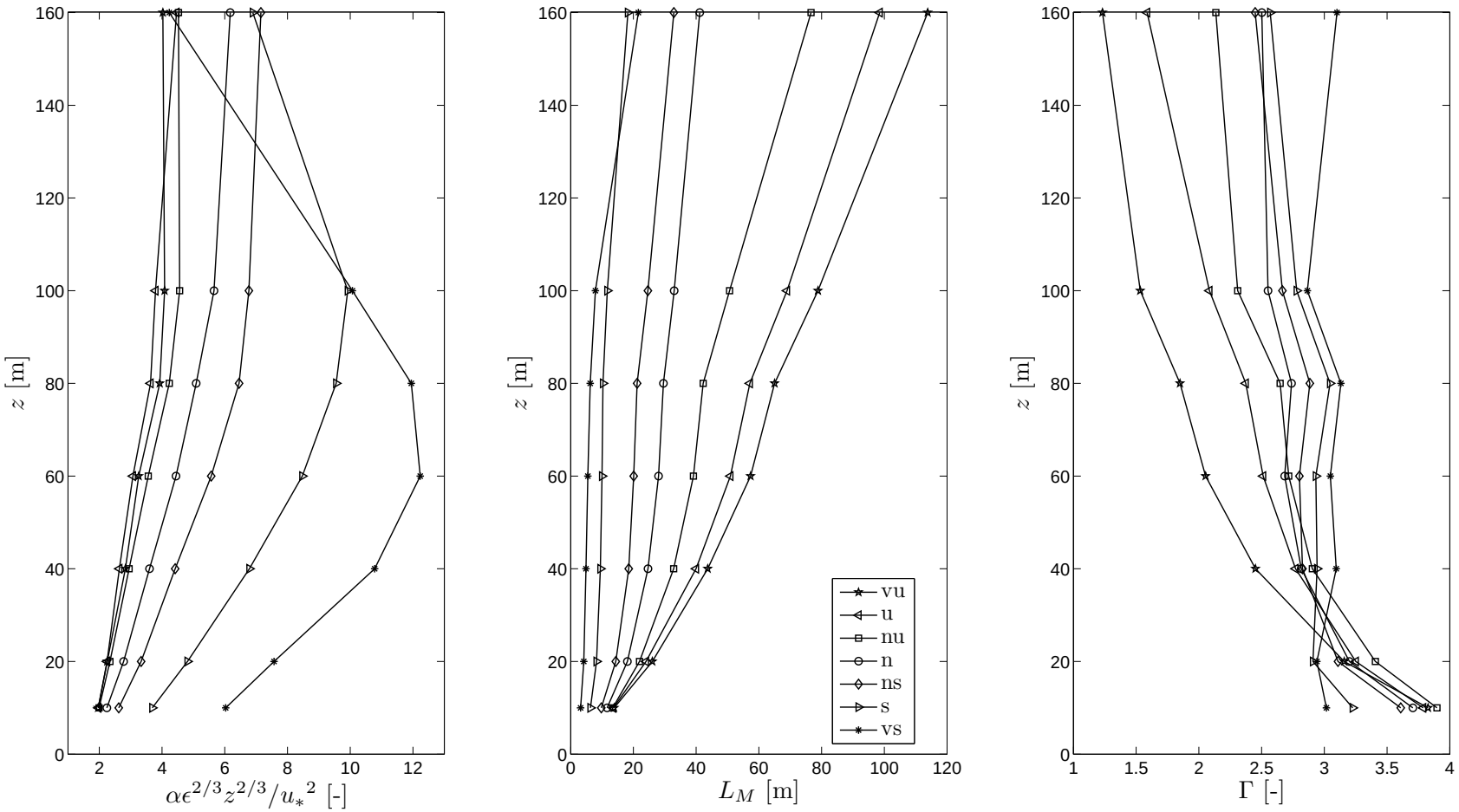
Stable spectra



Very stable spectra



Mann (1994) model results



Length scale relation for diabatic conditions: Mann (1994)

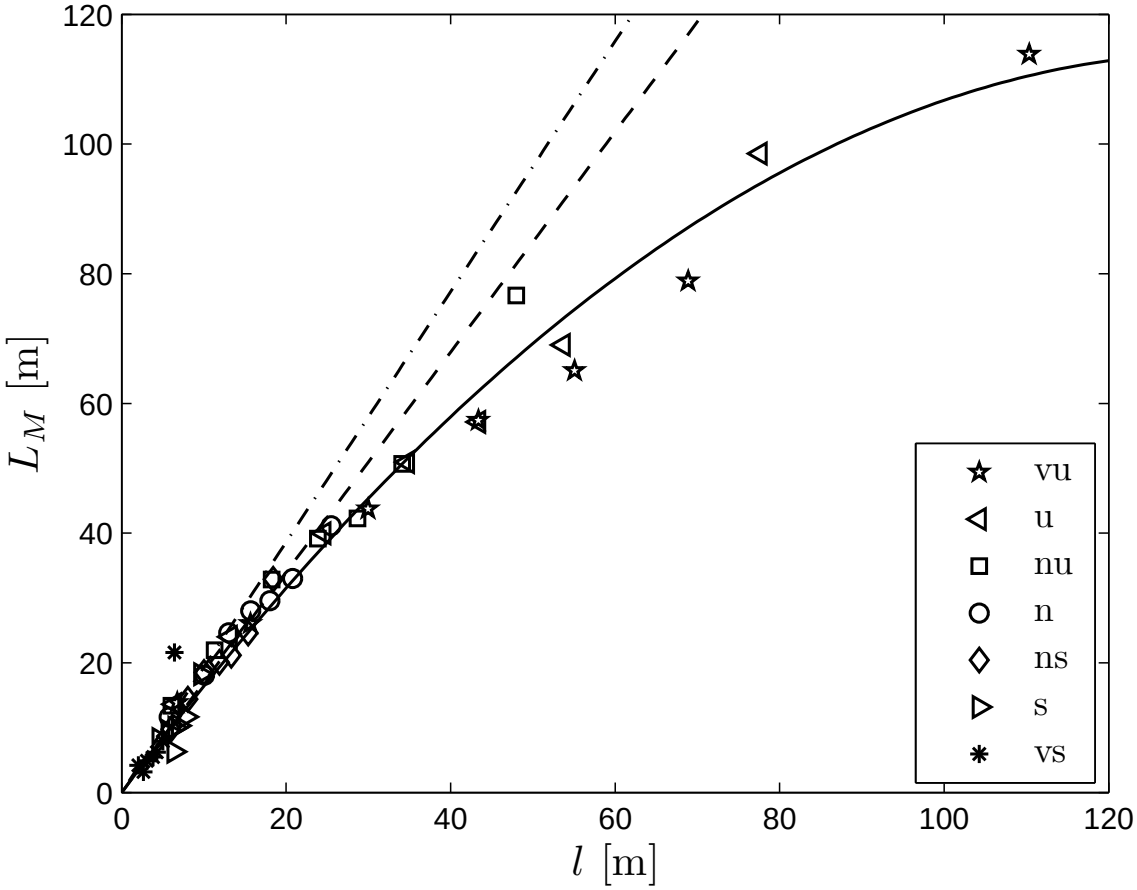
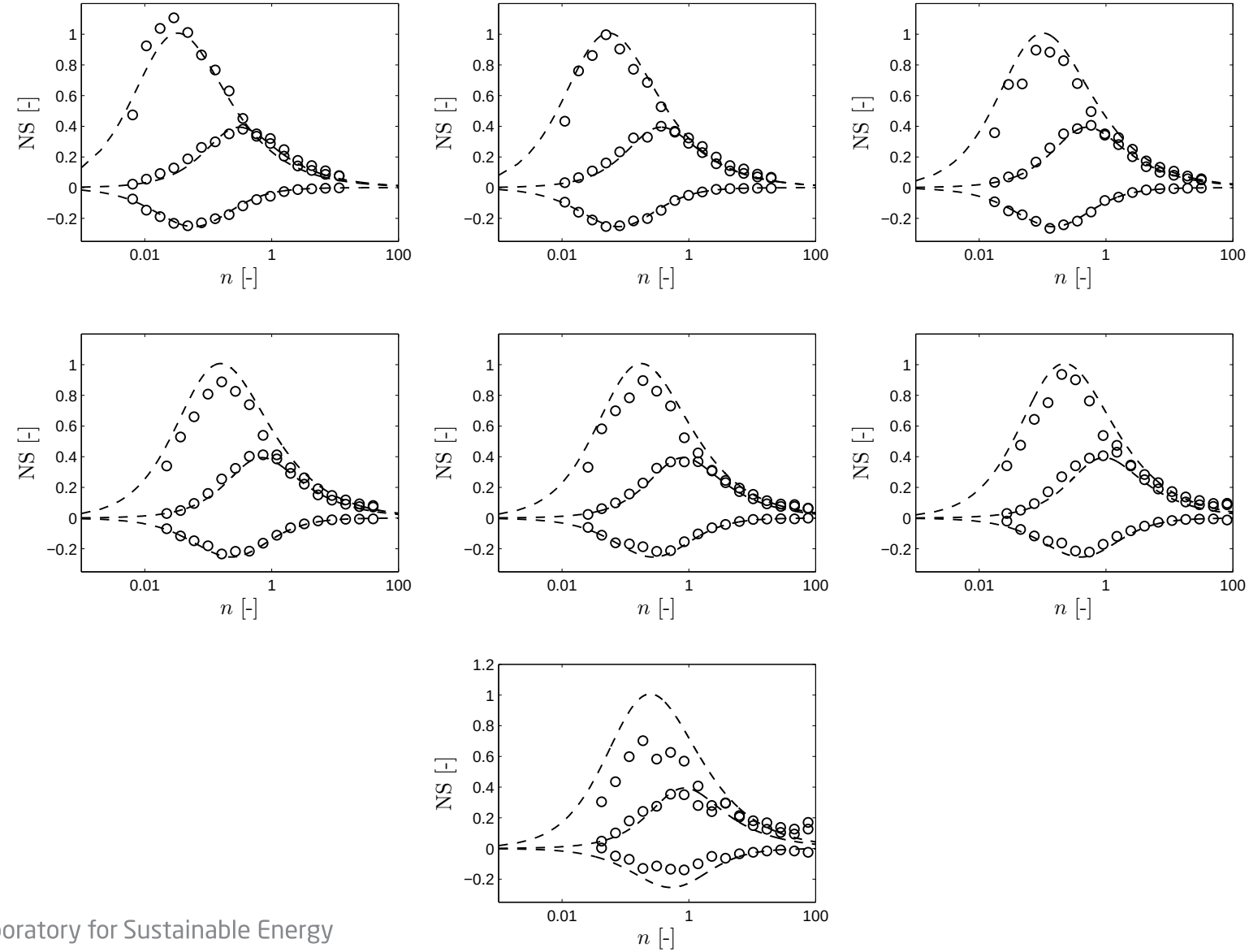


Figure: Fit $L_M = 1.702l - 0.006l^2$ (solid), $L_M = 1.93l$ (dash-dotted) and $L_M = 1.70l$ (dashed)

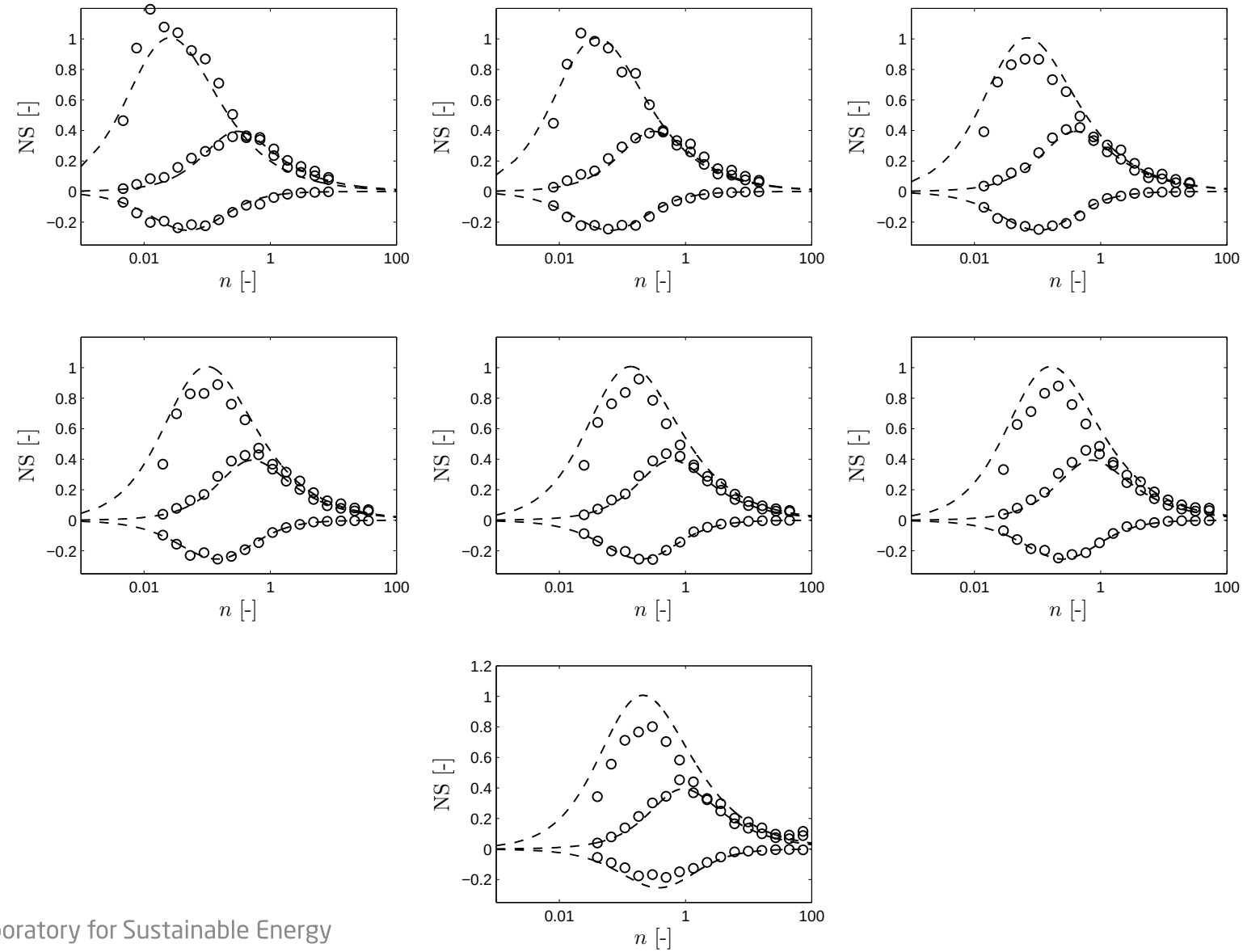
Results: adiabatic observations

Wind speed interval [m s^{-1}]	Wind speed class	u_{*o} [m s^{-1}]	z_o [m]	$\mathrm{Ro}_f \times 10^5$ [-]	z_i [m]	No. of 10-min data
3–5	n1	0.26	0.018	1.23	328	159
5–7	n2	0.38	0.018	1.73	466	233
7–9	n3	0.45	0.009	4.11	562	129
9–11	n4	0.54	0.008	5.91	674	75
11–13	n5	0.67	0.009	5.99	840	21

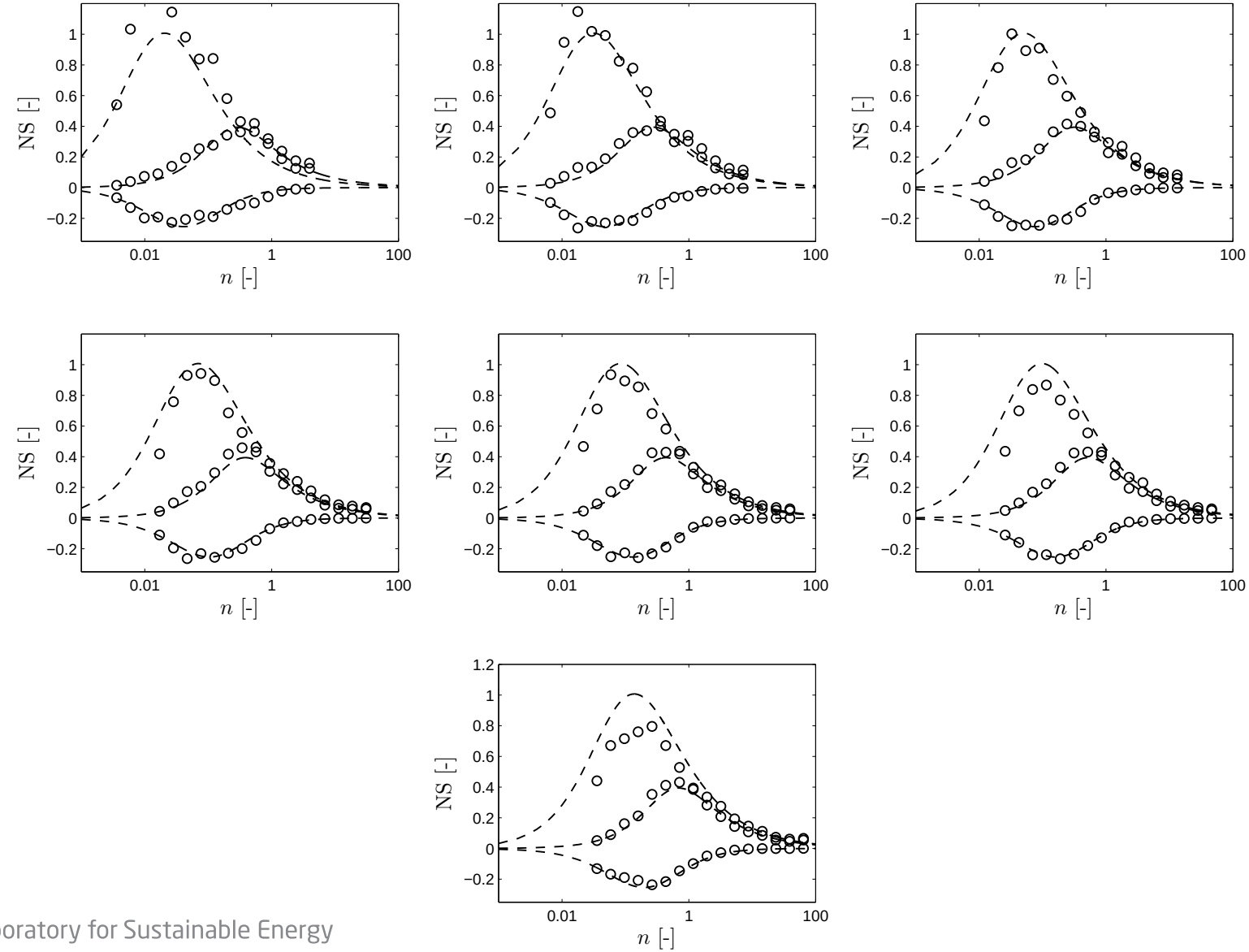
3–5 m/s spectra



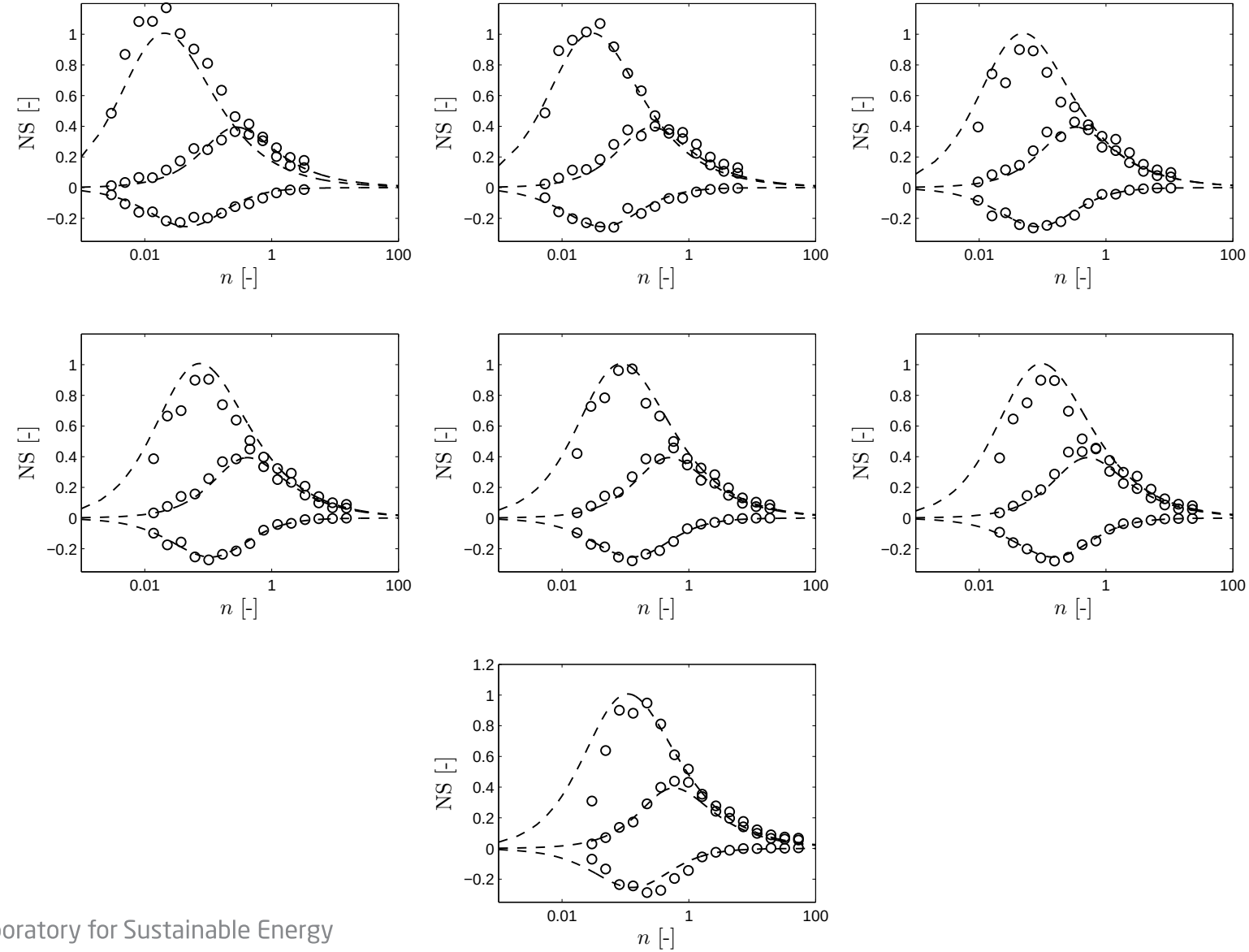
5–7 m/s spectra



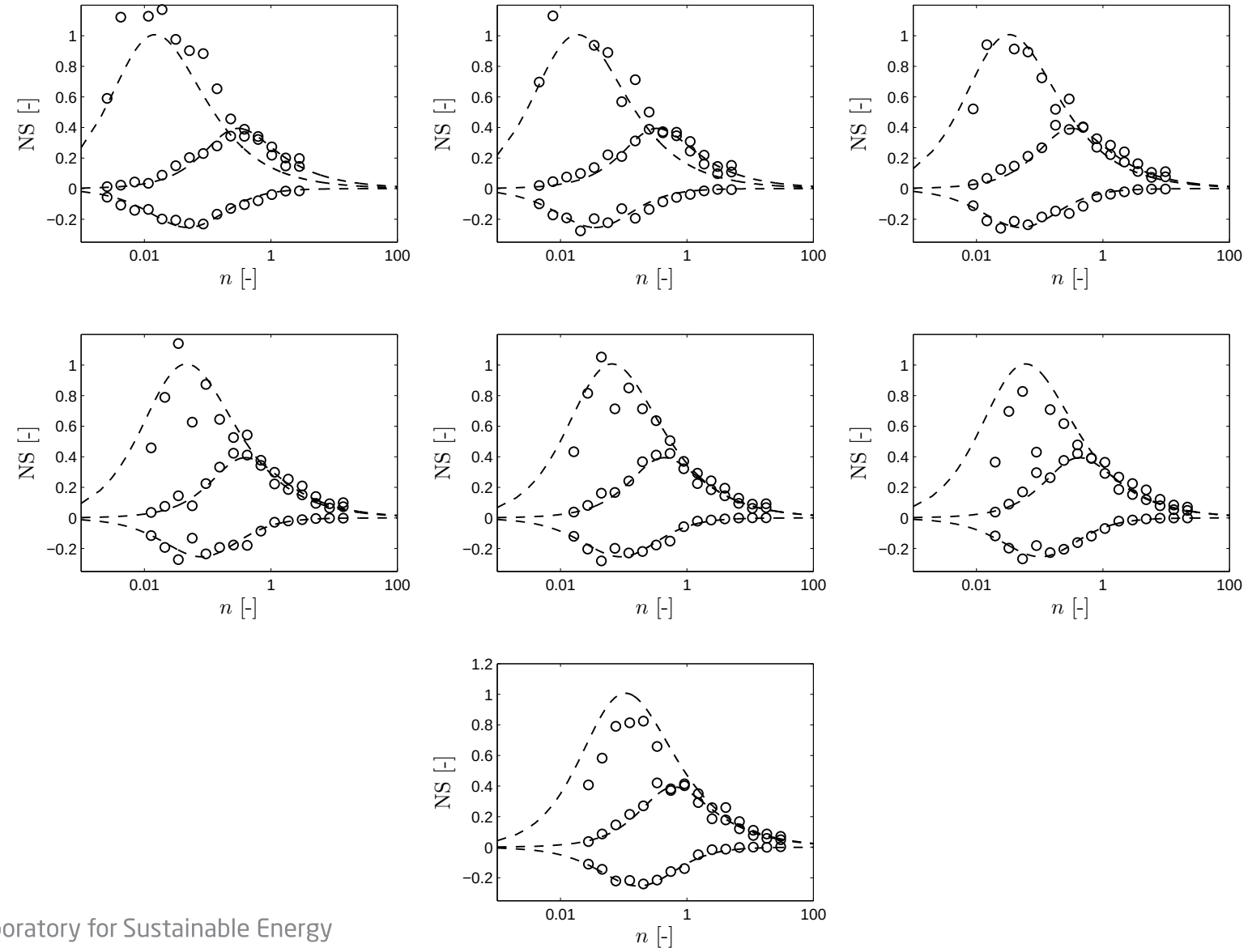
7–9 m/s spectra



9–11 m/s spectra



11–13 m/s spectra



Future work

- Include buoyancy on Coriolis force in the Rapid Distortion (M. Kelly, A. Chougule) ($\langle uv \rangle \neq 0$ even for neutral quite close to the surface)
- Compare models with NCAR LES data
- Compare with Høvsøre data, also cross-spectra

Conclusions

- Within the surface layer and for a range of stabilities, l is linearly proportional to $(\lambda_m)_w$ and to L_M from Mann (1994)
- Beyond surface layer too, except for very unstable conditions
- Stability and Coriolis force will be included in the future

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