Study of Turbulence Structure with Atmospheric Stratification

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Outline

- Introduction
- Turbulence structure under neutral stratification
- Analysis of anisotropy tensor
- Turbulence under uniform stratification and shear
- Summary and future work

Introduction

- Risø has produced a model for the three-dimensional spectrum of turbulence, Φ_{ij} (Mann,1994)
- Widely used in wind energy industry to simulate inflow turbulence for load calculations
- The Mann model incorporates Rapid Distortion Theory (RDT)
- Neutral surface layer
- Assumption of uniform mean shear
- Flat terrain and gently varying orography in neutral flow

The spectral velocity tensor;

$$\Phi_{ij}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int R_{ij}(\mathbf{r}) \exp(-\mathrm{i}\mathbf{k} \cdot \mathbf{r}) \mathrm{d}\mathbf{r}, \tag{1}$$

where $R_{ij}(\mathbf{r}) = \langle u_i(\mathbf{x}) u_j(\mathbf{x} + \mathbf{r}) \rangle$, is the covariance tensor, and $\langle \ \rangle$ denotes ensemble averaging.

$\Phi_{ij}(\mathbf{k})$ is modeled from the relation,

$$\frac{\langle dZ_i^*(\mathbf{k}(t), t) dZ_j(\mathbf{k}(t), t) \rangle}{dk_1 dk_2 dk_3} = \Phi_{ij}(\mathbf{k}(t), t), \tag{2}$$

where, $u_i = \int e^{i\mathbf{k}\cdot\mathbf{x}} dZ_i(\mathbf{k}, t)$ (Fourier-Stieltjes integral)

The RDT equation for neutral stratification

$$\frac{D}{Dt} dZ_i(\mathbf{k}, t) = \left(2\frac{k_i k_1}{k^2} - \delta_{i1}\right)\left(\frac{dU}{dz}\right) dZ_3(\mathbf{k}, t), \tag{3}$$

- The Mann spectral velocity tensor model contains only three adjustable parameters, determined from the one point measurements
- viz., L, Γ and $\alpha\epsilon^{2/3}$, where the empirical value of $\alpha \approx 1.7$

Analysis of anisotropy tensor

Anisotropy stress tensor ;

$$b_{ij} = \frac{R_{ij}}{2k} - \frac{1}{3}\delta_{ij}, \tag{4}$$

where, $k = \frac{1}{2}R_{ii}$, is turbulent kinetic energy, $R_{ij} = \langle u_i u_j \rangle$, is the Reynolds stress tensor.
The three principal invariants are;

$$\mathrm{I}_b = b_{ii}(=0) \tag{5}$$

$$\mathrm{II}_b = \frac{1}{2}\left[(b_{ii})^2 - b_{ii}^2\right] \tag{6}$$

$$\mathrm{III}_b = \frac{1}{6}(b_{ii})^3 - \frac{1}{2}b_{ii}b_{jj}^2 + \frac{1}{3}b_{ii}^3 = \det(\mathbf{b}), \tag{7}$$

OR..

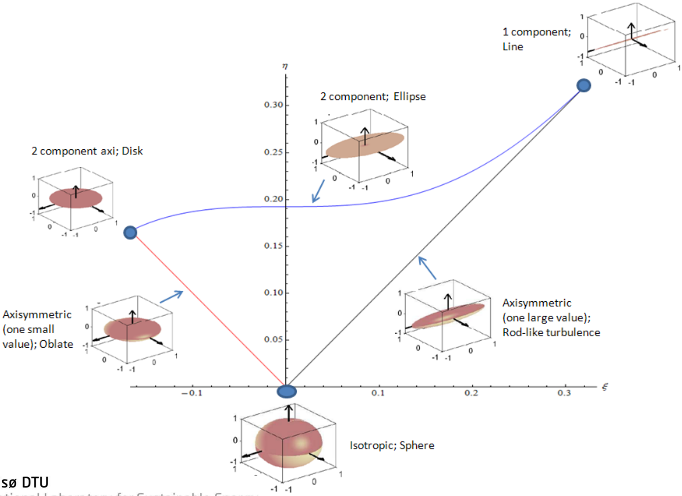
$$6\eta^2 = -2\mathrm{II}_b = b_{ii}^2 = b_{ij}b_{ji} \quad (8)$$

$$6\xi^3 = 3\mathrm{III}_b = b_{ii}^3 = b_{ij}b_{jk}b_{ki} \quad (9)$$

- Any point in the $\xi - \eta$ plane represents the state of anisotropy of the Reynolds stresses that can occur at any point and time in any turbulent flow
- In the Mann model, b_{ij} becomes only function of the Γ parameter

The Lumley triangle

(Source:A.J. Simonsen and P.-A. Krogstad, *Turbulent Stress Invariant Analysis: Clarification of Existing Terminology*)

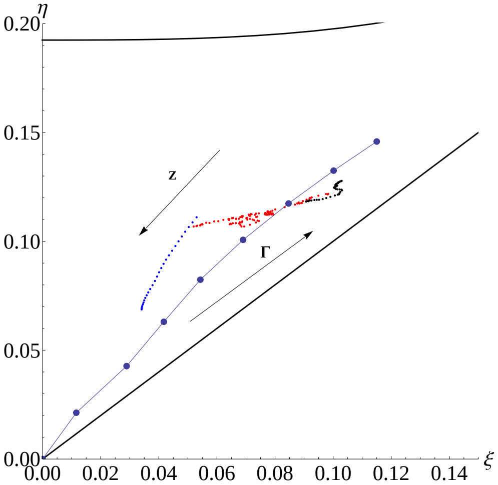


NCAR LES data

	Stable	Neutral	Unstable
Domain $[m^3]$	$1000 \times 1000 \times 400$	$2400 \times 2400 \times 1000$	$5120 \times 5120 \times 2048$
Δx, Δy $[m]$	2	4	10
Δz $[m]$	1	2.5	4
z_i $[m]$	188	616	1000
$\frac{z_i}{L}$	2	0	-10
u_g $[m/s]$ $(v_g = 0)$	8	5	10
Number of z levels $[m]$	101	41	27

LES data with Mann US model

.....; Neutral, ...; Stable, ...; Unstable, –●–; Mann US model



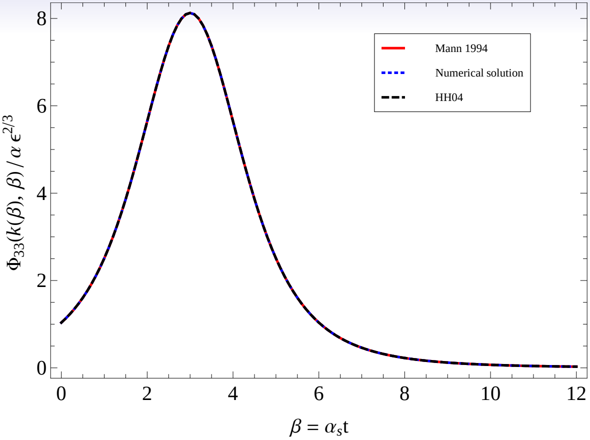
Turbulence under uniform stratification and shear

The governing equations of RDT;

$$\frac{D}{Dt}dZ_i(\mathbf{k},t) = \left(2\frac{k_ik_1}{k^2}-\delta_{i1}\right)\left(\frac{dU}{dz}\right)dZ_3(\mathbf{k},t) - \left(\frac{k_ik_3}{k^2}-\delta_{i3}\right)\frac{g}{T}d\Theta(\mathbf{k},t), \quad (10)$$

$$\frac{D}{Dt}d\Theta(\mathbf{k},t) = -N^2 dZ_3(\mathbf{k},t), \quad (11)$$

where, $N^2 = (g/T)dT/dz$



Φ_{33} as function of β from numerical calculation in comparison with $\Phi_{33}(k(\beta), \beta)$ from Mann (1994) and Hanazaki & Hunt (2004)

The initial,isotropic tensor is given as,

$$\Phi_{ij}(\mathbf{k}_0, 0) = \begin{bmatrix} \frac{k_2^2+k_{30}^2}{4\pi k_0^4} & -\frac{k_1 k_2}{4\pi k_0^4} & -\frac{k_1 k_{30}}{4\pi k_0^4} & 0 \\ -\frac{k_1 k_2}{4\pi k_0^4} & \frac{k_1^2+k_{30}^2}{4\pi k_0^4} & -\frac{k_2 k_{30}}{4\pi k_0^4} & 0 \\ -\frac{k_1 k_{30}}{4\pi k_0^4} & -\frac{k_2 k_{30}}{4\pi k_0^4} & \frac{k_1^2+k_2^2}{4\pi k_0^4} & 0 \\ 0 & 0 & 0 & \frac{2\alpha_s^2 Ri}{4\pi k_0^2}\frac{S(k_0)}{E(k_0)} \end{bmatrix} \times E(k_0)$$

where, $E(k_0)$ and $S(k_0)$ are form of spectral densities such that initial kinetic energy, $\mathrm{KE}_0 = \int_0^\infty E(k_0)\mathrm{d}k_0$ and initial potential energy, $\mathrm{PE}_0 = \int_0^\infty S(k_0)\mathrm{d}k_0$ and $k_0^4 = |\mathbf{k}_0|^4$.

Summary and Future work

- The analysis of the anisotropy tensor in the Lumley triangle represents a simple way of comparing stability dependant model
- The neutral case falls on/near Γ curve for Γ values ranging from 3 to 4
- The future task is to incorporate the eddy life time from the Mann spectral tensor model into the stability dependent RDT equations
- To solve and analyse numerically the spectra and co-spectra of all velocity components
- Comparisons with data will be made

Thank you