Low Order Numerical Model of the Inherent Wake Behind an Infinitely Long Row of Wind Turbines.

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Overview

- Introduction
- Methodology
- Results
- Conclusion and Outlook

Introduction

Investigate the wake interaction within a wind farm in order to optimize farm layout: power production and load reduction.

Starting point:

Investigate the inherent turbulence generated by the wind turbines operation within a wind farm and construct a low order model of the wake.

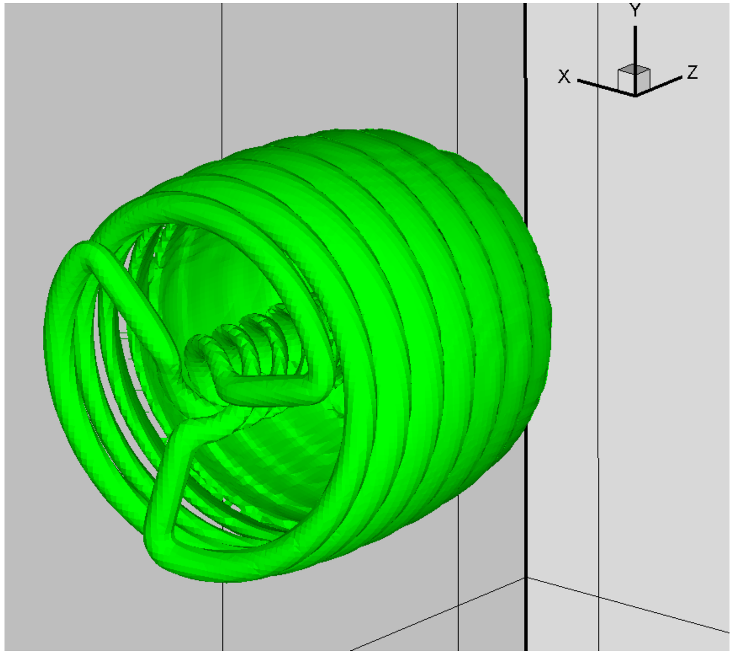
Methodology

- EllipSys3D: finite volume, multi-block NS solver, parallel, non-dimensional

- LES turbulence model

- Actuator Line method (Sørensen and Shen, 2002):
  Includes effect of both tip and root vortices

- Internal Body forces: Flex5 - Tjæreborg Wind Turbine



Methodology

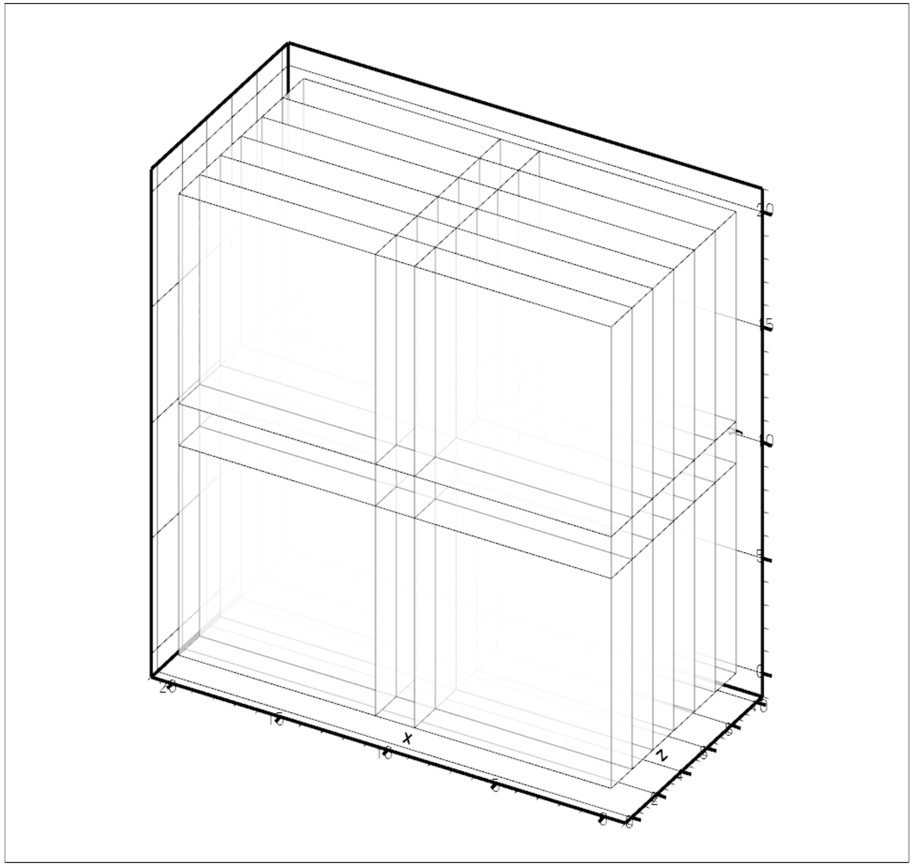
Mesh Setup

- 20R x 20R x 10R

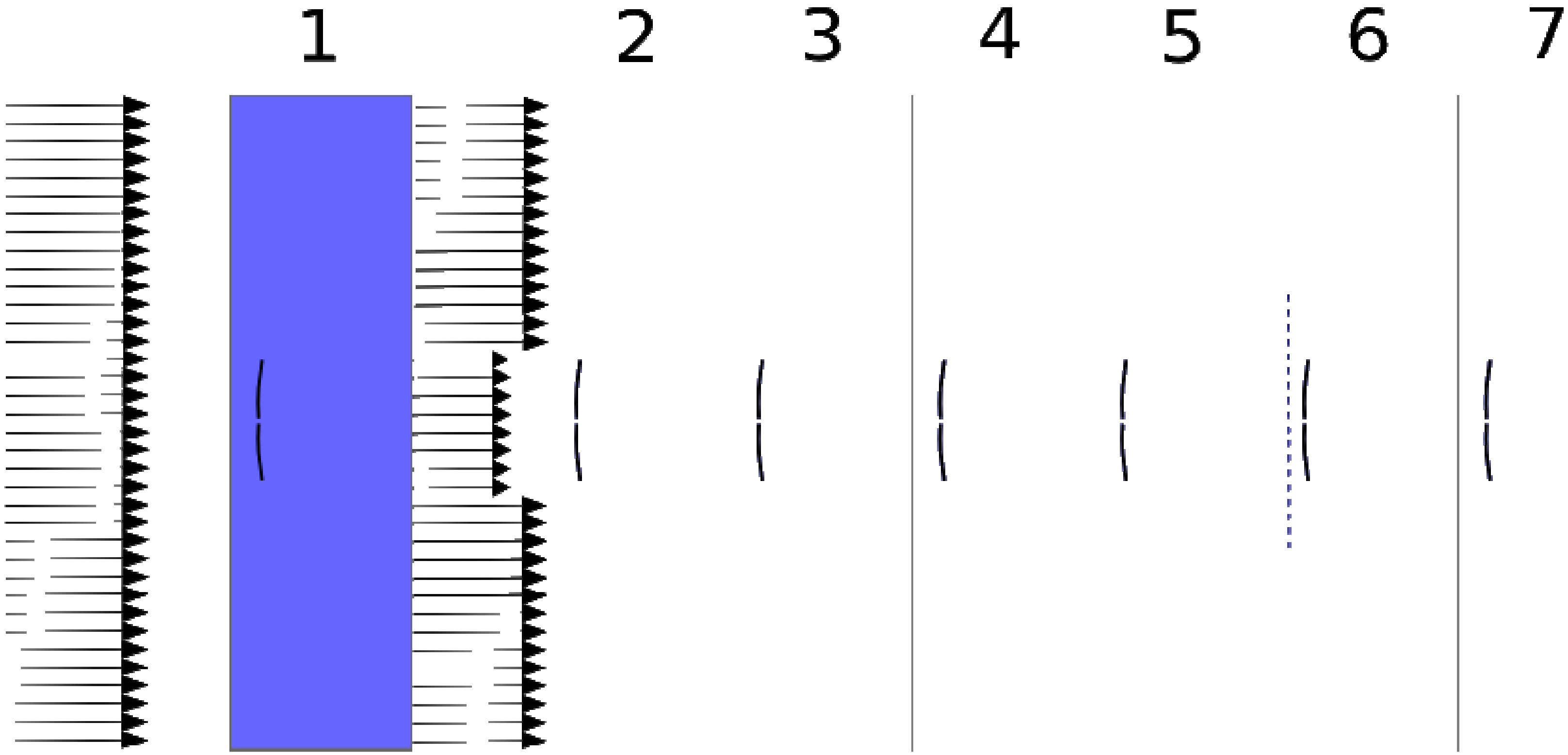
- 54 blocks of 48^3 grid points
  (6 10^6 points)
  >> Blade resolution: 19 cells

- Uniform Inflow
  >> (10m/s in physical space)

- Cyclic BCs
  >> Decrease domain, but infinite



Methodology



Methodology

Low Order Model

(Citriniti & George, 2000)

1. FFT in time (subtracted mean)
   >> window => assume periodic

2. FFT in θ (subtracted mean)
   >> Rotational => Continous

3. POD
   >> Random Stationary Process

4. IFFT in θ (added mean)

5. IFFT in time (added mean)

POD overview:

Snapshots:

$$\hat{\hat{\mathbf{U}}}' = [\hat{\hat{\mathbf{u}}}'_{\mathbf{1}} \hat{\hat{\mathbf{u}}}'_{\mathbf{2}} \dots \hat{\hat{\mathbf{u}}}'_{\mathbf{N}}]$$

Auto-covariance matrix

$$\hat{\hat{\mathbf{R}}} = \hat{\hat{\mathbf{U}}}'^{T} \hat{\hat{\mathbf{U}}}'$$

Solve eigenvalue problem with eigenvalues $\hat{\hat{\mathbf{G}}}$

$$\hat{\hat{\mathbf{R}}}\hat{\hat{\mathbf{G}}} = \hat{\hat{\mathbf{G}}}\hat{\hat{\mathbf{\Lambda}}}$$

where Λ is a diagonal eigenvalue matrix with eigenvalues $[\lambda_1 \lambda_2 \dots \lambda_{n-1}]$.

The orthonormal basis matrix is given as:

$$\hat{\hat{\mathbf{\Phi}}} = (\hat{\hat{\mathbf{U}}}'\hat{\hat{\mathbf{G}}})\hat{\hat{\mathbf{\Lambda}}}^{-1/2}$$

The amplitudes are found from:

$$\hat{\hat{\mathbf{A}}} = \hat{\hat{\mathbf{\Phi}}}^{T} \hat{\hat{\mathbf{U}}}'$$

Then, the snapshots can be reconstructed:

$$\tilde{\hat{\hat{\mathbf{u}}}}_{\mathbf{j}} = \bar{\hat{\hat{\mathbf{u}}}} + \sum_{k=0}^{K-1} \phi_k a_{kj}$$

Methodology

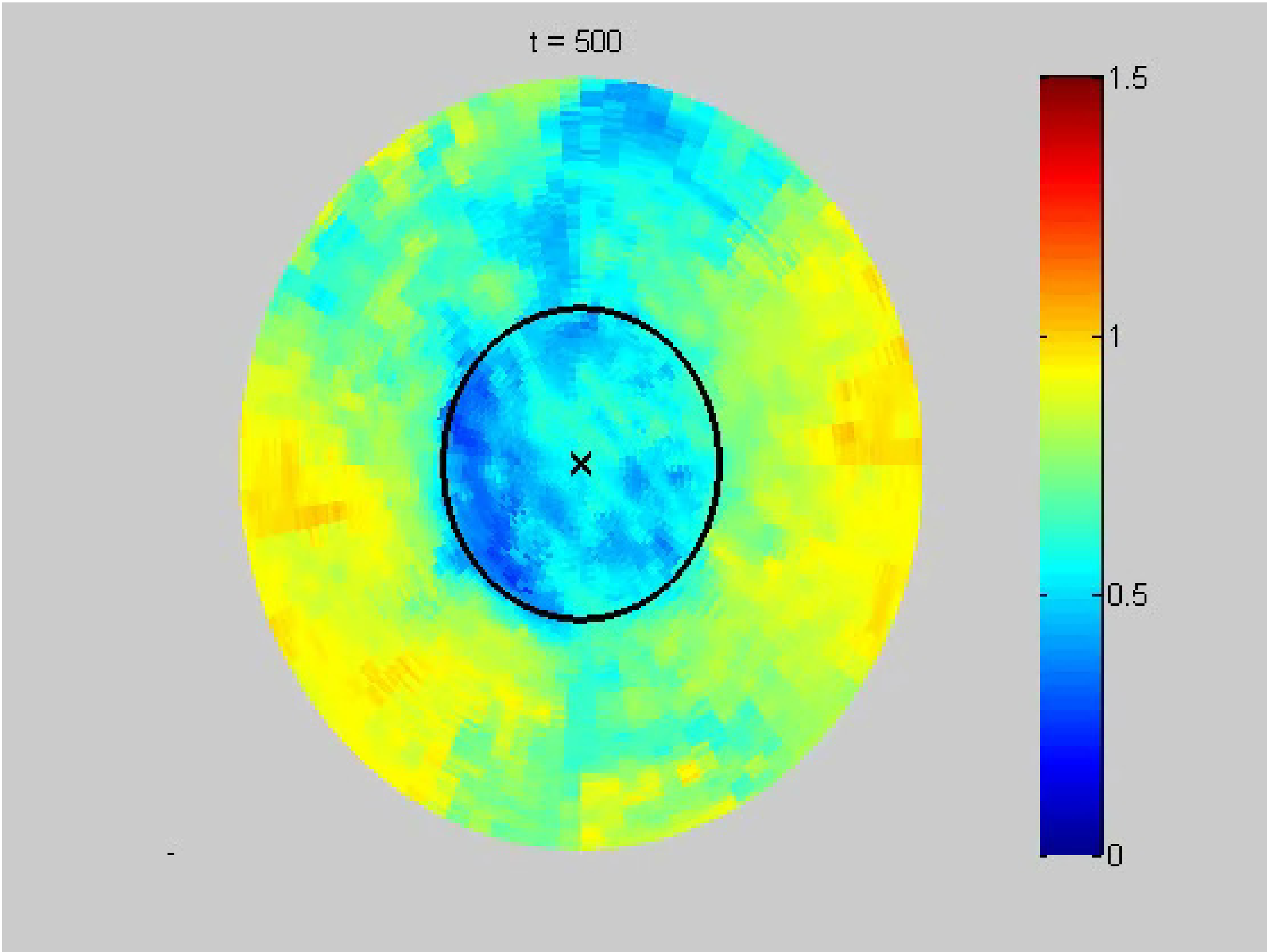
$$u(r,m,f) \approx \tilde{u}(r,m,f) = \sum_{f=0}^{F-1} \left(\sum_{m=0}^{M-1} \left(\sum_{k=0}^{K-1} \hat{\hat{a}}_k(m,f)\phi_i^k(r,m,f) \right) e^{im\theta} + \bar{\hat{u}} \right) e^{i2\pi\frac{ft}{T}} + \bar{u}$$

POD

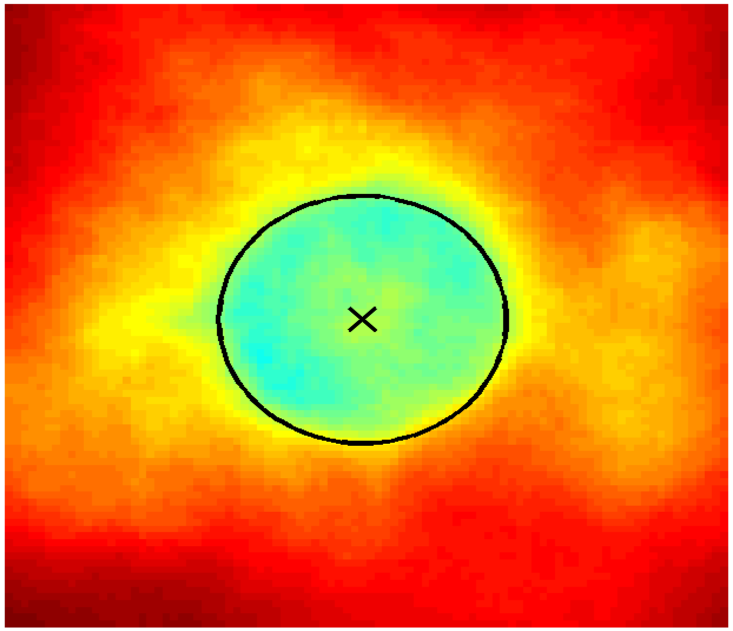
IFFT in θ

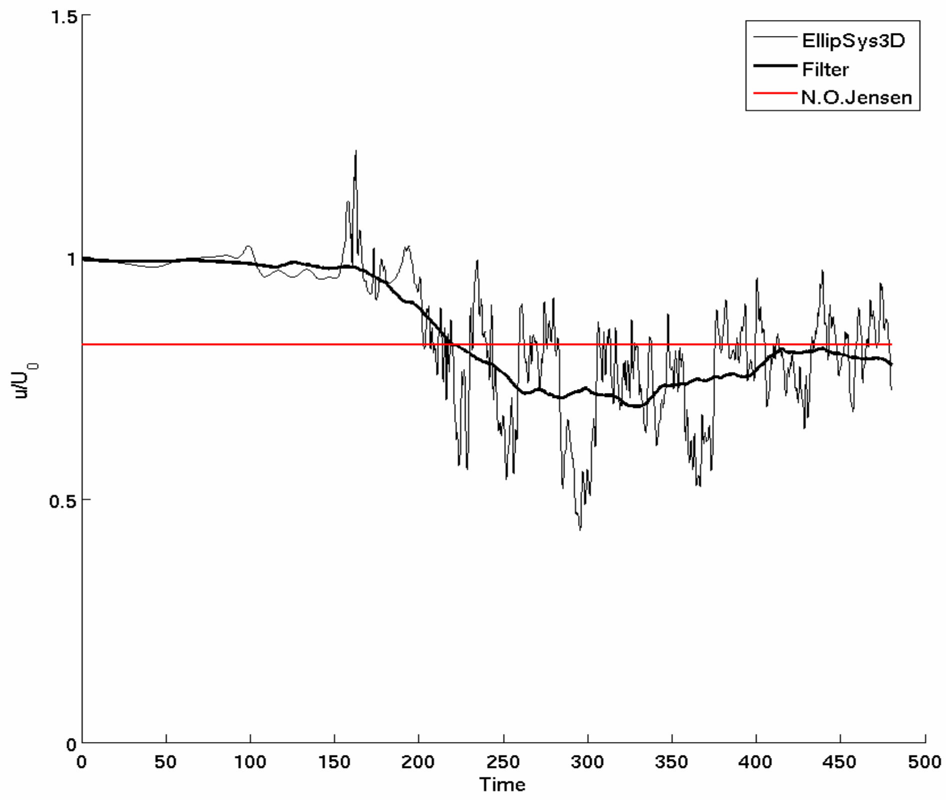
IFFT in t

Results

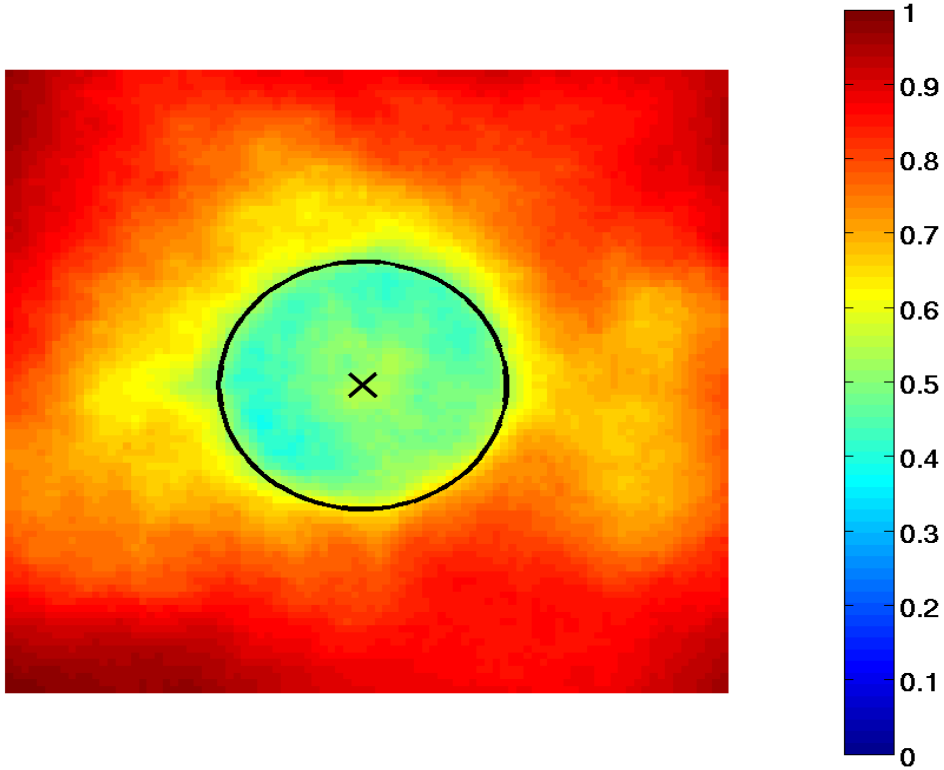


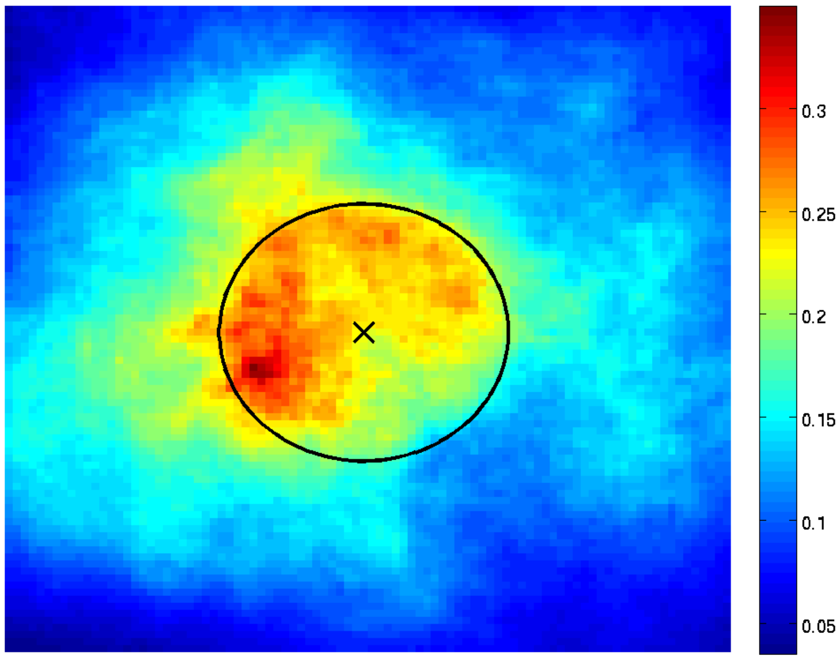
Results



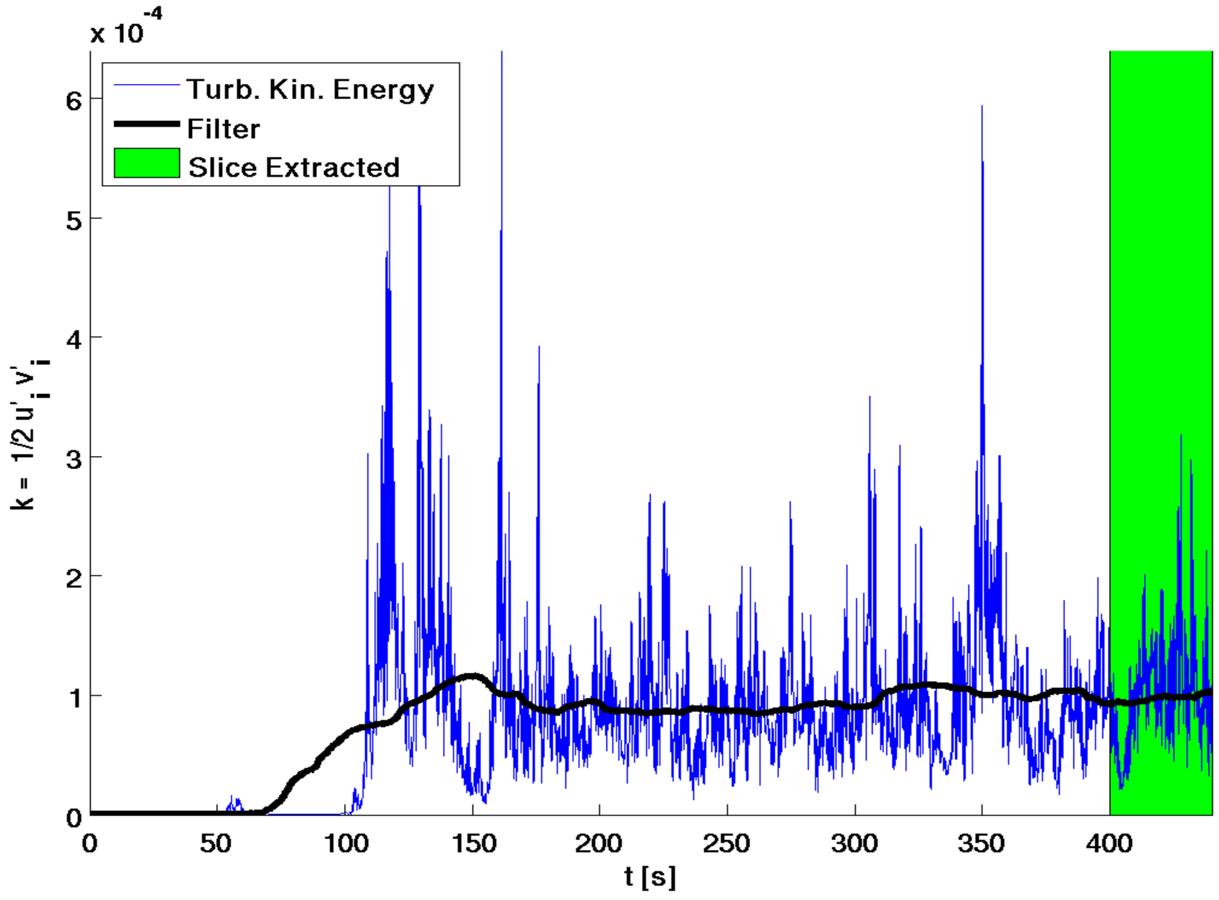

EllipSys3D
Filter
N.O.Jensen
1.5
1
0.5
0
u/U_0
0
50
100
150
200
250
300
350
400
450
500
Time

Results



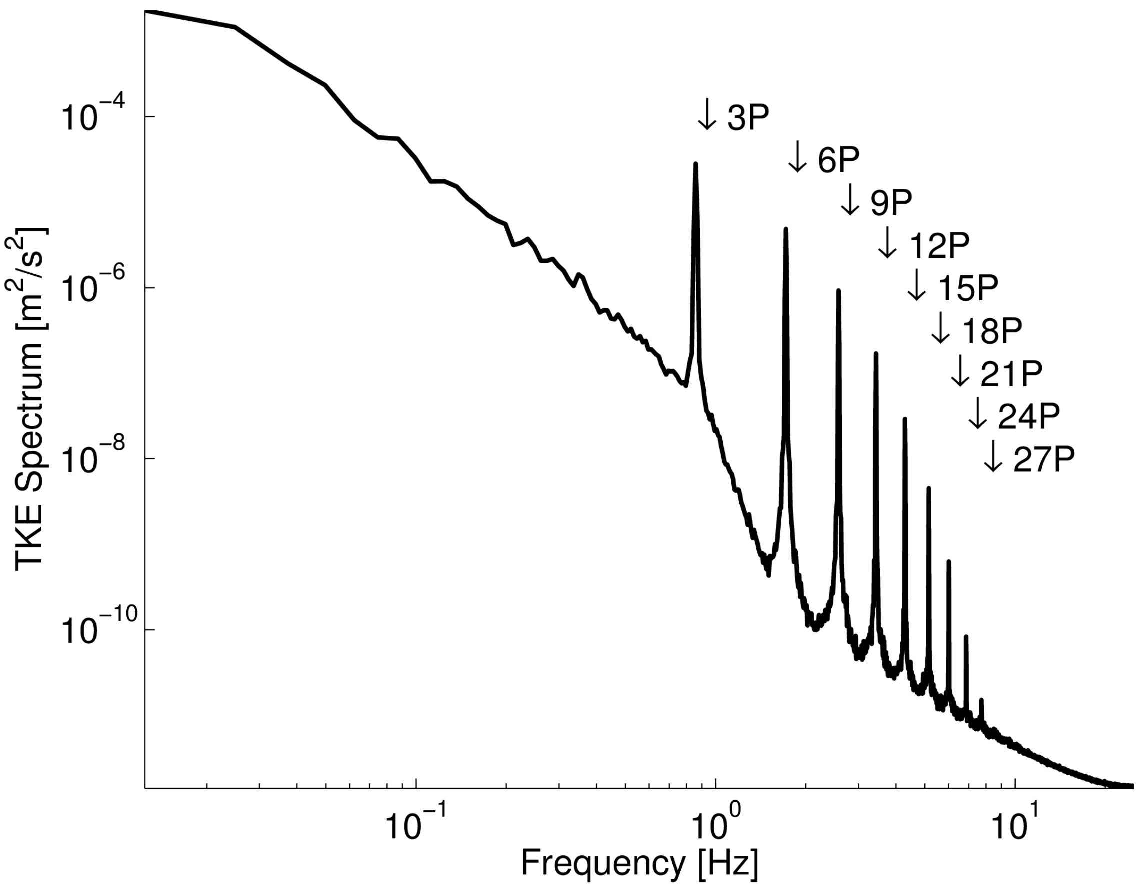


Results



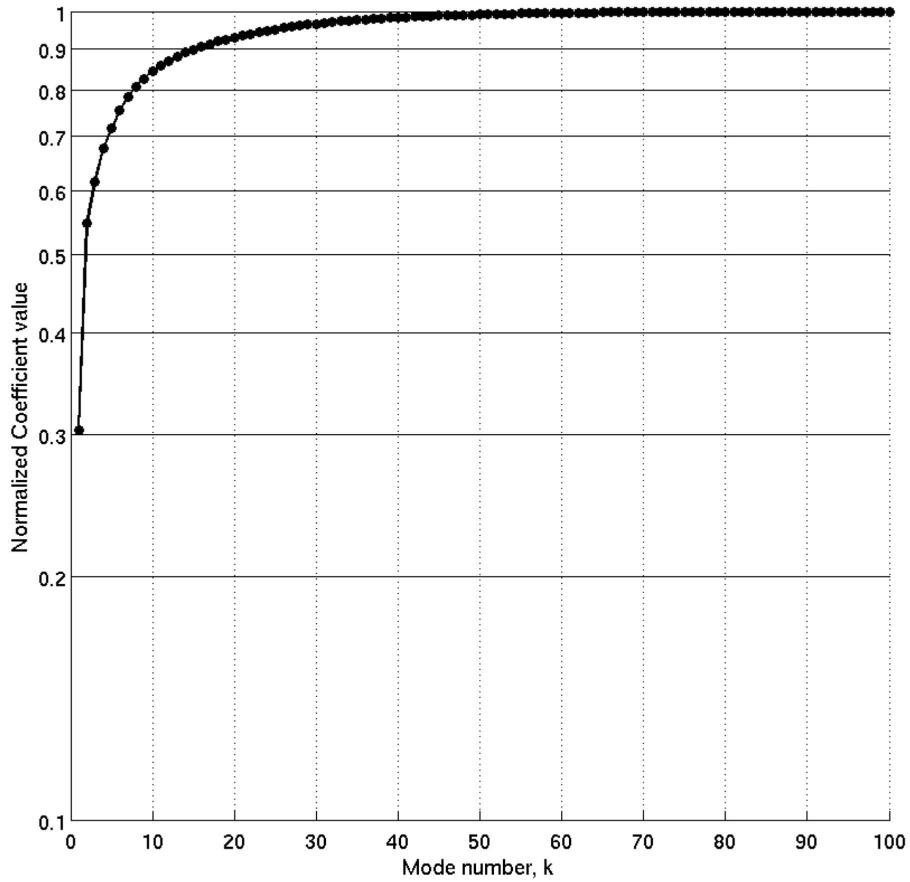
x 10^-4
Turb. Kin. Energy
Filter
Slice Extracted
k = 1/2 u'_i v'_i
0
1
2
3
4
5
6
0
50
100
150
200
250
300
350
400
t [s]

Results



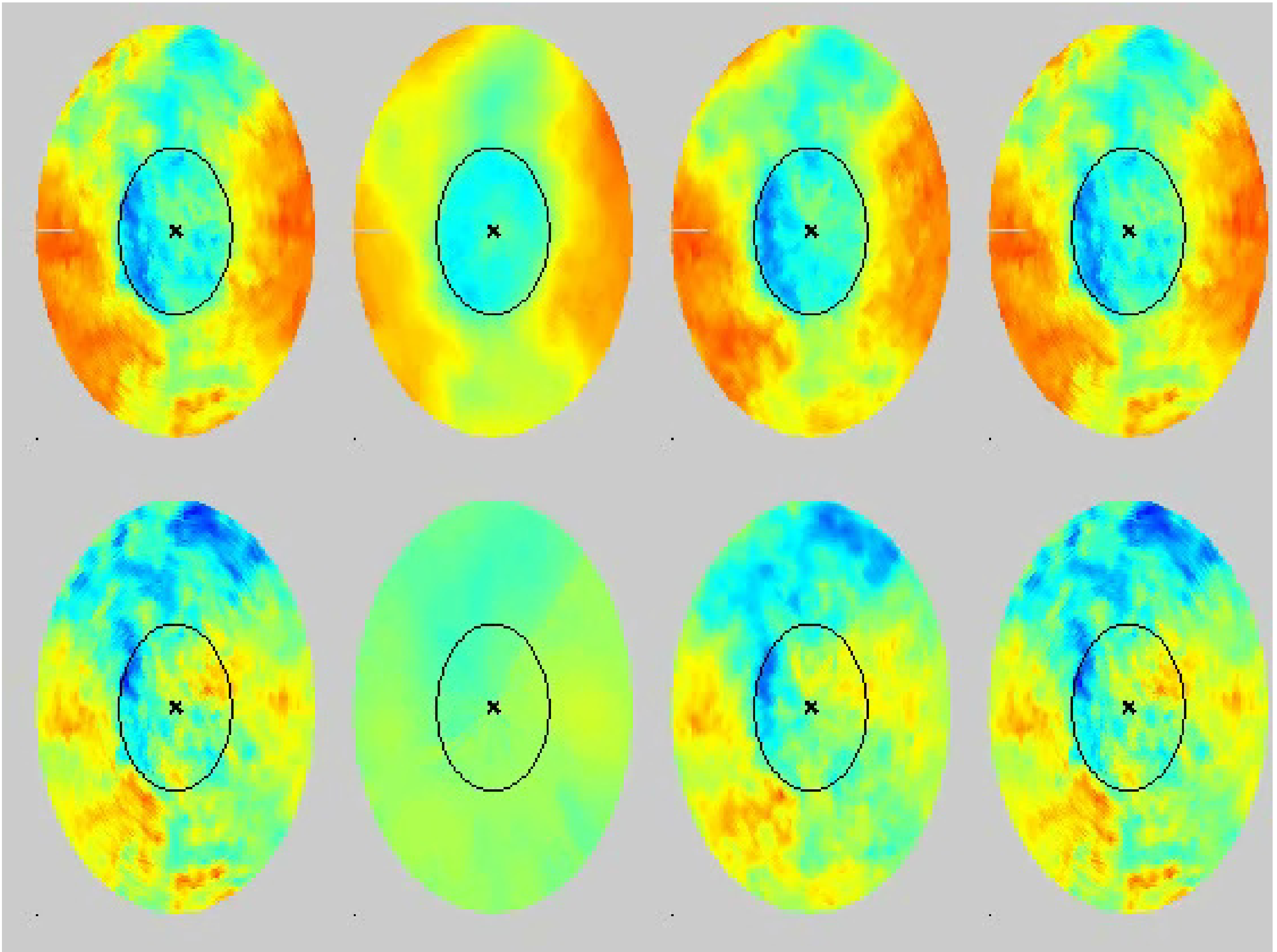
TKE Spectrum [m^2/s^2]
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-1}
10^{0}
10^{1}
Frequency [Hz]
↓ 3P
↓ 6P
↓ 9P
↓ 12P
↓ 15P
↓ 18P
↓ 21P
↓ 24P
↓ 27P

Results

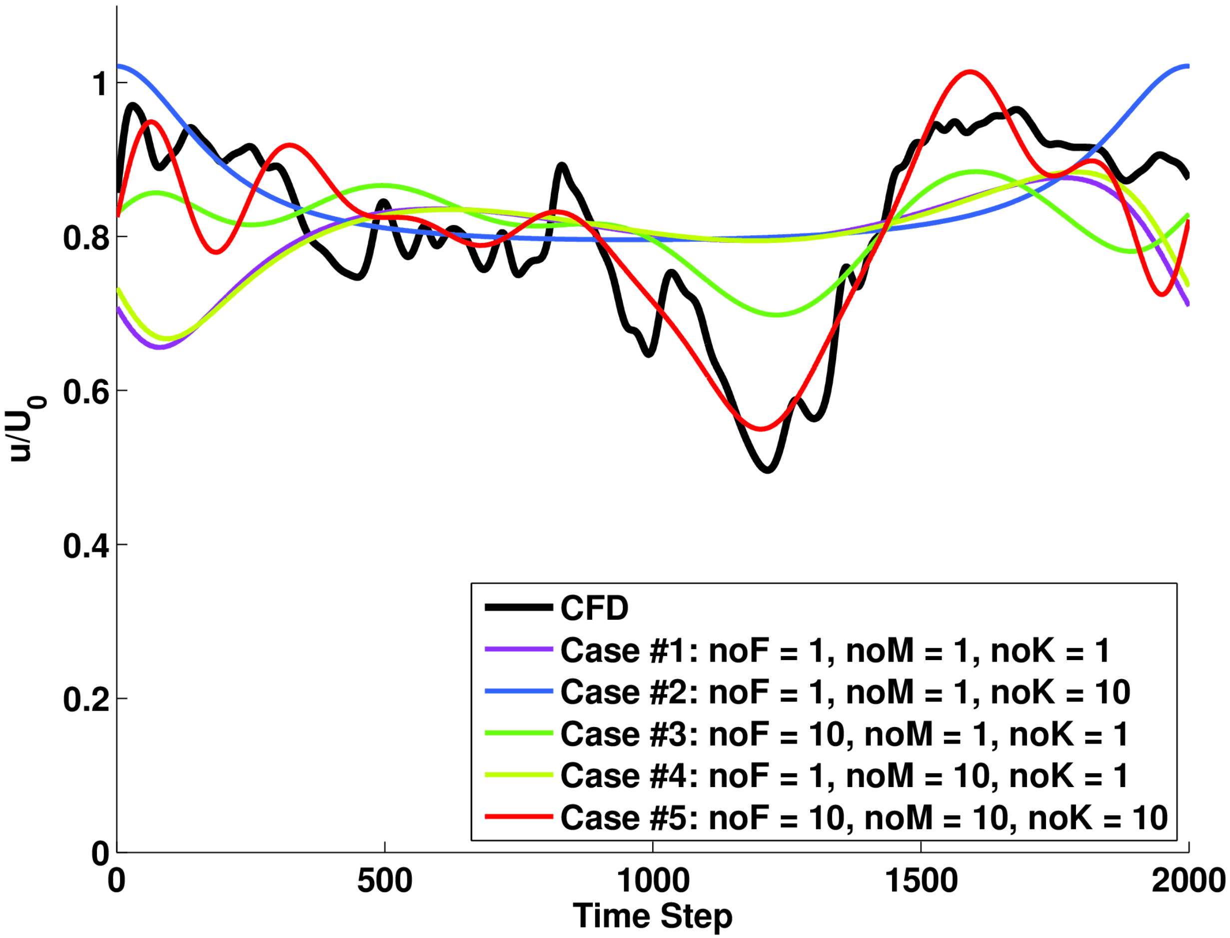


Normalized Coefficient value
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60
70
80
90
100
Mode number, k

Results

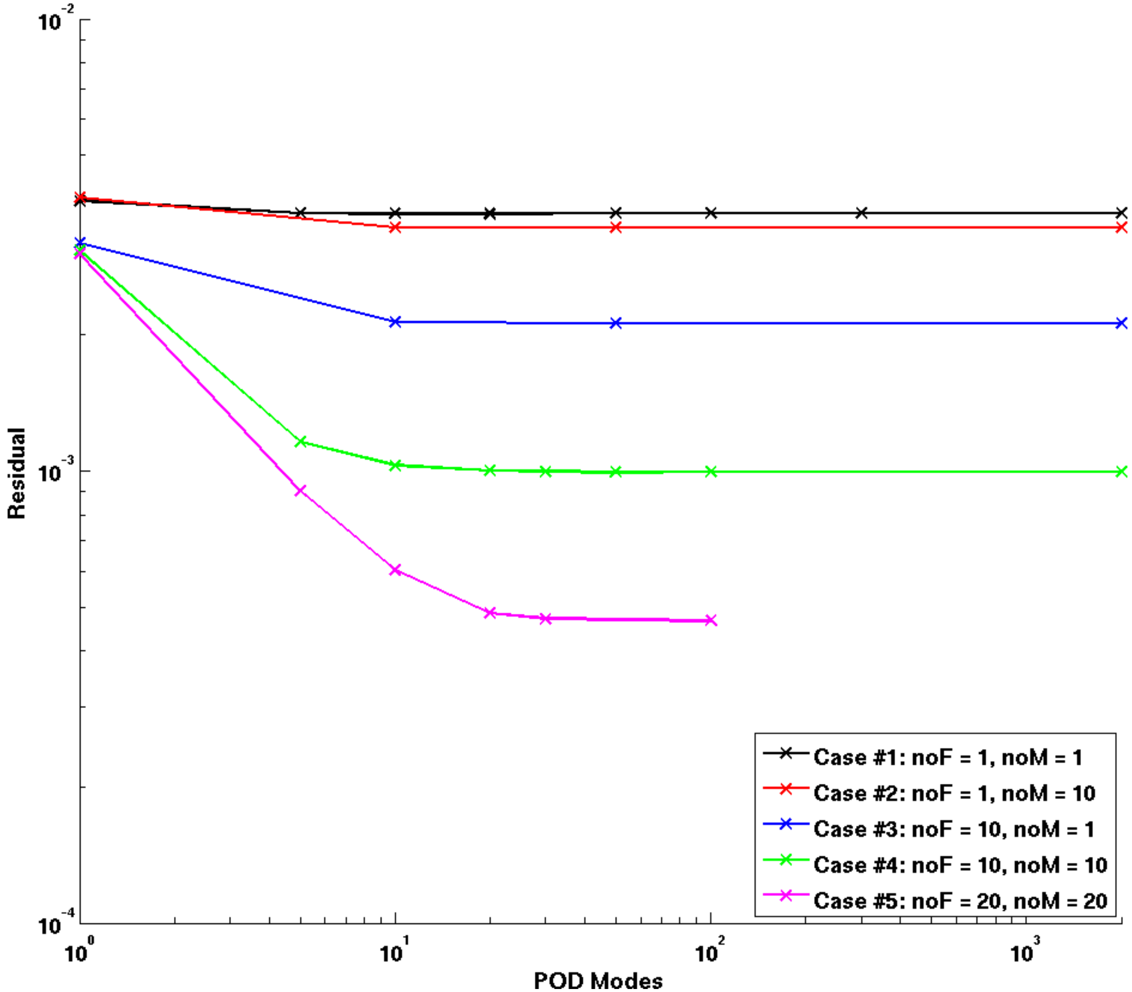


Results



1
0.8
0.6
0.4
0.2
0
u/U0
0
500
1000
1500
2000
Time Step
CFD
Case #1: noF = 1, noM = 1, noK = 1
Case #2: noF = 1, noM = 1, noK = 10
Case #3: noF = 10, noM = 1, noK = 1
Case #4: noF = 1, noM = 10, noK = 1
Case #5: noF = 10, noM = 10, noK = 10

Results



10^{-2}
10^{-3}
10^{-4}
Residual
10^{0}
10^{1}
10^{2}
10^{3}
POD Modes
Case #1: noF = 1, noM = 1
Case #2: noF = 1, noM = 10
Case #3: noF = 10, noM = 1
Case #4: noF = 10, noM = 10
Case #5: noF = 20, noM = 20

Conclusion and Outlook

- CFD computations using cyclic BCs

- Low order model of wake of wind turbines operating within a wind farm

- Parametric study to extend LOW: domain length, ambient turbulence, shear

Thank you for your attention.