Turbulence and Spectra for wind field simulation

Jakob Mann

Risø DTU, Roskilde, Denmark

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Outline of Part I: Turbulence structure

1. Turbulence for wind turbine load modeling
   - Basic properties and assumptions
   - Validity of assumptions

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   - Rapid Distortion Theory and the Mann model
   - Testing the model

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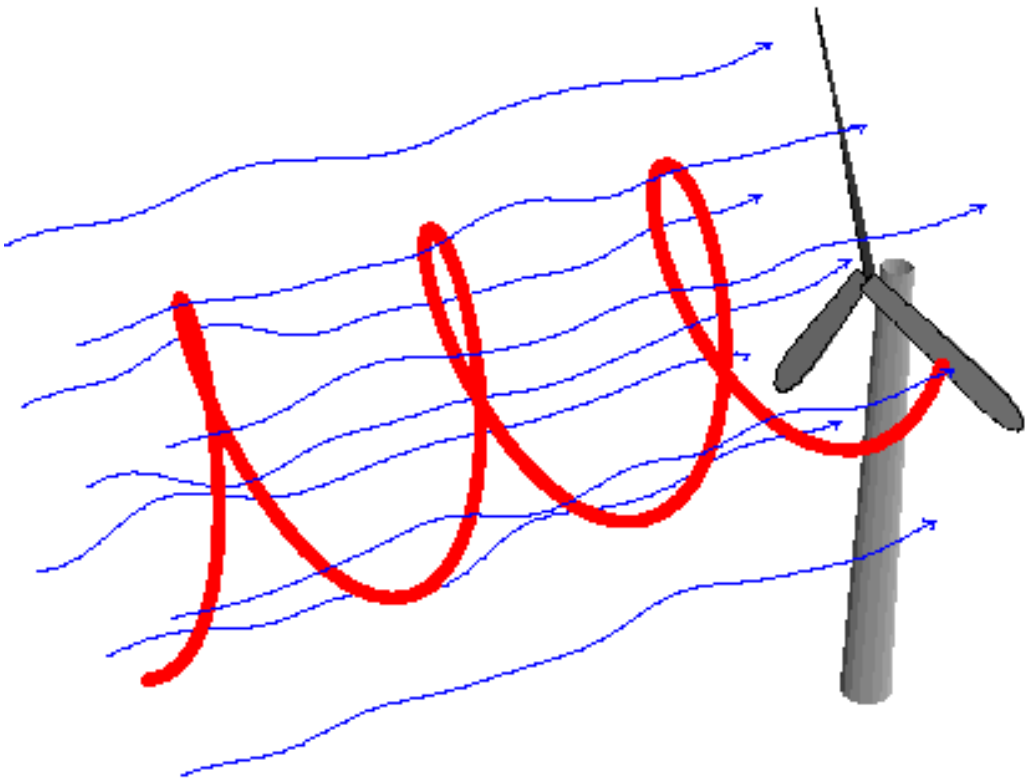
   - Description by a spectral tensor
   - Rapid Distortion Theory and the Mann model
   - Testing the model
3. Parameter variations
   - Høvsøre site
   - Diabatic observations
   - Extensions to complex terrain

Turbulence for wind turbine load modeling



The purpose is to describe spatial and temporal fluctuations with relevance for wind turbine load calculations.

Stationarity and homogeneity

A stochastic process $X(t)$ is completely described in term of all joint probabilities

$$p(x_1, t_1; x_2, t_2; ...; x_n, t_n) \quad \text{for all } n$$

or equivalently (under some conditions) all moments

$$\langle X(t_1)X(t_2)...X(t_n)\rangle$$

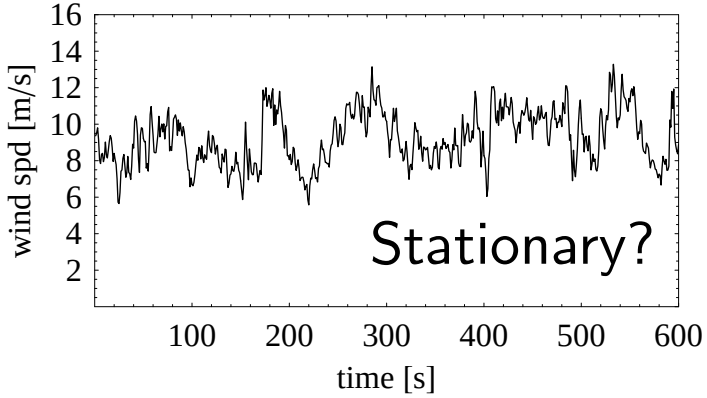
It is *stationary* if

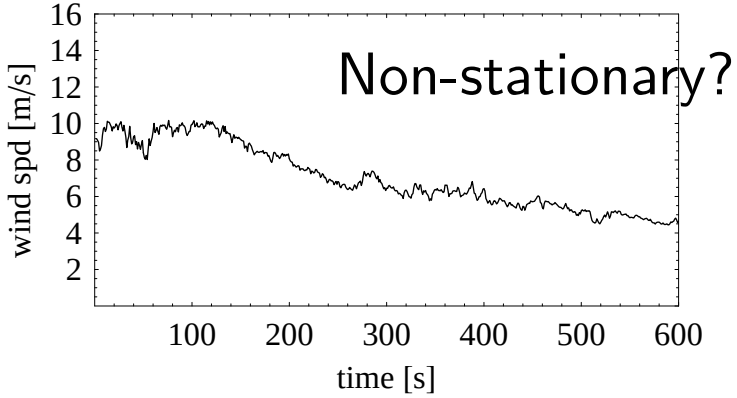
$$p(x_1, t_1; x_2, t_2; ...; x_n, t_n) = p(x_1, t_1 + t; x_2, t_2 + t; ...; x_n, t_n + t)$$

or

$$\langle X(t_1)X(t_2)...X(t_n)\rangle = \langle X(t_1 + t)X(t_2 + t)...X(t_n + t)\rangle \quad \forall t$$

Atmospheric time series and stationarity





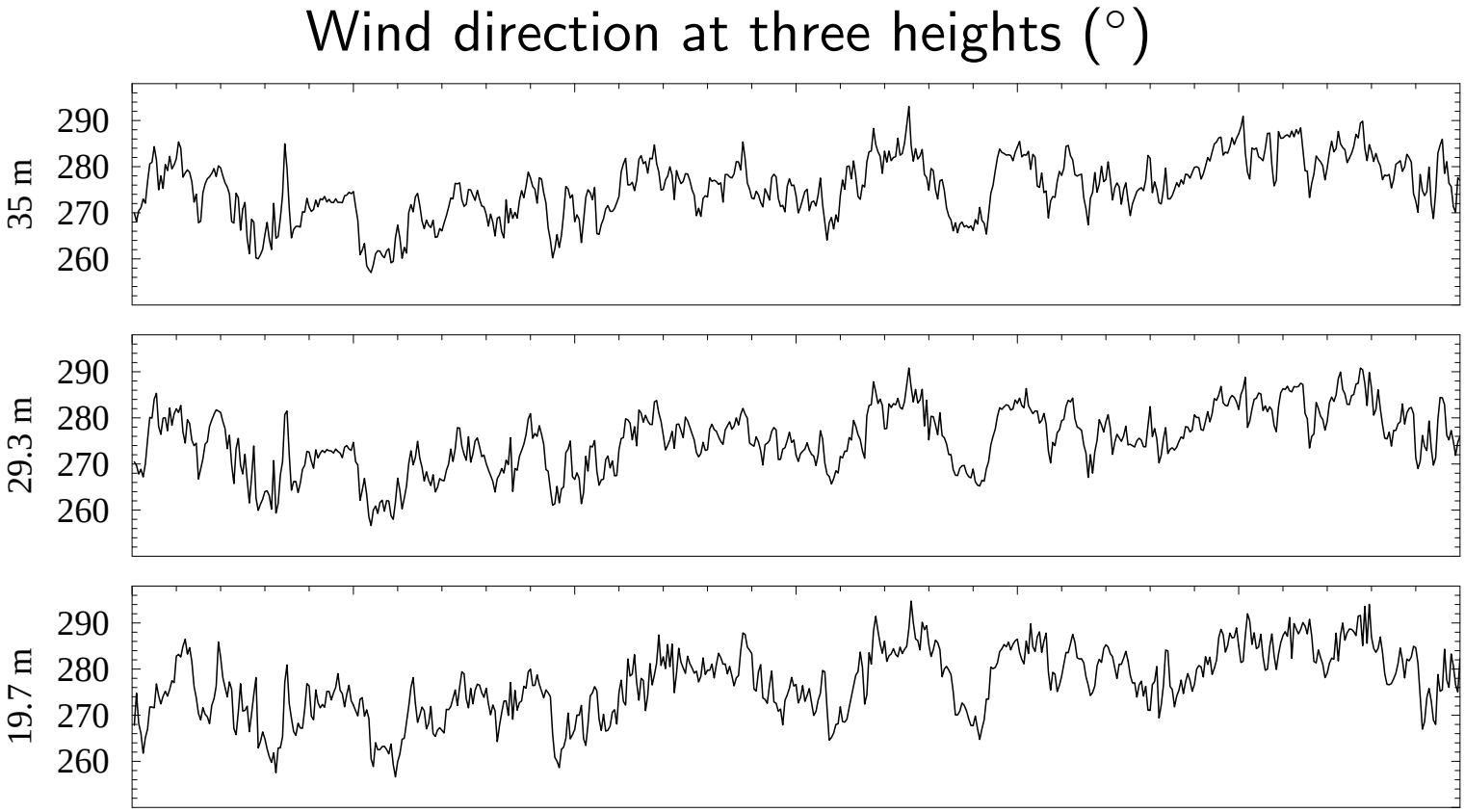
Homogeneity

A stochastic field $X(\mathbf{x})$ is *homogeneous* if

$$\langle X(\mathbf{x}_1)X(\mathbf{x}_2)...X(\mathbf{x}_n)\rangle = \langle X(\mathbf{x}_1+\mathbf{r})X(\mathbf{x}_2+\mathbf{r})...X(\mathbf{x}_n+\mathbf{r})\rangle$$

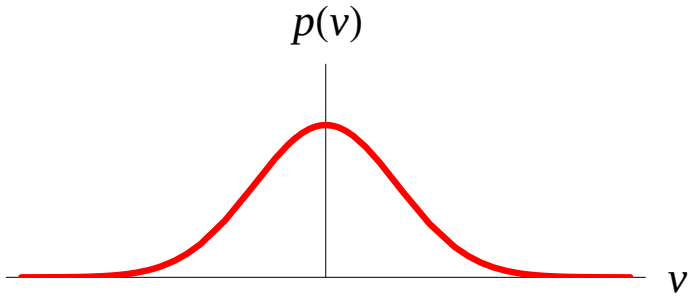
i. e. "stationary in space."

Homogeneity: Example



A Gaussian variable

$$p(v) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\frac{v^2}{\sigma^2}\right)$$



The zero mean gaussian variable v is simulated by the Box-Müller method.

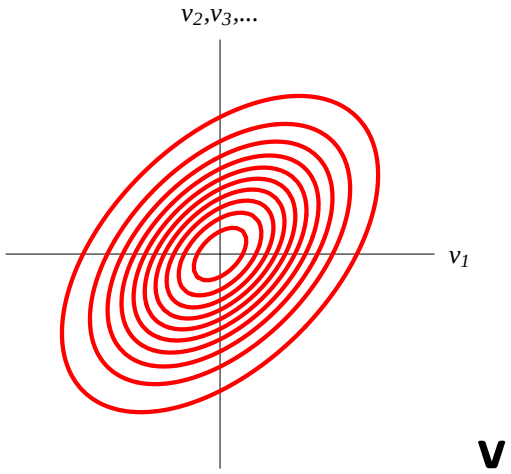
An n-dimensional Gaussian variable

$$p(\mathbf{v}) = \frac{1}{\sqrt{(2\pi)^n \det(\mathbf{R})}} \exp\left(-\frac{1}{2}\mathbf{v}\cdot\mathbf{Q}\mathbf{v}\right)$$

$$\mathbf{v} = \{v_1, v_2, ..., v_n\},\ \langle\mathbf{v}\rangle = \mathbf{0}$$

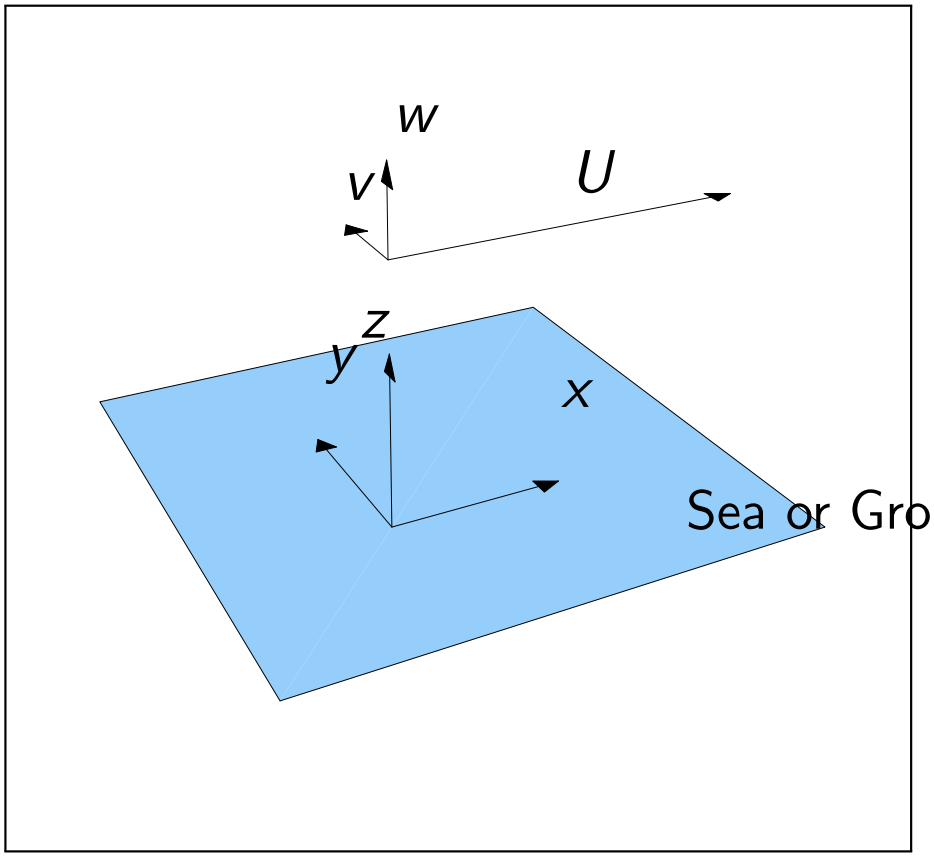
$$\mathbf{R} = \sigma^2 \begin{Bmatrix} 1 & \rho_1 & \rho_2 & \cdots \\ \rho_1 & 1 & \rho_1 & \\ \rho_2 & \rho_1 & 1 & \\ \vdots & & & \ddots \end{Bmatrix}$$

$$\mathbf{Q} = \mathbf{R}^{-1}$$



$\mathbf{v}$ is simulated by Fourier techniques, essentially treating the eigenvectors of $\mathbf{R}$ independently.

Coordinate systems



U: Mean wind speed.
u: **Longitudinal** fluctuations.
v: **Transversal** fluctuations.
w: **Vertical** fluctuations.
x_i: Space coordinates.

The logarithmic velocity profile

- ρ Air density (kg/m^3)
- τ Frictional force on a unit are of the surface (kg m^{-1} s^{-2})
- z Distance from the surface.

The only combination giving the dimension of velocity gradient is

$$\frac{dU}{dz} = \text{const}\sqrt{\frac{\tau}{\rho z^2}} = \frac{u_*}{\kappa z}, \tag{1}$$

where the *friction velocity* u_* is defined by

$$\tau = \rho u_*^2$$

and $\kappa \approx 0.4$ is the dimensionless *von Kármán constant*. Other turbulence quantities relate to $u_*^2 = -\langle uw \rangle$:

$$\sigma_u \approx 2.4 u_* \quad \sigma_v \approx 1.9 u_* \quad \sigma_w \approx 1.25 u_*$$

The logarithmic velocity profile

Solving (1) we get

$$U(z) = \frac{u_*}{\kappa} \log\left(z/z_0\right)$$

where the *roughness length* z_0 is an integration constant.

The influence of stability

Stable and unstable flows

A much used parameter in fluid dynamics

$$Ri = \frac{g}{T}\frac{d\Theta/dz}{(dU/dz)^2},$$

where Θ is the mean *potential* temperature.

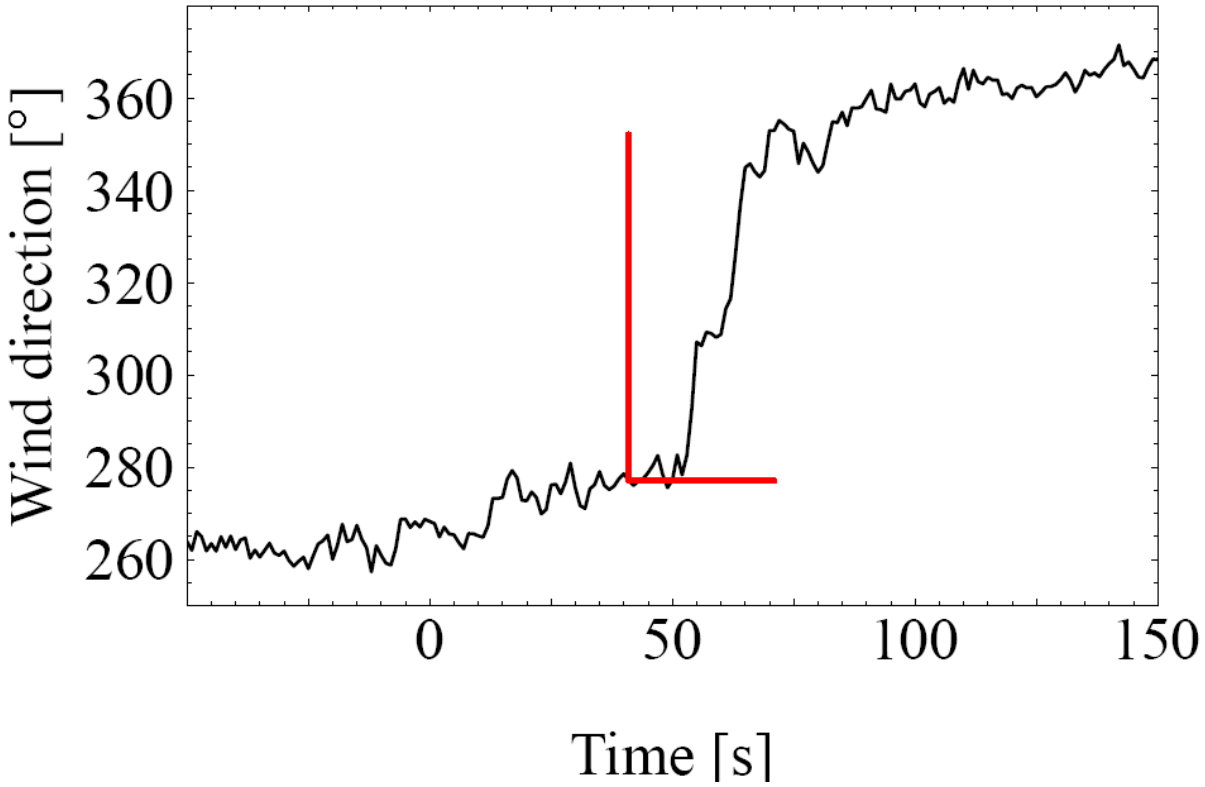
In surface layer meteorology the parameter z/L where

$$L = \frac{T}{g\kappa}\frac{u_*^3}{\langle w\theta\rangle} \tag{2}$$

is the *Monin-Obukhov length*, is widely used. Departures from neutral profiles can, at least close to the ground, be written as empirical functions of z/L.

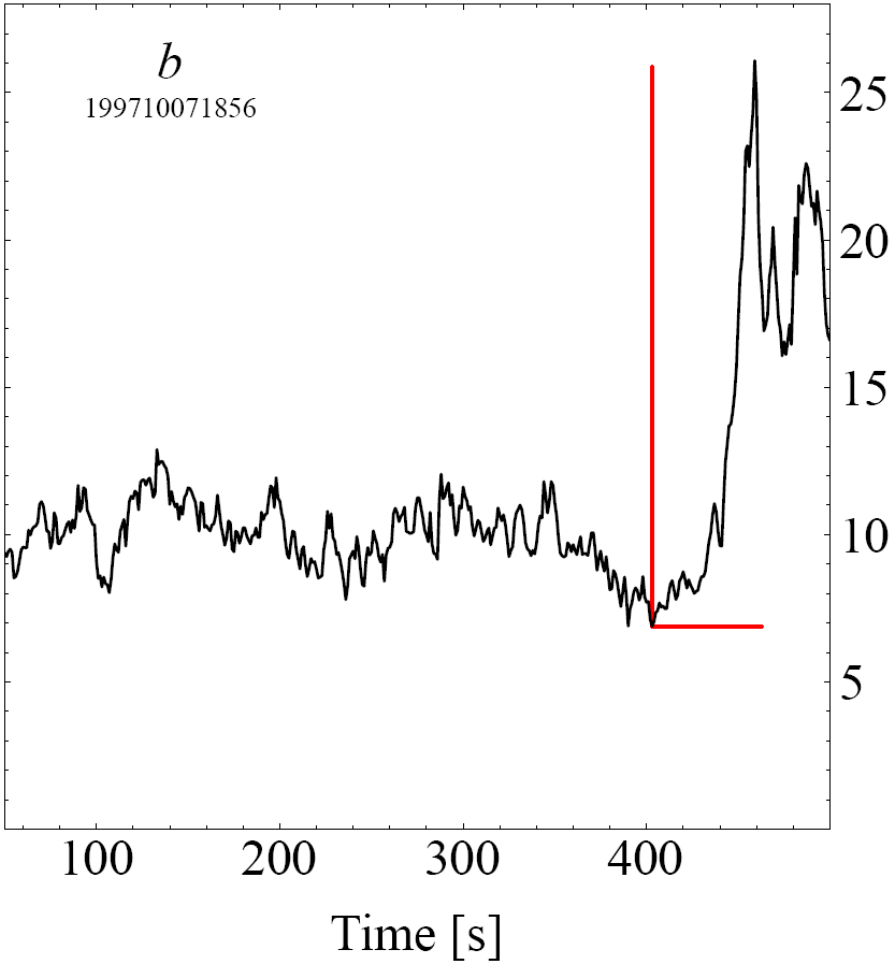
The finite height of the boundary layer will limit the linear growth of the eddy diffusivity making the profile look stable. Analysis of profiles from Høvsøre up to 160 m confirm this.

Are homogeneity, stationarity, gaussianity and neutral atmospheric stratification valid?



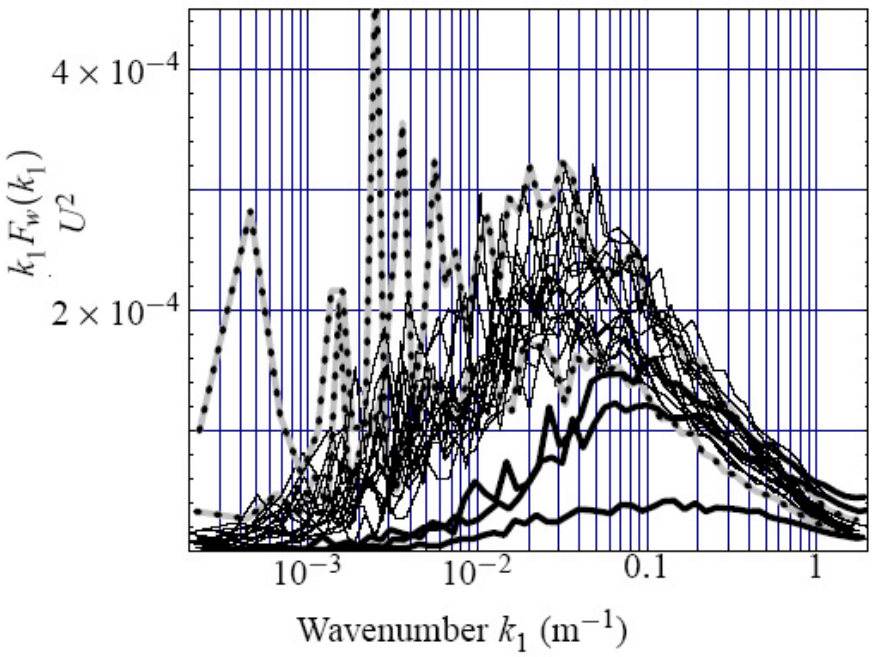
Lack of stationarity at an off-shore location. Wind speed constant ≈ 15 m/s in $\Delta\theta = 75^\circ$ in 30 s.

Stationarity?



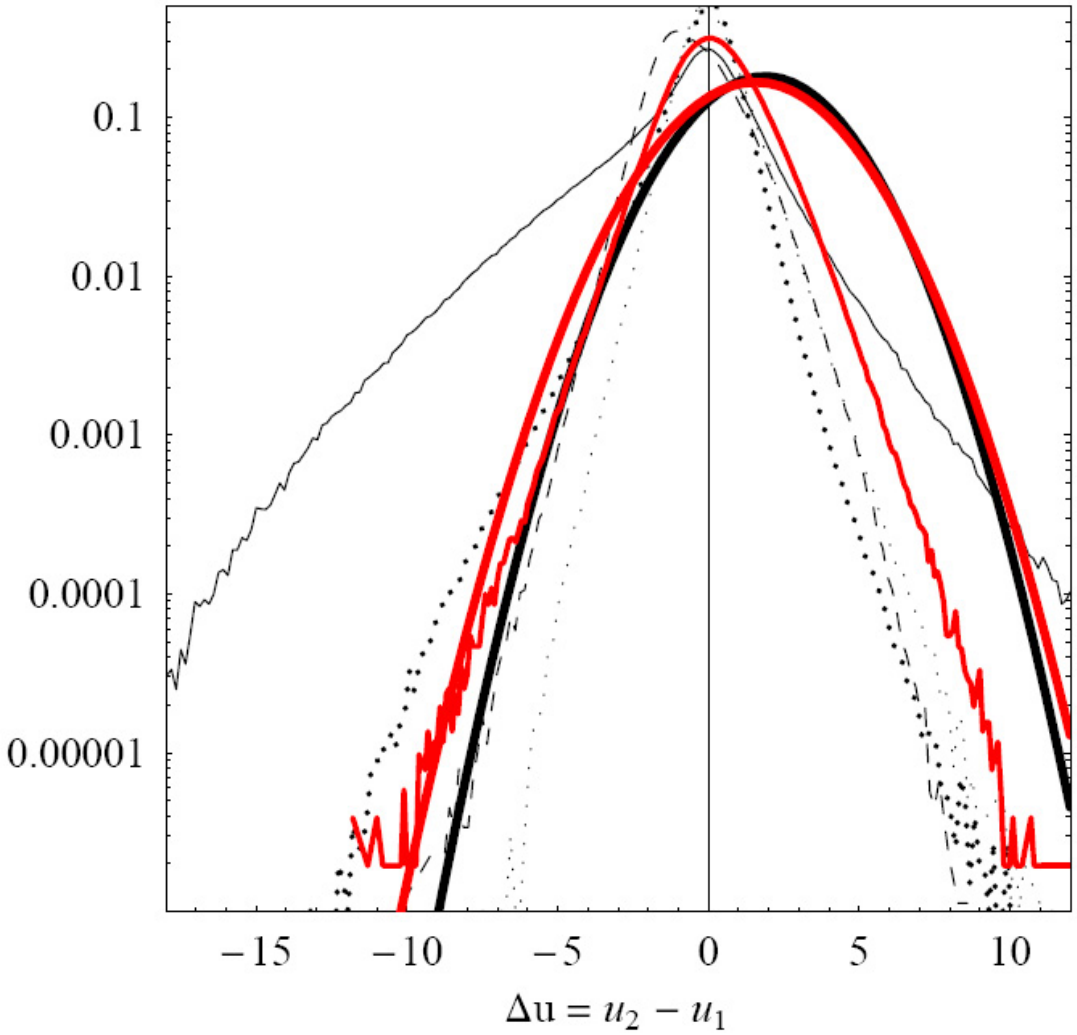
Off-shore frontal passage: $\Delta u = 19\mathrm{m/s}$ in $\Delta t_u = 60\mathrm{s}$.

Independence of stability at strong winds?



Spectra of w from the Great Belt Coherence Experiment. Mean wind speeds are between 16 and 20 m/s and directions are in a narrow interval around the South. Dashed spectra have slightly unstable stratification, gray have stable, and the thin have neutral.

Gaussianity



Pdf of instantaneous velocity differences between two heights within the rotor plane. Smooth curves are expectations from standards.

Technical preliminaries of the Mann model

Suppose the velocity field is homogeneous.
Taylor's frozen turbulence hypothesis

$$\tilde{\mathbf{u}}(x, y, z, t) = \tilde{\mathbf{u}}(x - Ut, y, z, 0)$$

Covariance tensor

$$R_{ij}(\mathbf{r}) = \langle u_i(\mathbf{x}) u_j(\mathbf{x} + \mathbf{r}) \rangle$$

For $\mathbf{r} = 0$ the diagonal elements of R_{ij} are $\sigma_u^2, \sigma_v^2, \sigma_w^2$. For $|\mathbf{r}| \to \infty$ $R_{ij} \to 0$.

Technical preliminaries of the Mann model

Spectral tensor

$$\Phi_{ij}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int R_{ij}(\mathbf{r}) \exp(-\mathrm{i}\mathbf{k}\cdot\mathbf{r})\mathrm{d}\mathbf{r}$$

One-dimensional spectrum

$$F_i(k_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{ij}(x,0,0)\mathrm{e}^{-\mathrm{i}k_1 x}\mathrm{d}x$$

Cross-spectrum

$$\chi_{ij}(k_1,\Delta y,\Delta z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{ij}(x,\Delta y,\Delta z)\mathrm{e}^{-\mathrm{i}k_1 x}\mathrm{d}x$$

Coherence

$$\mathrm{coh}_{ij}(k_1,\Delta y,\Delta x) = \frac{|\chi_{ij}(k_1,\Delta y,\Delta z)|^2}{F_i(k_1)F_j(k_1)}$$

Symmetries

From *symmetries* it is possible to determine if some cross-spectra are real, purely imaginary or zero. The symmetry group of a turbulent field is the set of all orthonormal transformations T for which the second order statistics of $u_i(\mathbf{x})$ is the same as $T_{ij}u_j(T\mathbf{x})$. Consequences for the correlation tensor:

$$\begin{aligned} R_{ij}(\mathbf{r}) &\equiv \langle u_i(\mathbf{x})u_j(\mathbf{x}+\mathbf{r})\rangle \\ &= \langle T_{ik}u_k(T\mathbf{x})T_{jl}u_l(T\mathbf{x}+T\mathbf{r})\rangle \\ &= T_{ik}\langle u_k(T\mathbf{x})u_l(T\mathbf{x}+T\mathbf{r})\rangle T_{jl} \\ &= T_{ik}R_{kl}(T\mathbf{r})T_{jl} \end{aligned}$$

Symmetries

Example

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow$$

$$R_{23}(x,0,z) = -R_{23}(x,0,z) \Rightarrow R_{23}(x,0,z) = 0$$

Properties of the spectral tensor

$$\Phi_{ij}(\mathbf{k}) \equiv \frac{1}{(2\pi)^3} \int R_{ij}(\mathbf{r}) \exp\left(-\mathrm{i}\mathbf{k}\cdot\mathbf{r}\right) \mathrm{d}\mathbf{r}$$

$$\Updownarrow$$

$$R_{ij}(\mathbf{r}) = \int \Phi_{ij}(\mathbf{k}) \exp\left(\mathrm{i}\mathbf{k}\cdot\mathbf{r}\right) \mathrm{d}\mathbf{k}$$

$$R_{ij}(\mathbf{r}) = R_{ji}(-\mathbf{r}) \Rightarrow \Phi_{ij}(\mathbf{k}) = \Phi^*_{ji}(\mathbf{k}),$$

where * denotes complex conjugation.

$$R_{ij}(\mathbf{r}) = T_{ik} R_{kl}(T\mathbf{r}) T_{jl} \Leftrightarrow \Phi_{ij}(\mathbf{k}) = T_{ik} \Phi_{kl}(T\mathbf{k}) T_{jl}$$

where * also denotes the adjoint, i.e. in the case of a real matrix the transpose.

Incompressibility

$$\nabla \cdot \mathbf{u}(\mathbf{x}) = \frac{\partial u_i}{\partial x_i} = 0$$

$\Leftrightarrow k_i u_i(\mathbf{k}) = 0.$

$$\frac{\partial}{\partial r_j} R_{ij}(\mathbf{r}) = \frac{\partial}{\partial r_j} \langle u_i(\mathbf{x}) u_j(\mathbf{x}+\mathbf{r}) \rangle = \left\langle u_i(\mathbf{x}) \frac{\partial}{\partial r_j} u_j(\mathbf{x}+\mathbf{r}) \right\rangle = 0$$

$\Leftrightarrow k_j \Phi_{ij}(\mathbf{k}) = 0$. This property implies zero direct backscatter of acoustical beams under neutral stratification.

Isotropy

All orthonormal transformations T leaves the statistics of the velocity field $\mathbf{u}$ unchanged.
For a moment think of a scalar field $\theta(\mathbf{x})$ and consider the second order statistics $R(\mathbf{r}) = \langle \theta(\mathbf{x})\theta(\mathbf{x}+\mathbf{r})\rangle$. Isotropy here implies that $R(\mathbf{r})$ can only depend on $r = |\mathbf{r}|$.
An isotropic, symmetric second order tensor as the velocity correlation function can only depend on

$$\delta_{ij} \qquad\qquad k_i k_j$$

We are left with

$$\Phi_{ij}(\mathbf{k}) = f_1(k)\delta_{ij} + f_2(k)k_i k_j$$

$$\Phi_{ij}(\mathbf{k}) = \frac{E(k)}{4\pi k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)$$

where $E(k)$ is half the variance of the wind velocity fluctuations whose magnitude of the wave vector is in the range $(k, k + \mathrm{d}k)$.

Kolmogorov and von Kármán

Kolmogorov (1941) (dimensional analysis) for large k but still smaller than the wave length corresponding to the viscous scale.

$$E(k) = \alpha \varepsilon^{2/3} k^{5/3}$$

The value of "the spectral Kolmogorov constant" α is ≈ 1.7.
Implies

$$F_{22}(k_1) = F_{33}(k_1) = \frac{4}{3} F_{11}(k_1)$$

as in the IEC standard.
Isotropy implies $\sigma_u^2 = \sigma_v^2 = \sigma_w^2$. Also $\chi_{13} = 0$.
Von Kármán proposed

$$E(k) = \alpha \varepsilon^{2/3} L^{5/3} \frac{(Lk)^4}{(1 + (Lk)^2)^{17/6}}$$

Navier-Stokes equations and higher order moments

$$\mathbf{a} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}$$

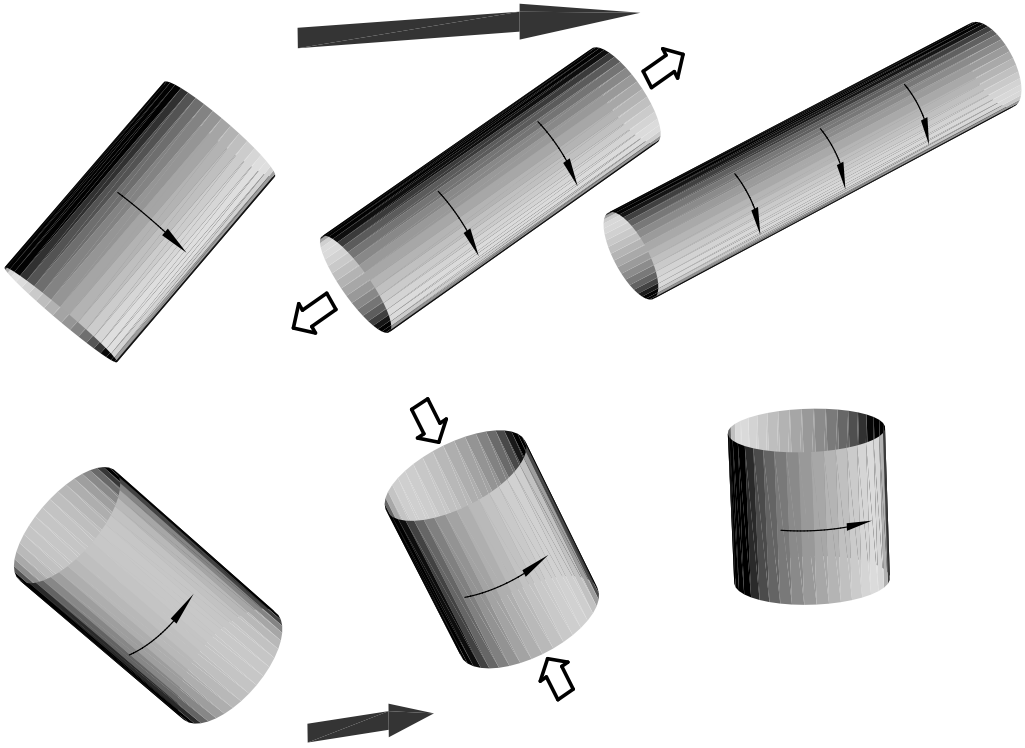
From this it can be shown (Kolmogorov, 1941) that

$$\left\langle \delta u_{\parallel}(\mathbf{r})^3 \right\rangle = -\frac{4}{5} \varepsilon r$$

which is in direct conflict with gaussianity.

Rapid distortion theory

Rapid distortion theory (RDT) was originally formulated to calculate turbulence in a wind tunnel contraction.
It was later used to model the response of turbulence to shear.

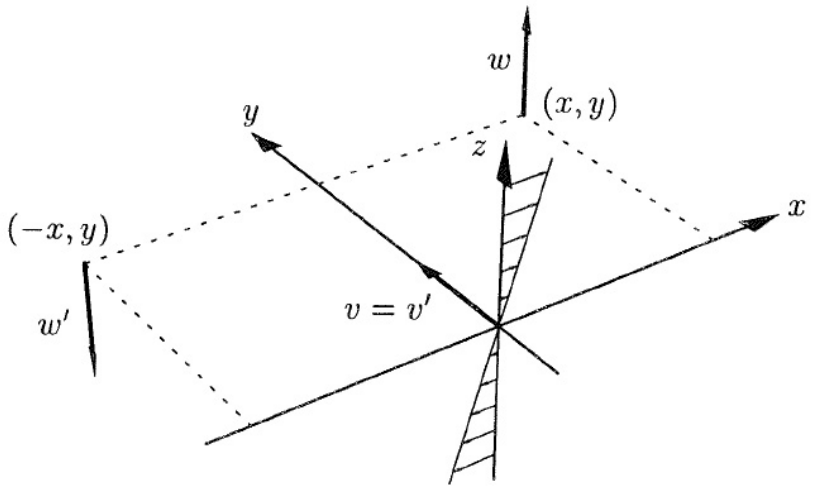


The basic idea is to divide the flow into a mean and a fluctuating part. In the Navier-Stokes equations the interaction (or products) between fluctuating parts are ignored. This allows for a Fourier transform of the equations, resulting in linear differential equations with no coupling between wave-vectors.

The Mann model

Symmetry group of the Mann model

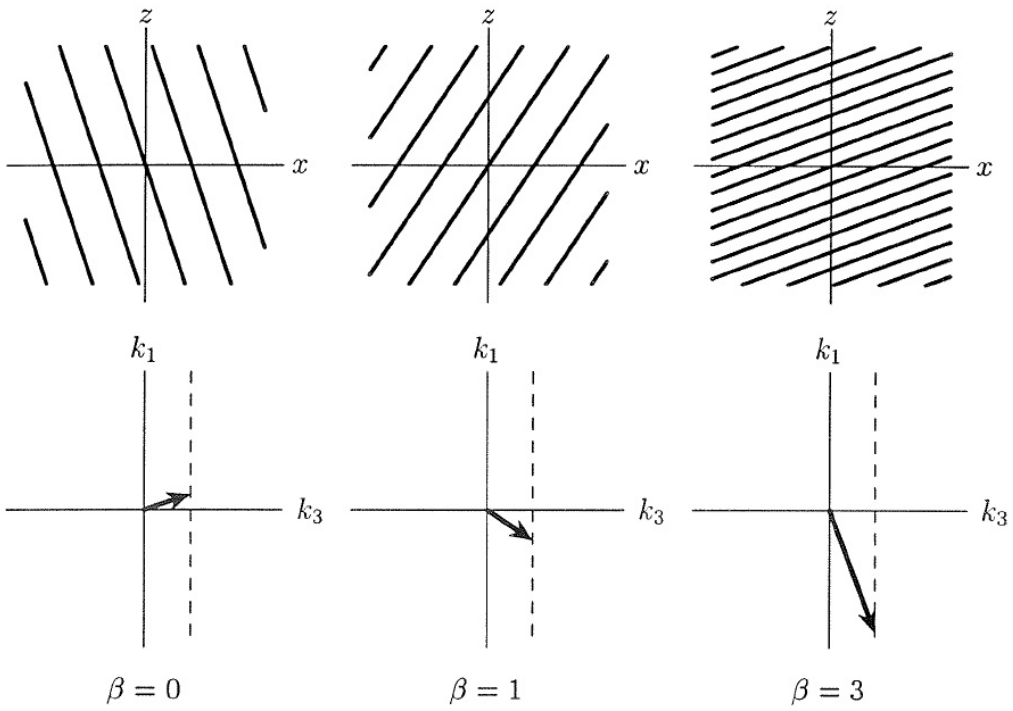
$$\left\{ I, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, -I \right\}$$



The linearization is unrealistic; stretched 'eddies' will break up (interaction between fluctuations). Equilibrium is postulated where eddies of size $\propto |\mathbf{k}|^{-1}$ are stretched by the shear over a time proportional to their life time τ. In the inertial subrange $\tau \propto k^{-2/3}$. We introduce a parameter Γ, such that the dimensionless life time, β, can be written as $\beta \equiv \frac{\mathrm{d}U}{\mathrm{d}z}\tau = \Gamma(kL)^{-\frac{2}{3}}$. A more general model of the dimensionless eddy life time β outside the inertial subrange is established in Mann (1994).

Evolution of individual wave-vectors:

$$\mathbf{k}_0 = (k_1, k_2, k_{30}) \qquad \text{with } k_{30} = k_3 + \beta k_1$$



'Initial condition' $\mathrm{d}\mathbf{Z}^{\mathrm{iso}}(\mathbf{k}_0)$ has the statistics of the isotropic von Kármán tensor. The sheared tensor is then given by

$$\mathrm{d}\mathbf{Z}(\mathbf{k}) = \begin{bmatrix} 1 & 0 & \zeta_1 \\ 0 & 1 & \zeta_2 \\ 0 & 0 & k_0^2/k^2 \end{bmatrix} \mathrm{d}\mathbf{Z}^{\mathrm{iso}}(\mathbf{k}_0)$$

where

$$\zeta_1 = C_1 - k_2 C_2/k_1 \ , \quad \zeta_2 = k_2 C_1/k_1 + C_2$$

with

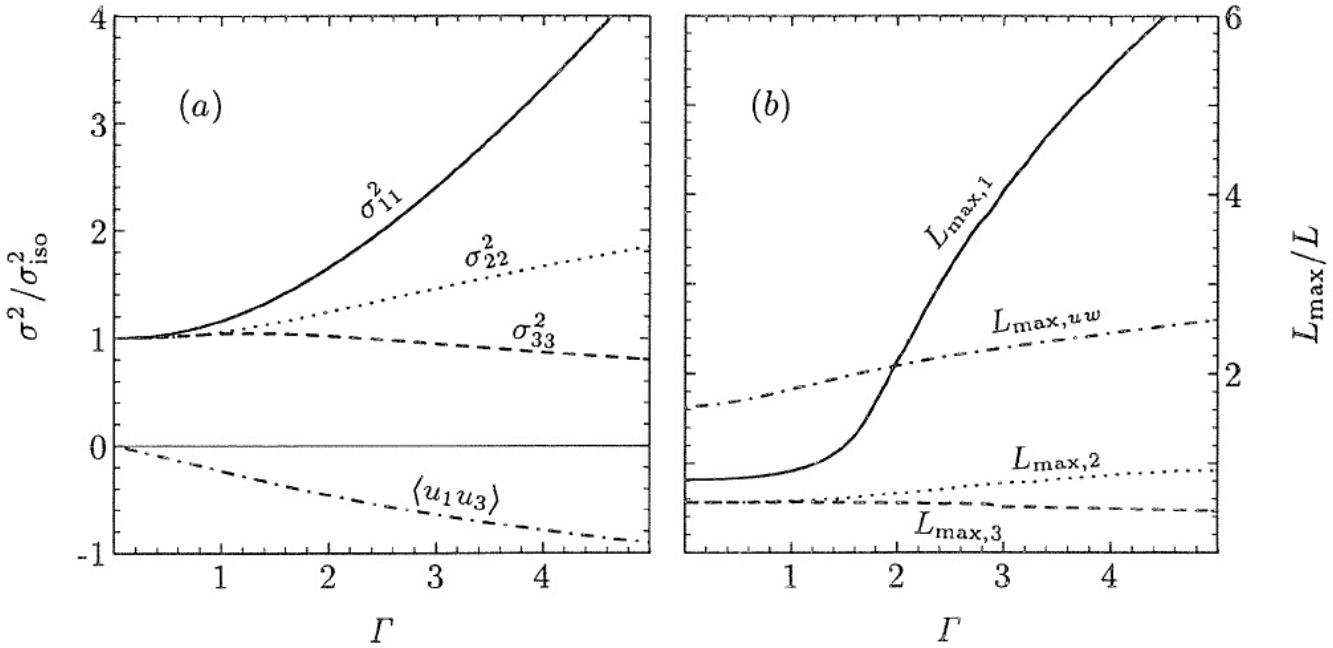
$$C_1 = \frac{\beta k_1^2 (k_0^2 - 2k_{30}^2 + \beta k_1 k_{30})}{k^2 (k_1^2 + k_2^2)}$$

and

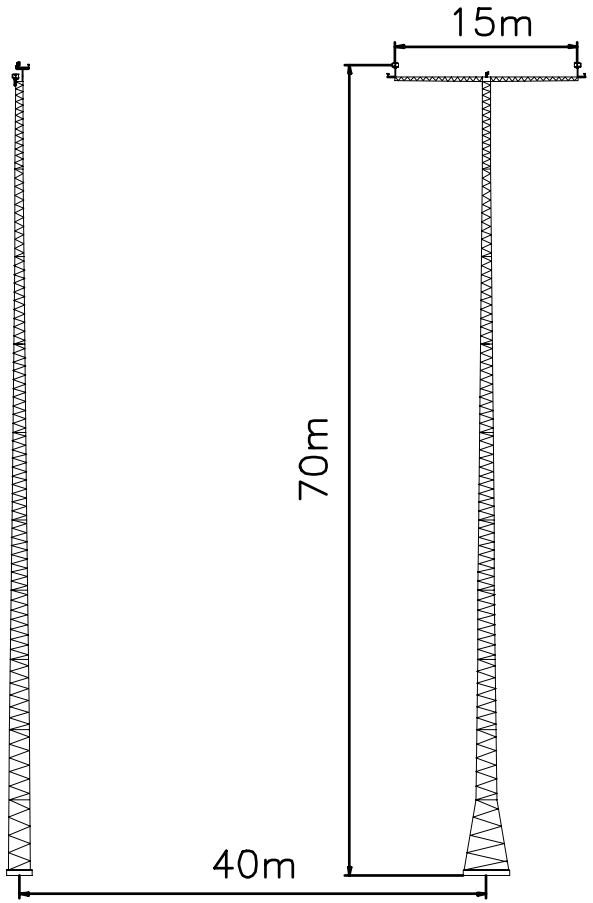
$$C_2 = \frac{k_2 k_0^2}{(k_1^2 + k_2^2)^{\frac{3}{2}}} \arctan \left[\frac{\beta k_1 (k_1^2 + k_2^2)^{\frac{1}{2}}}{k_0^2 - k_{30} k_1 \beta} \right],$$

Compared to the isotropic tensor the extra parameter Γ implies

- $\sigma_u^2 > \sigma_v^2 > \sigma_w^2$
- $\langle uw \rangle < 0$
- Length scale of u much larger than w



Comparison with data: Great Belt Coherence Experiment



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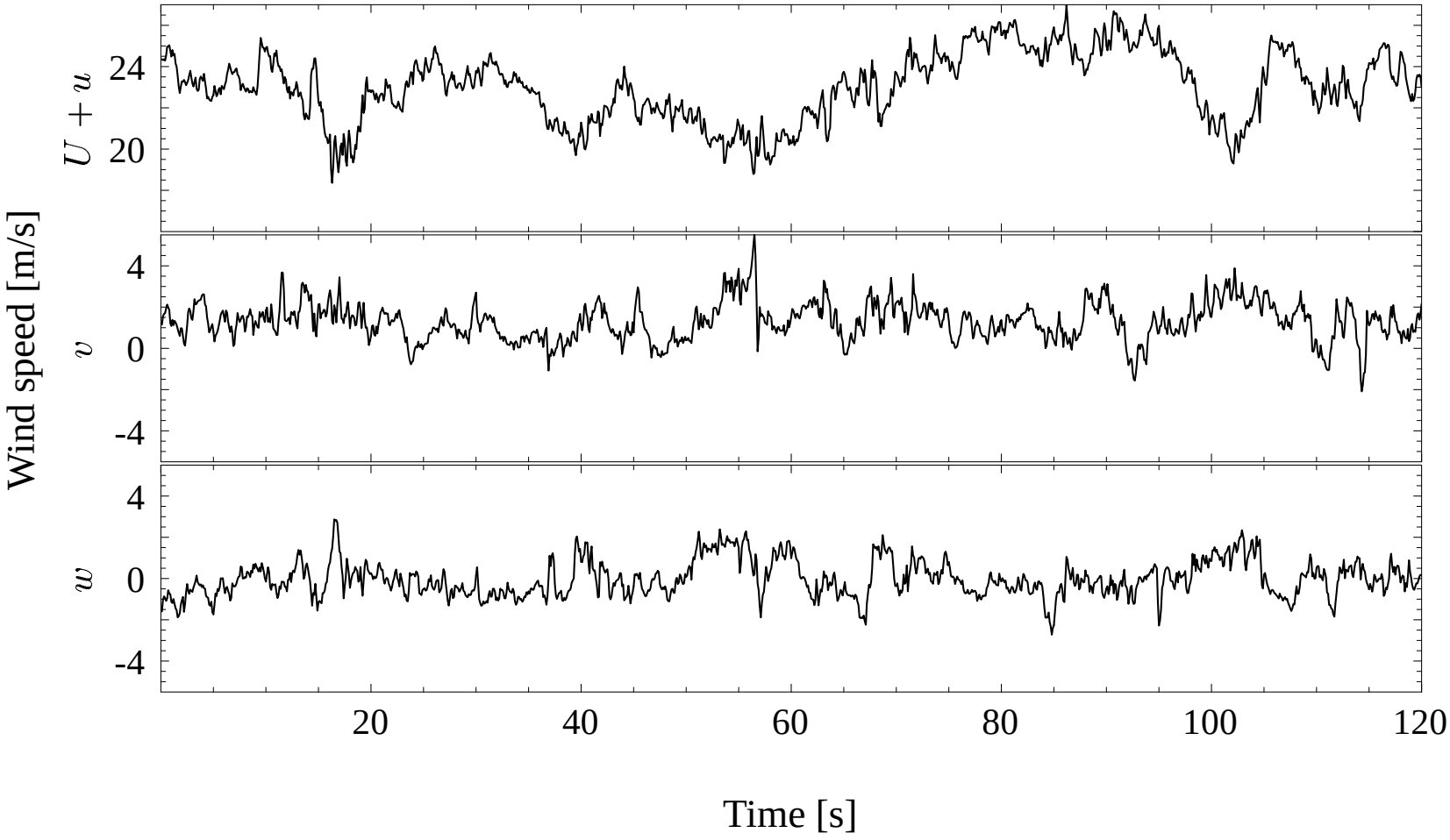
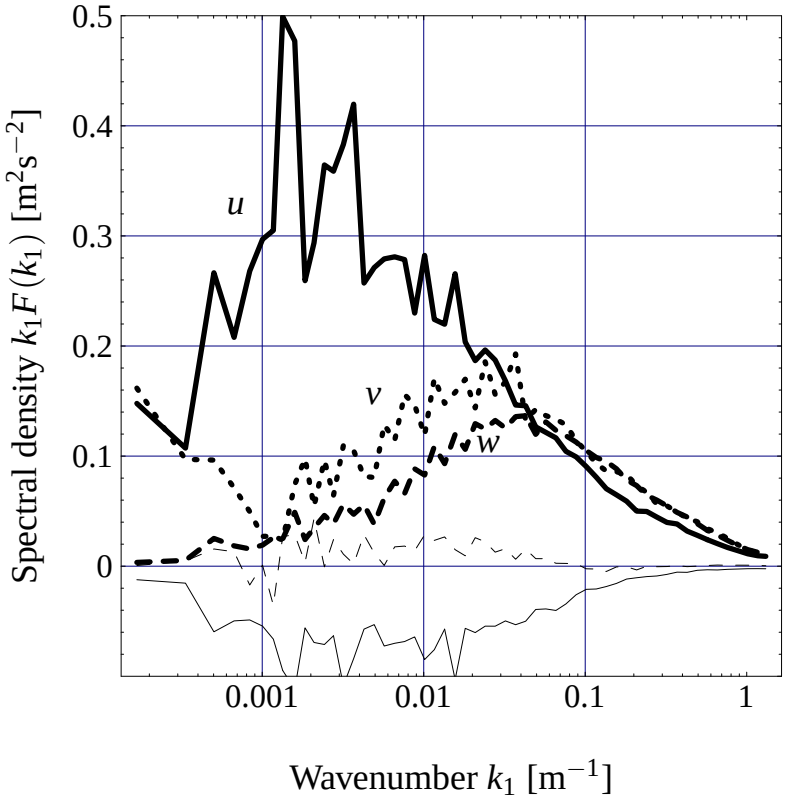
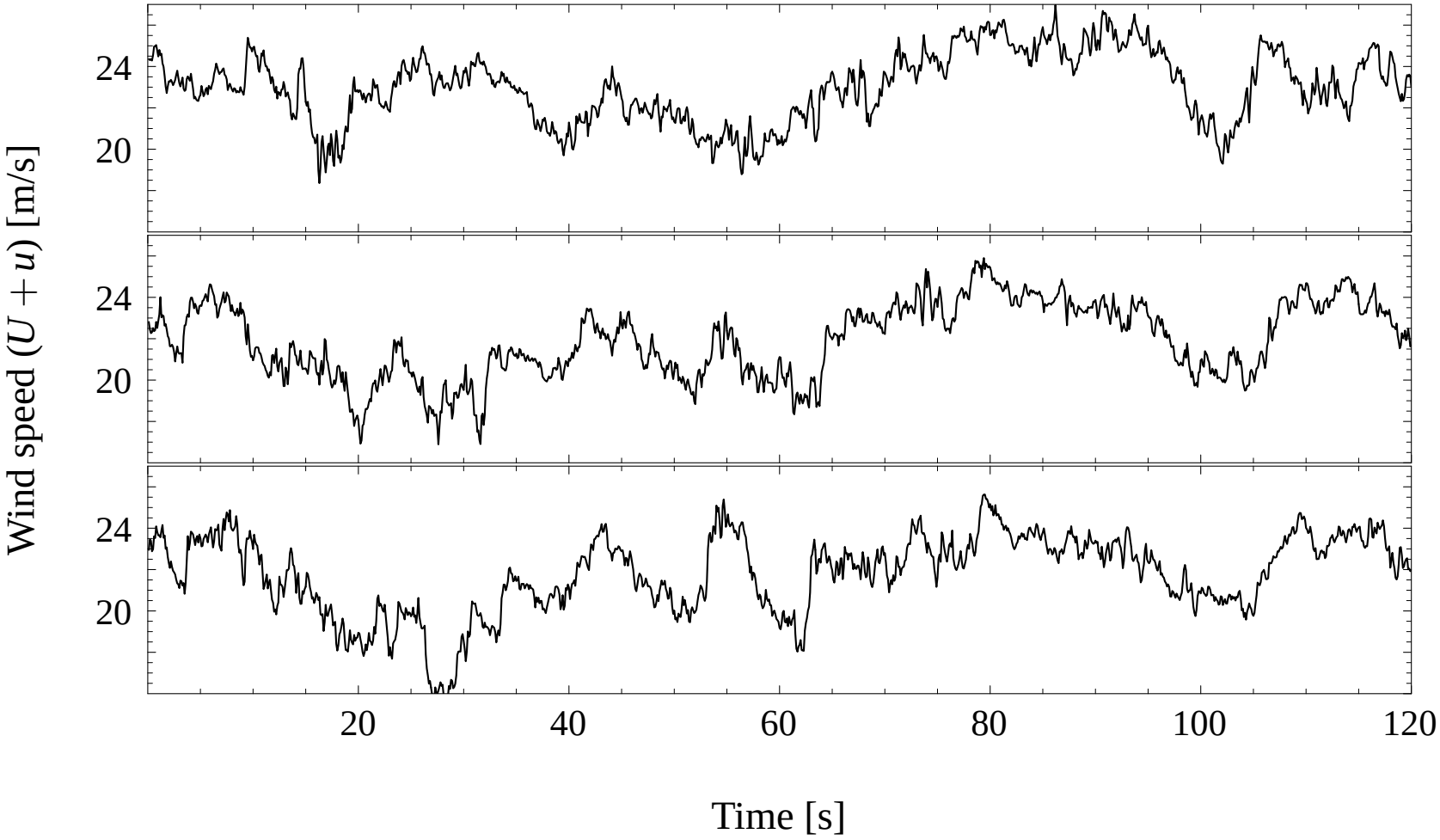


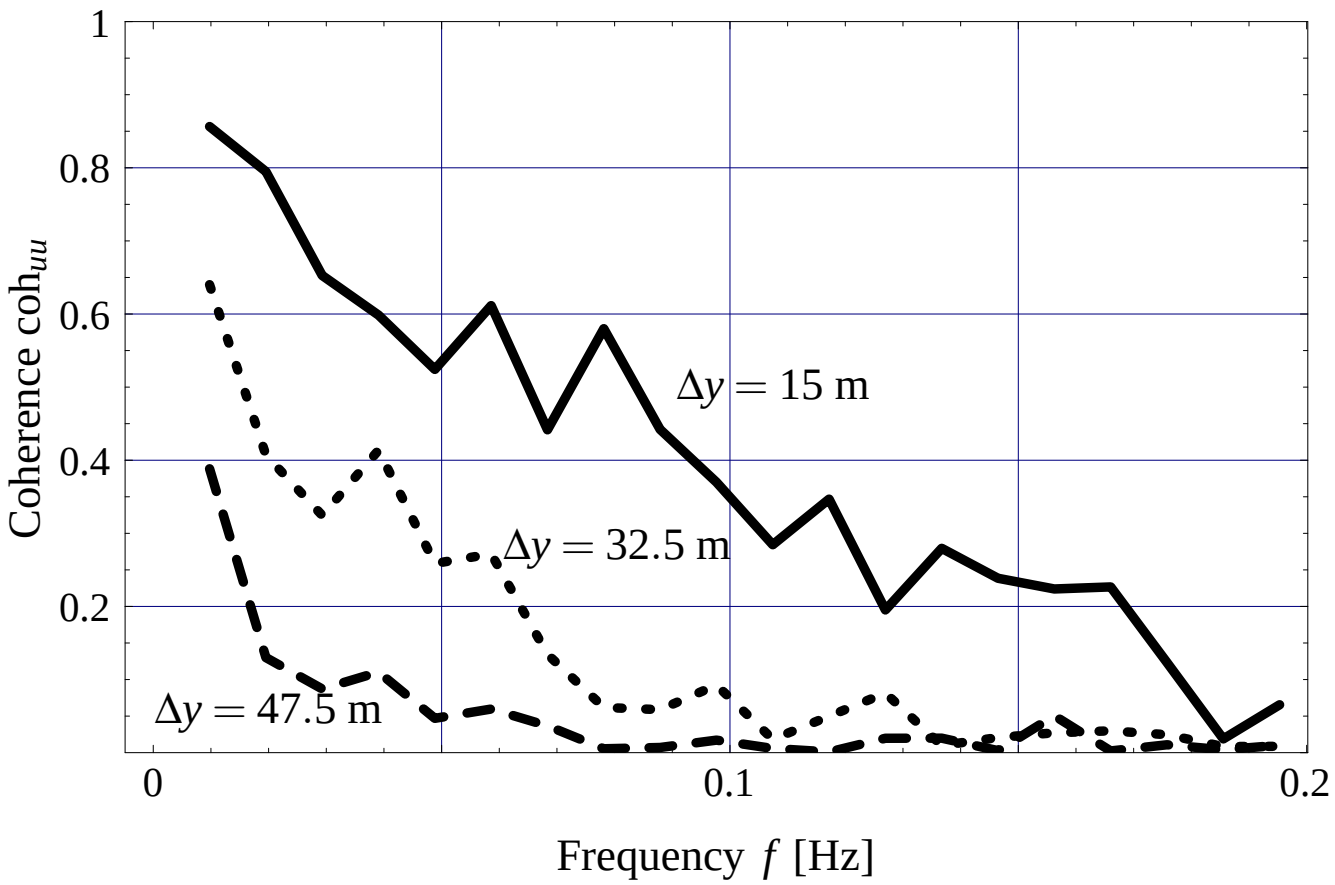
Spectra and one-point cross-spectra



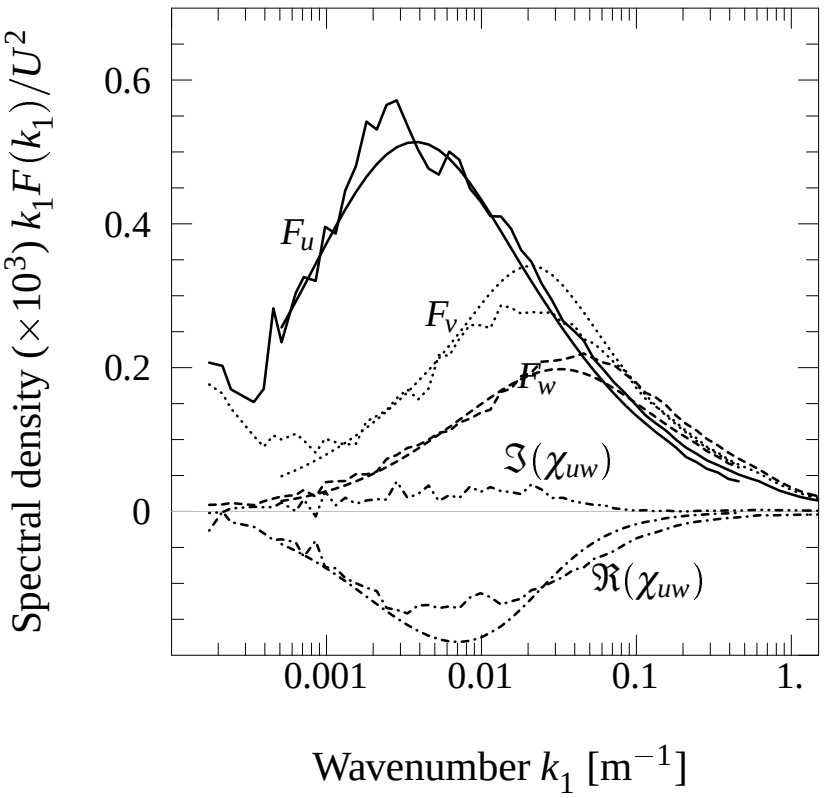
One 2 hour run.


Wind speed $(U+u)$ [m/s]
24
20
24
20
24
20
20
40
60
80
100
120
Time [s]

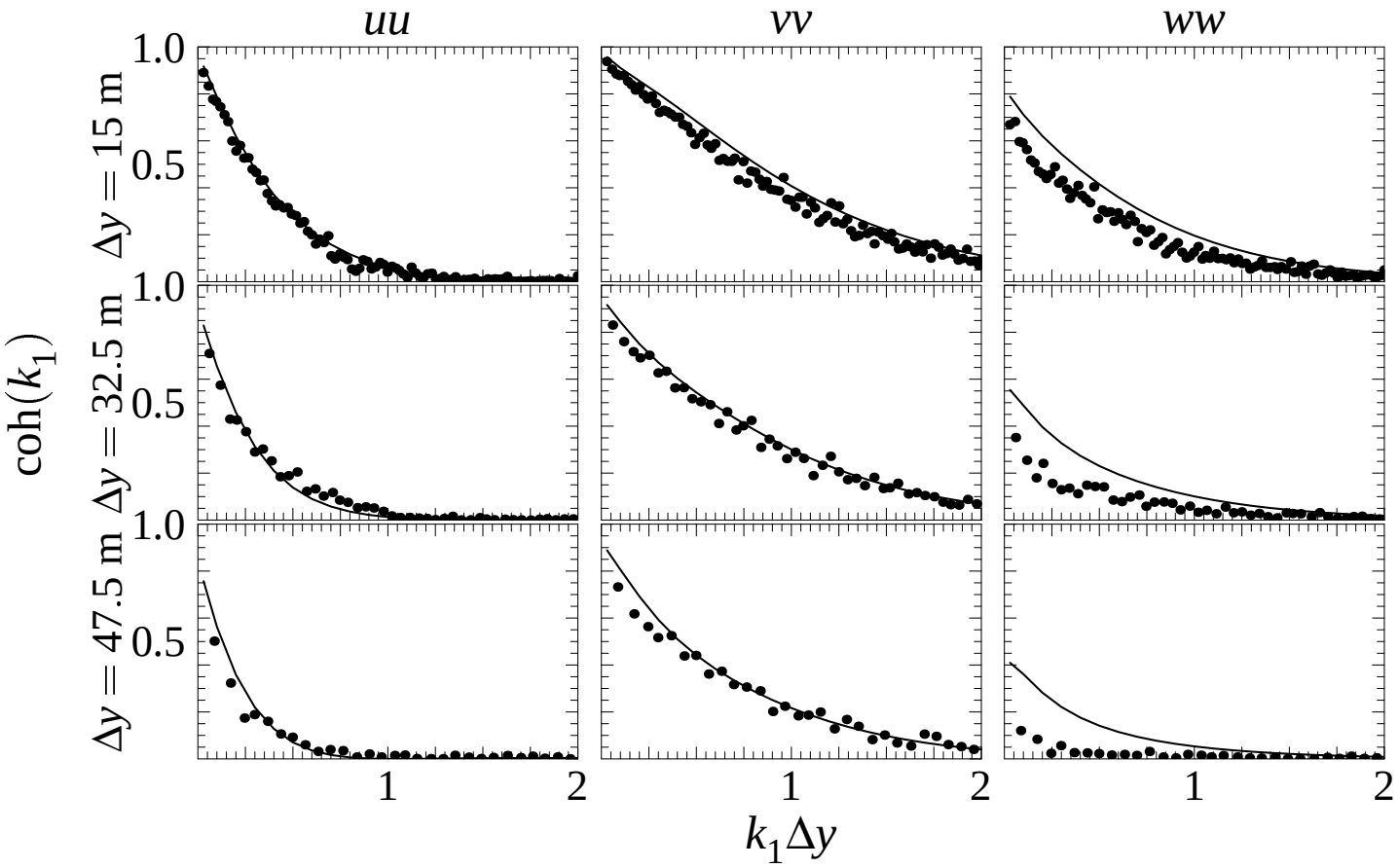
Normalized two-point cross-spectra: coherences



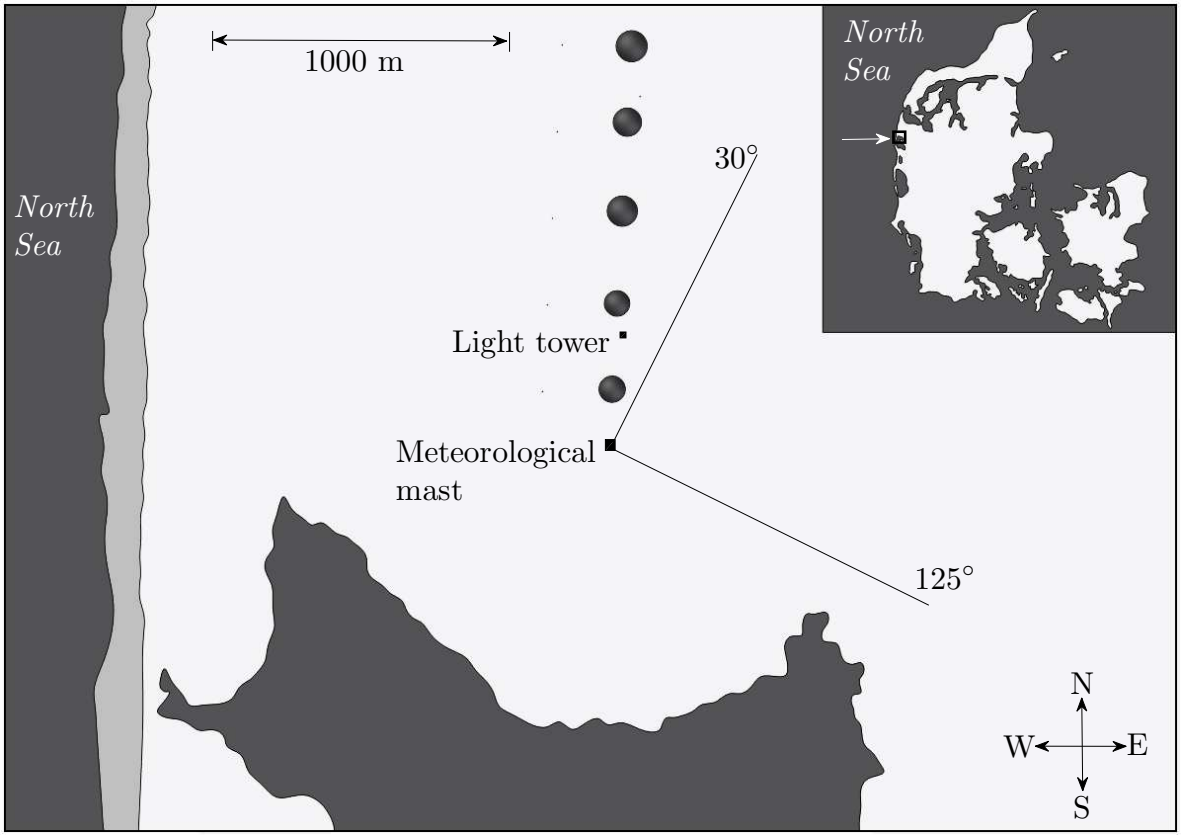
Average of all neutral runs



Predicted coherences



Site and measurements at Høvsøre



- 20 Hz Sonics at 10, 20, 40, 60, 80. 100 and 160 m
- 10-min time series collected for ∼1 year

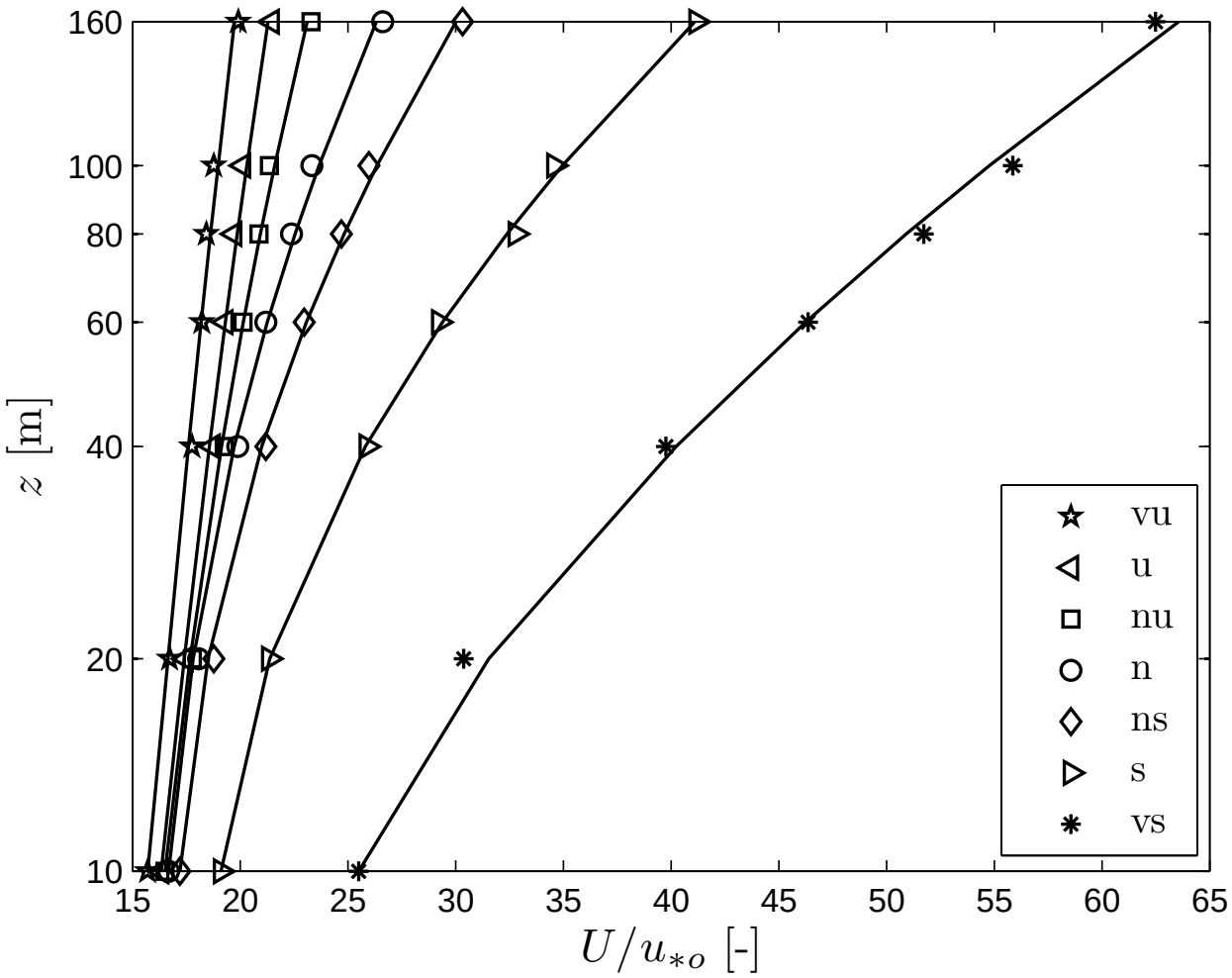
Results: diabatic observations

Obukhov length interval [m]	Atmospheric stability class	L [m]	u_{*o} [m s^{-1}]	z_o [m]	z_i [m]	No. of 10-min data
$-100 \leq L \leq -50$	Very unstable (vu)	-74	0.35	0.013	600	397
$-200 \leq L \leq -100$	Unstable (u)	-142	0.41	0.012	600	459
$-500 \leq L \leq -200$	Near unstable (nu)	-314	0.40	0.012	550	292
$\lvert L \rvert \geq 500$	Neutral (n)	5336	0.39	0.013	488	617
$200 \leq L \leq 500$	Near stable (ns)	318	0.36	0.012	451	439
$50 \leq L \leq 200$	Stable (s)	104	0.26	0.008	257	1144
$10 \leq L \leq 50$	Very stable (vs)	28	0.16	0.002	135	704

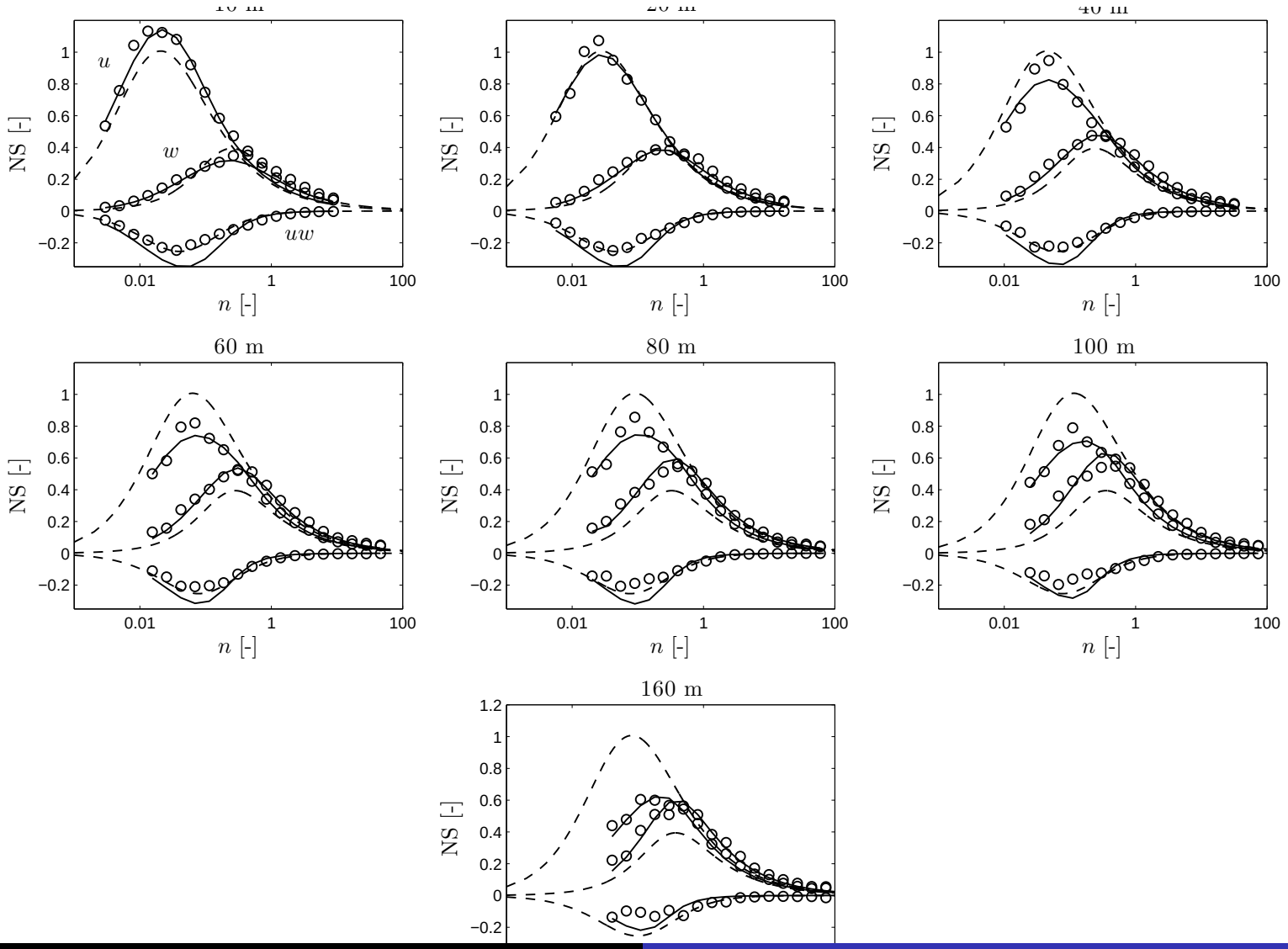
- $z_i = C \frac{u_{*o}}{|f_c|}$ for neutral and stable conditions
- z_o from

$$U = \frac{u_{*o}}{\kappa} \left[\ln \left(\frac{z}{z_o} \right) - \psi_m \right] \tag{3}$$

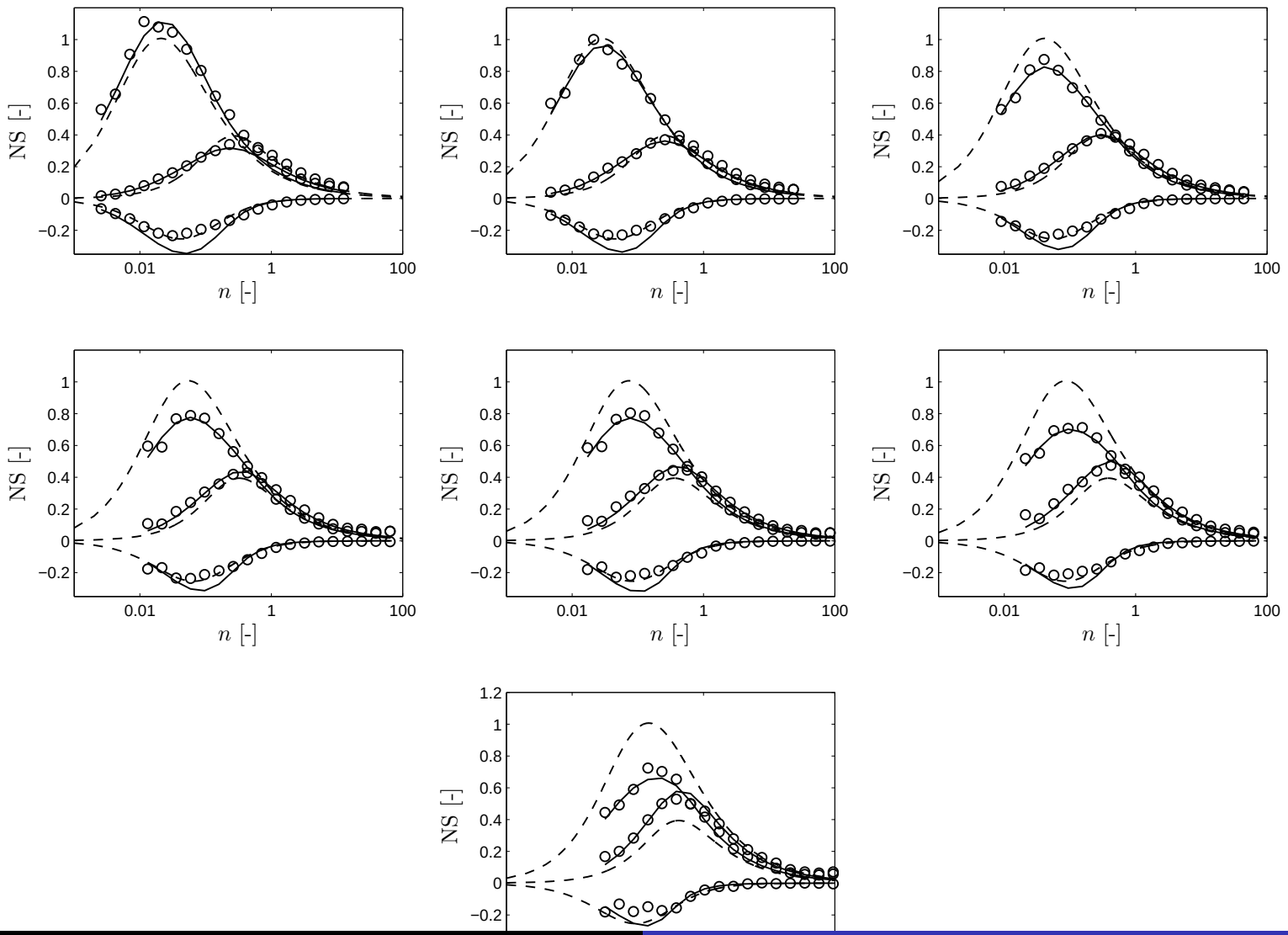
Results: Mean wind profiles for various atmospheric stabilities



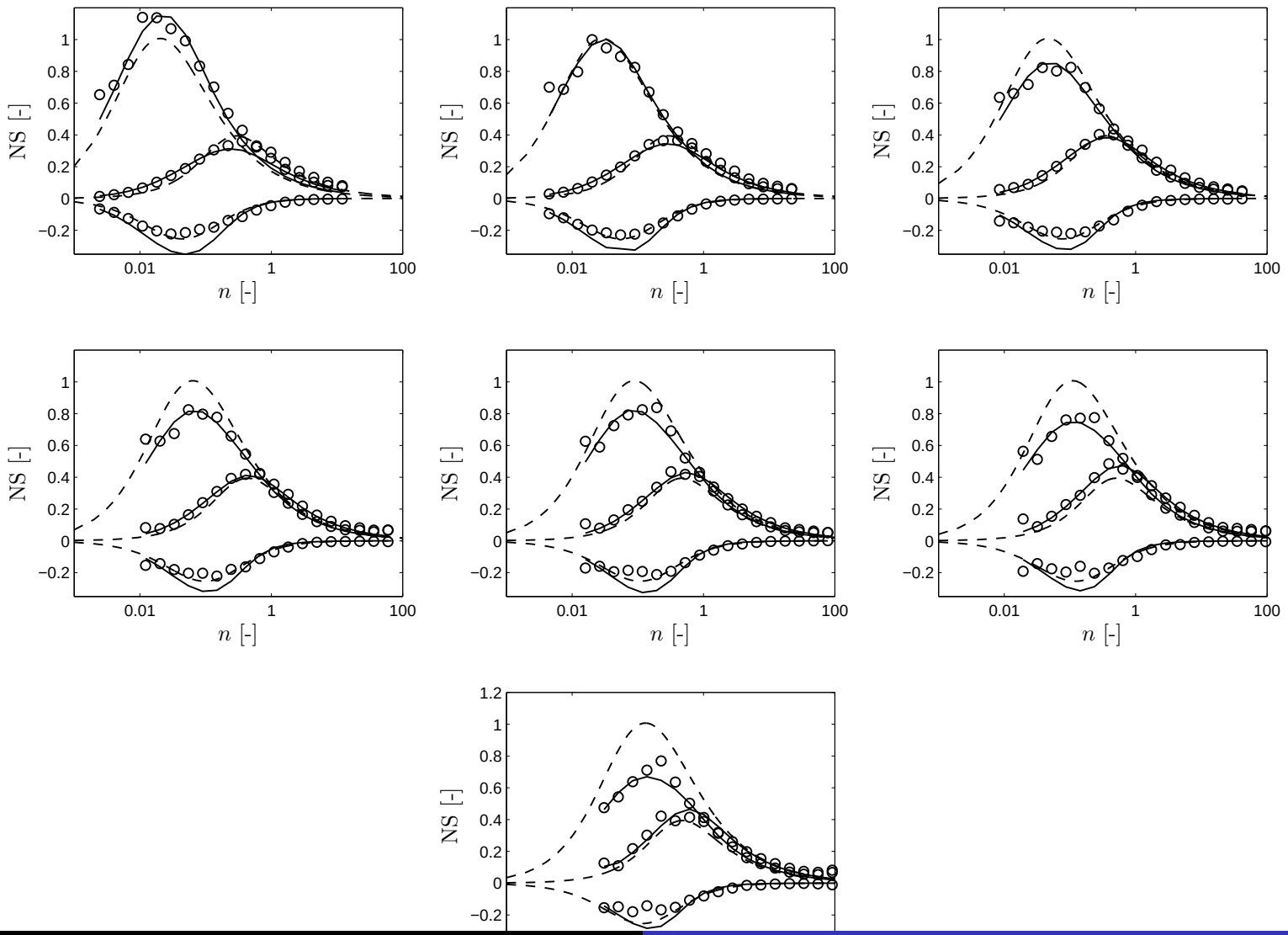
Very unstable spectra



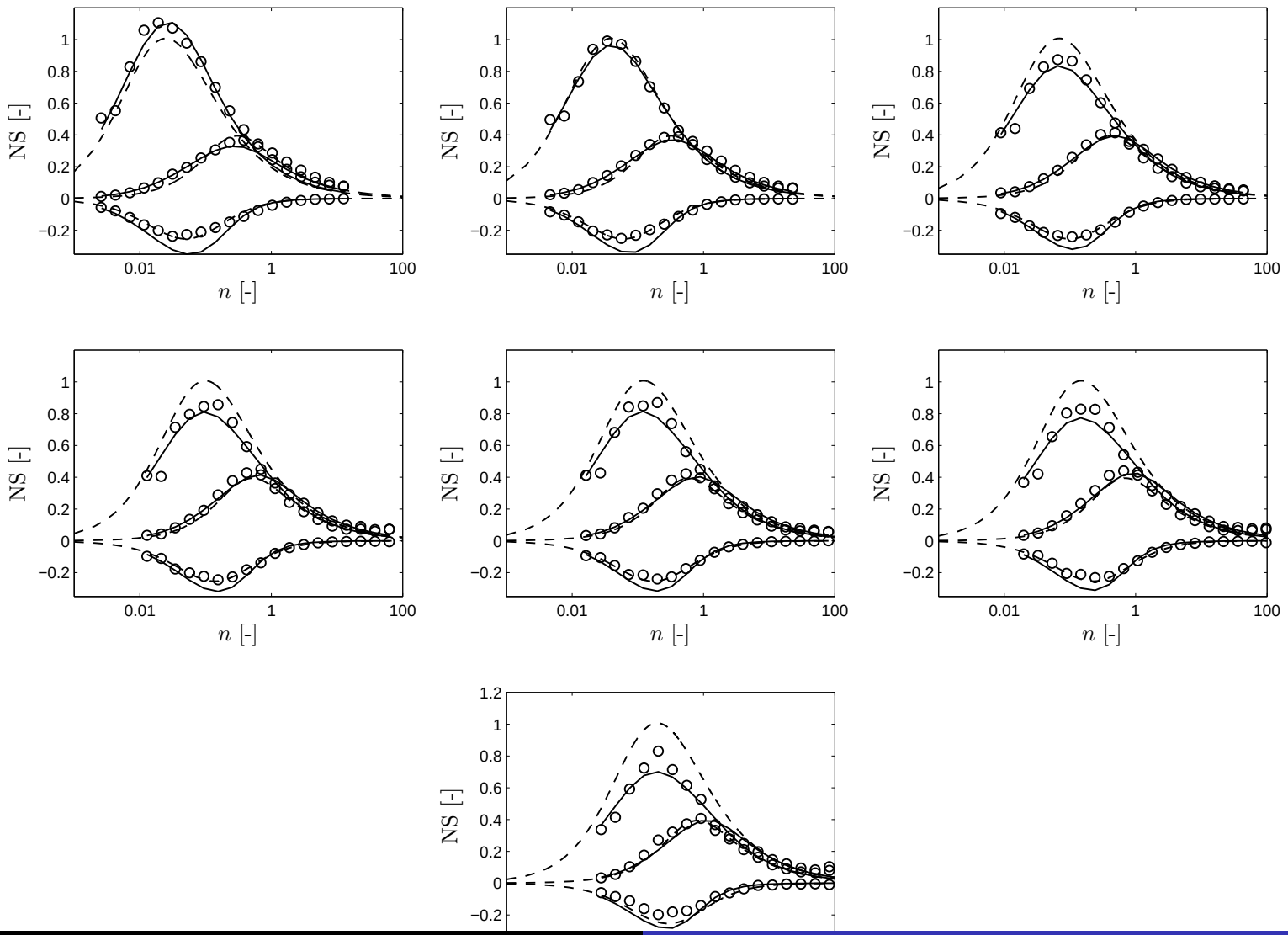
Unstable spectra



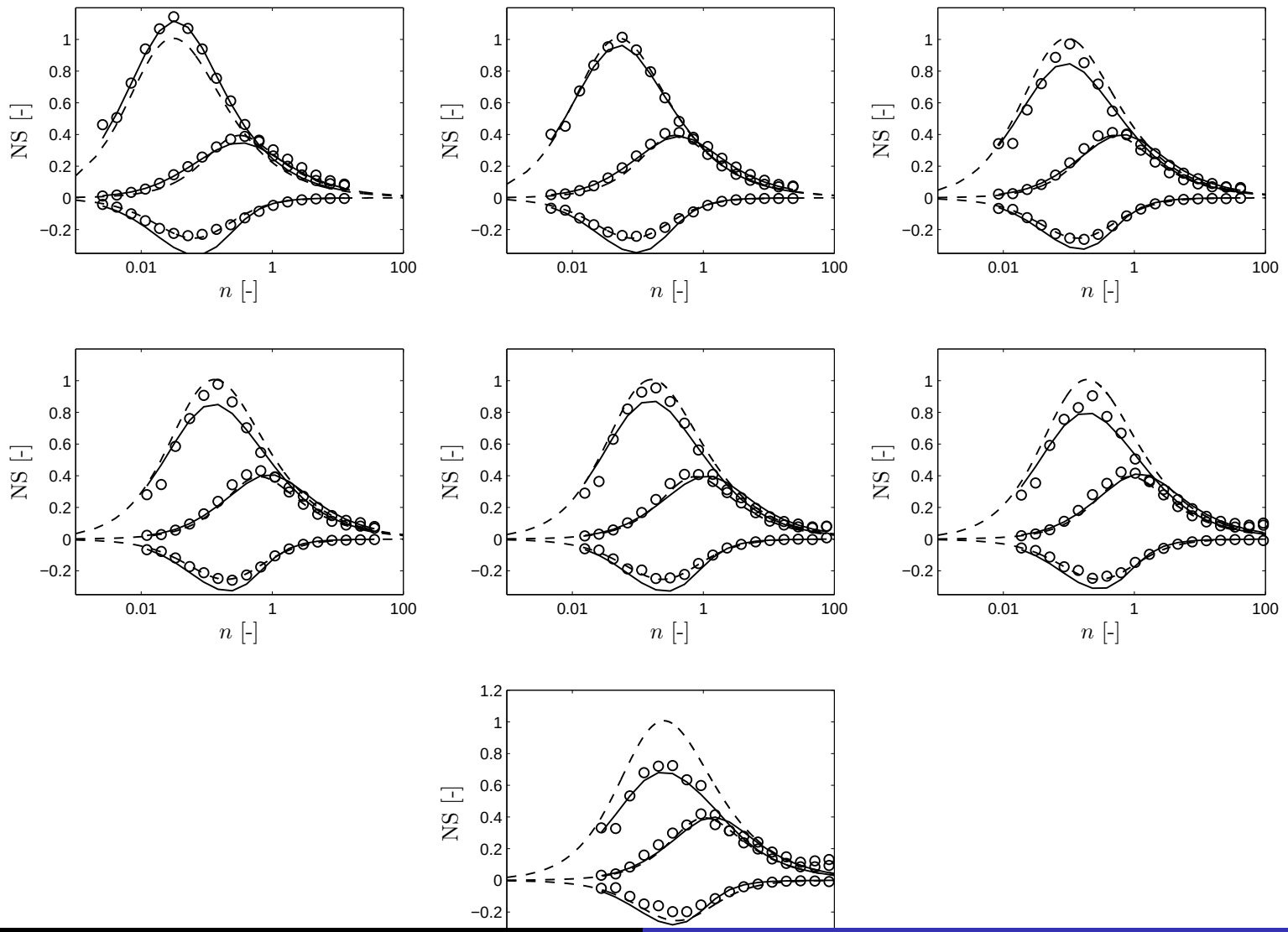
Near unstable spectra



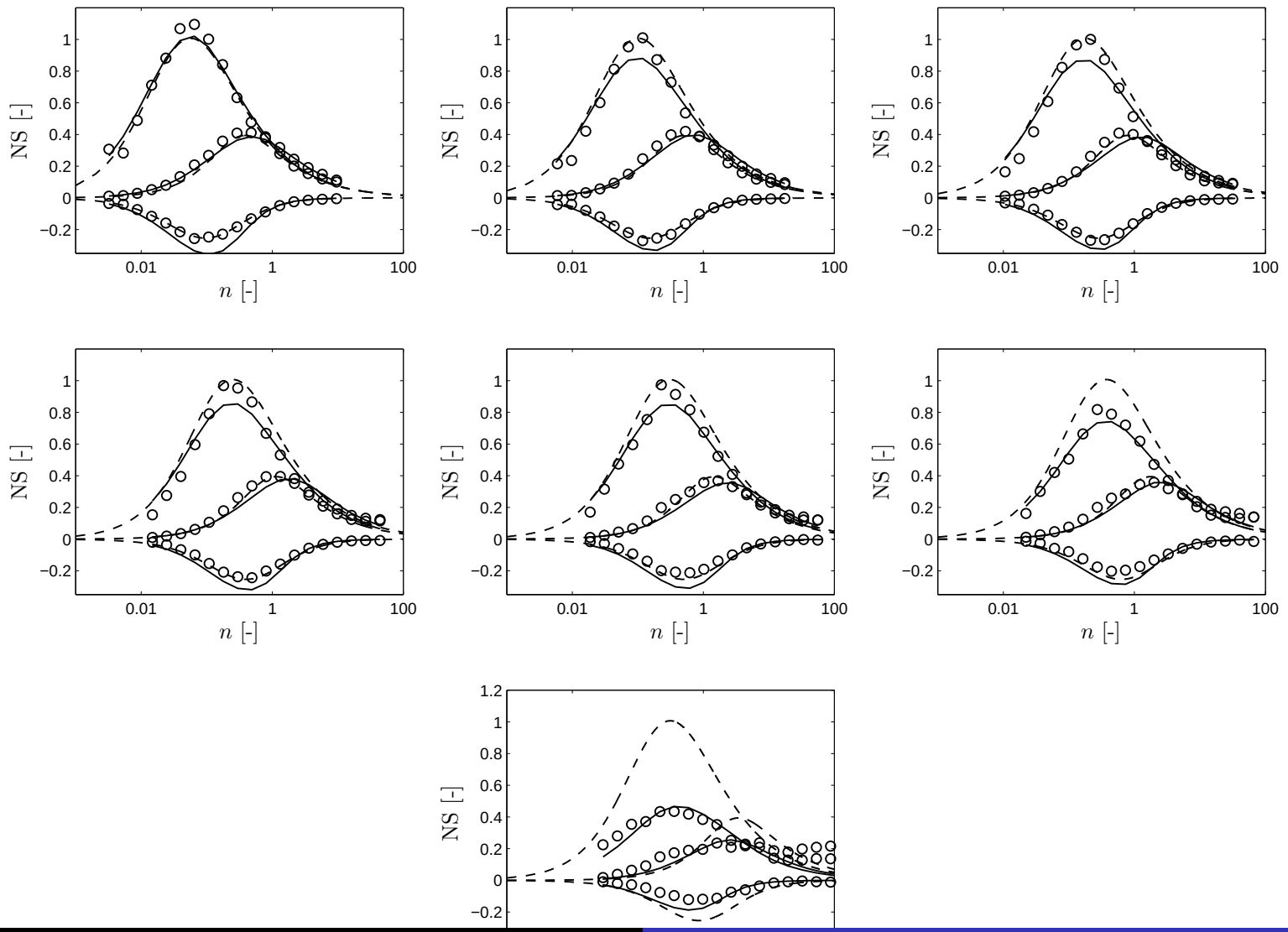
Neutral spectra



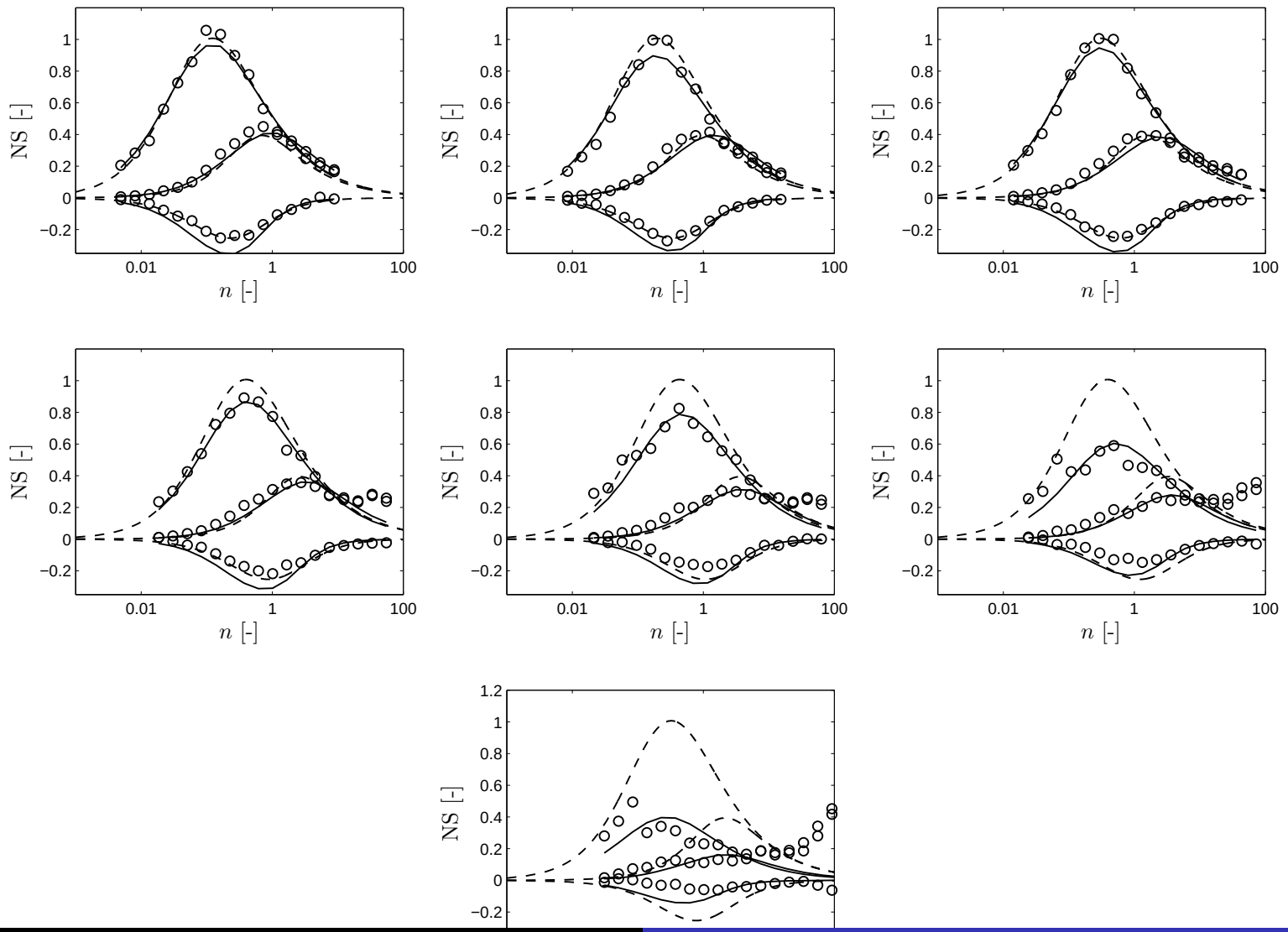
Near stable spectra



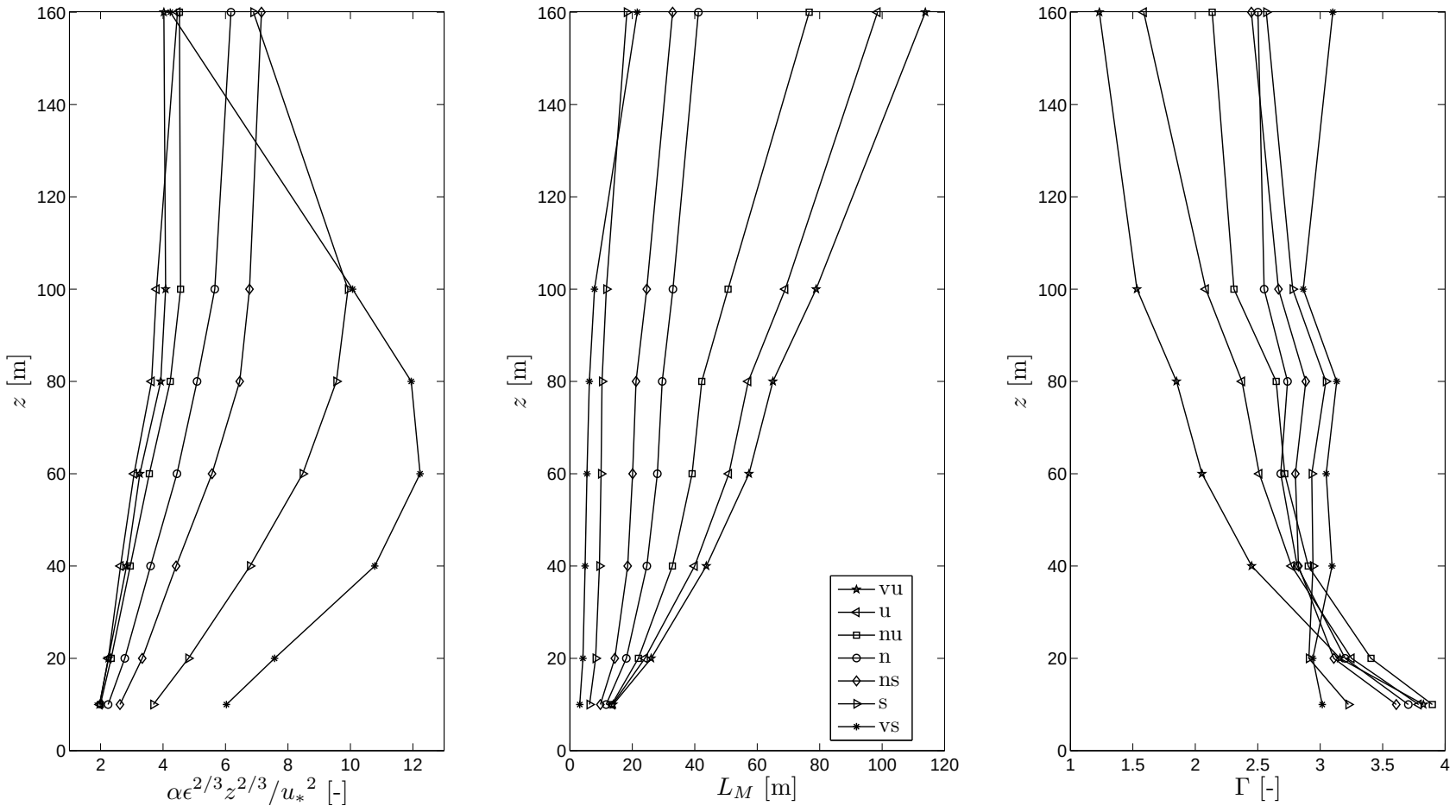
Stable spectra



Very stable spectra



Mann (1994) model parameters



Turbulence in complex terrain

Two complications compared to flat terrain:

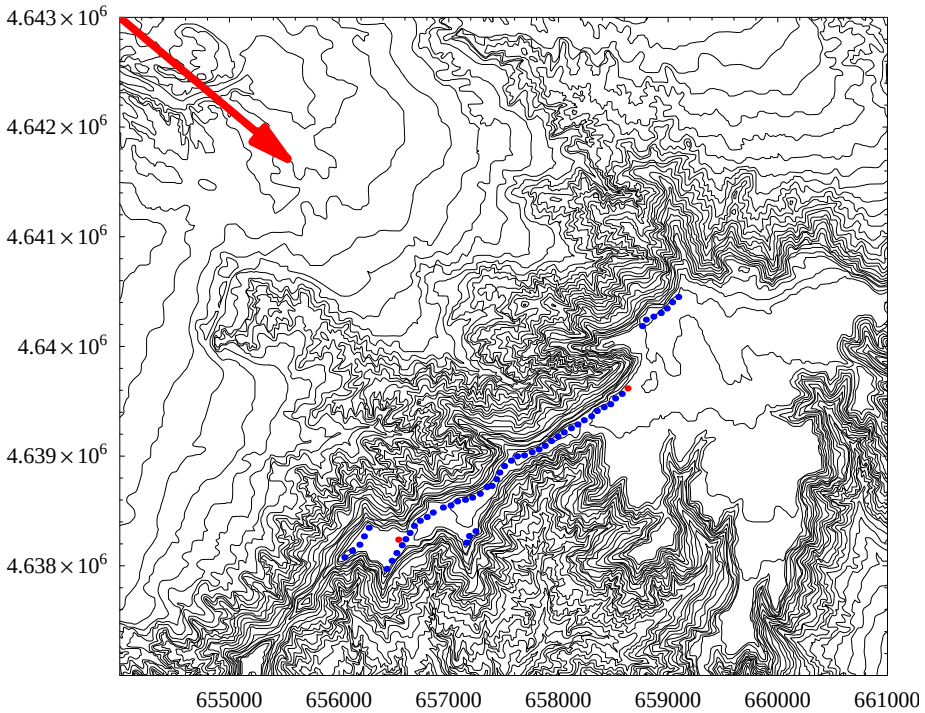
- varying roughness
- orography

These are incorporated in WAsP Engineering, but only for moderately complex terrain: The basic limitation:

> *If there are extended areas within a radius of 3 to 4 km from the site of interest with slopes of more than 20° to 25°, then turbulence may be much larger than calculated. In this situation measurements at the site may be required.*

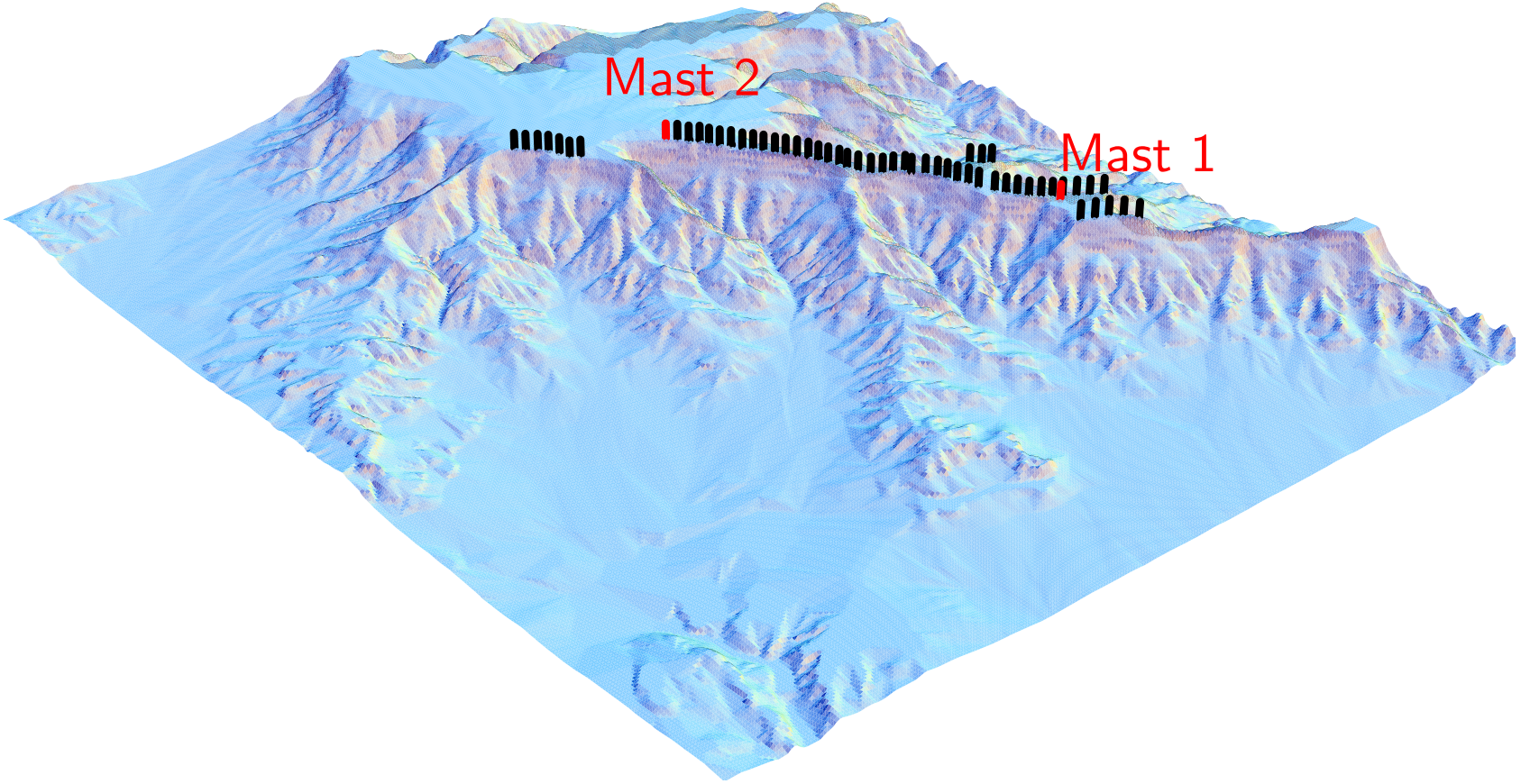
Really complex terrain

A table mountain in Spain

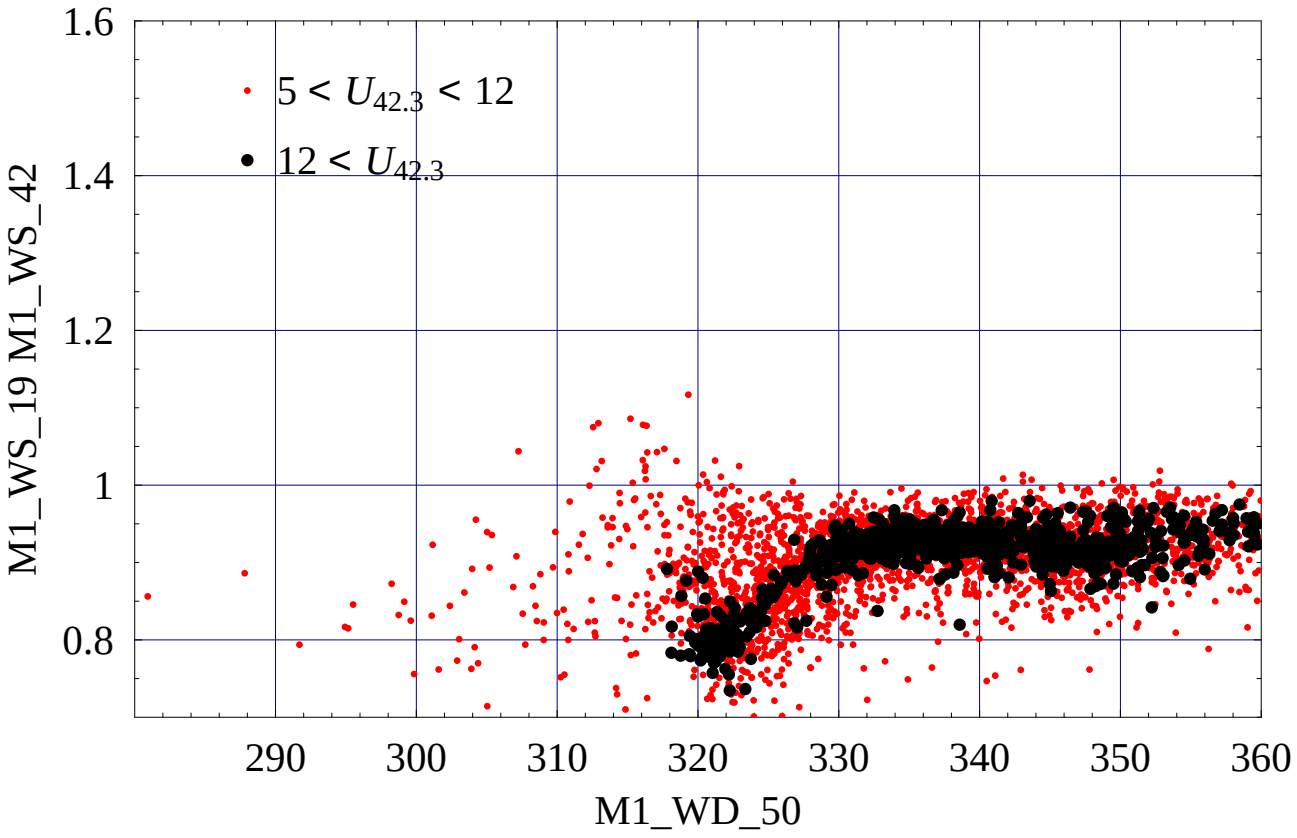


2 masts and 54 turbines.

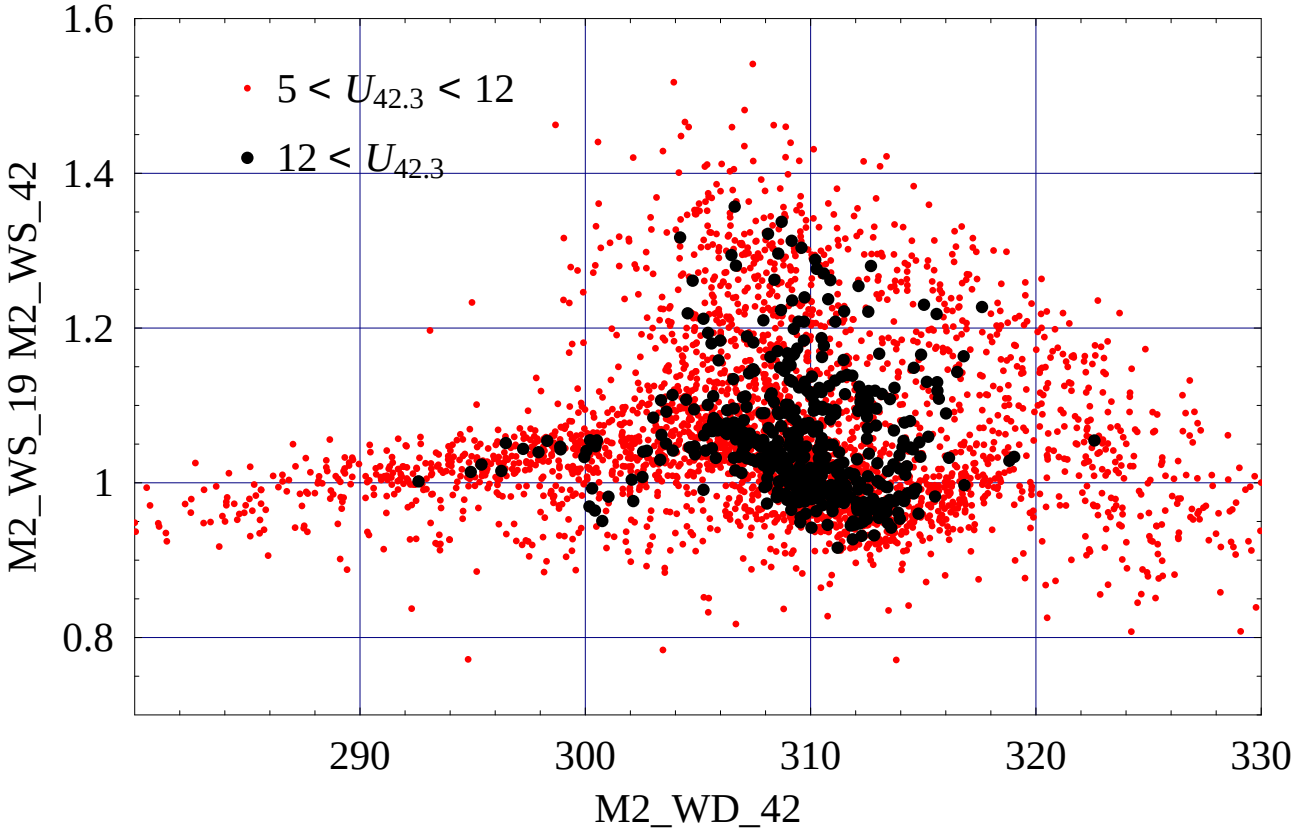
Stretched a factor of 2



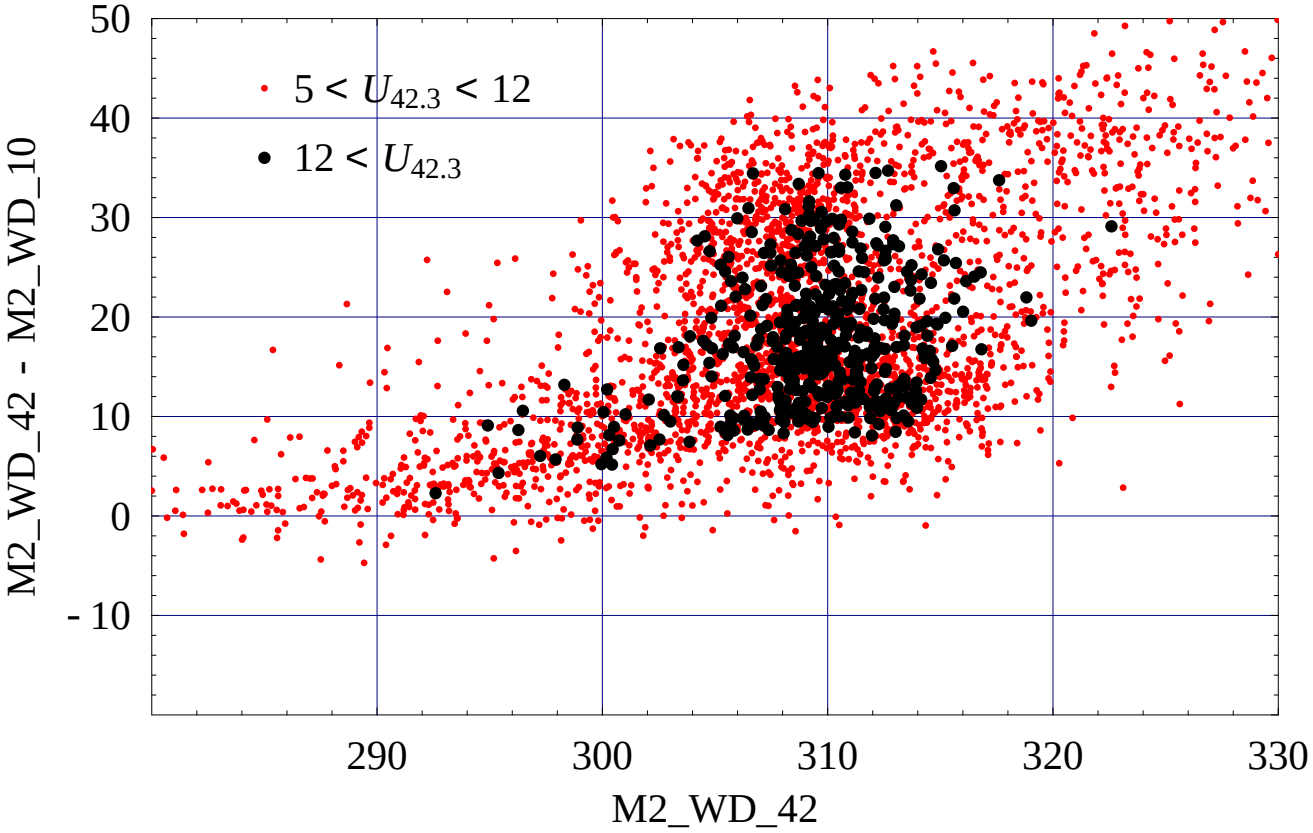
Mast 1: Ratio of between wind speed at lower tip and hub height



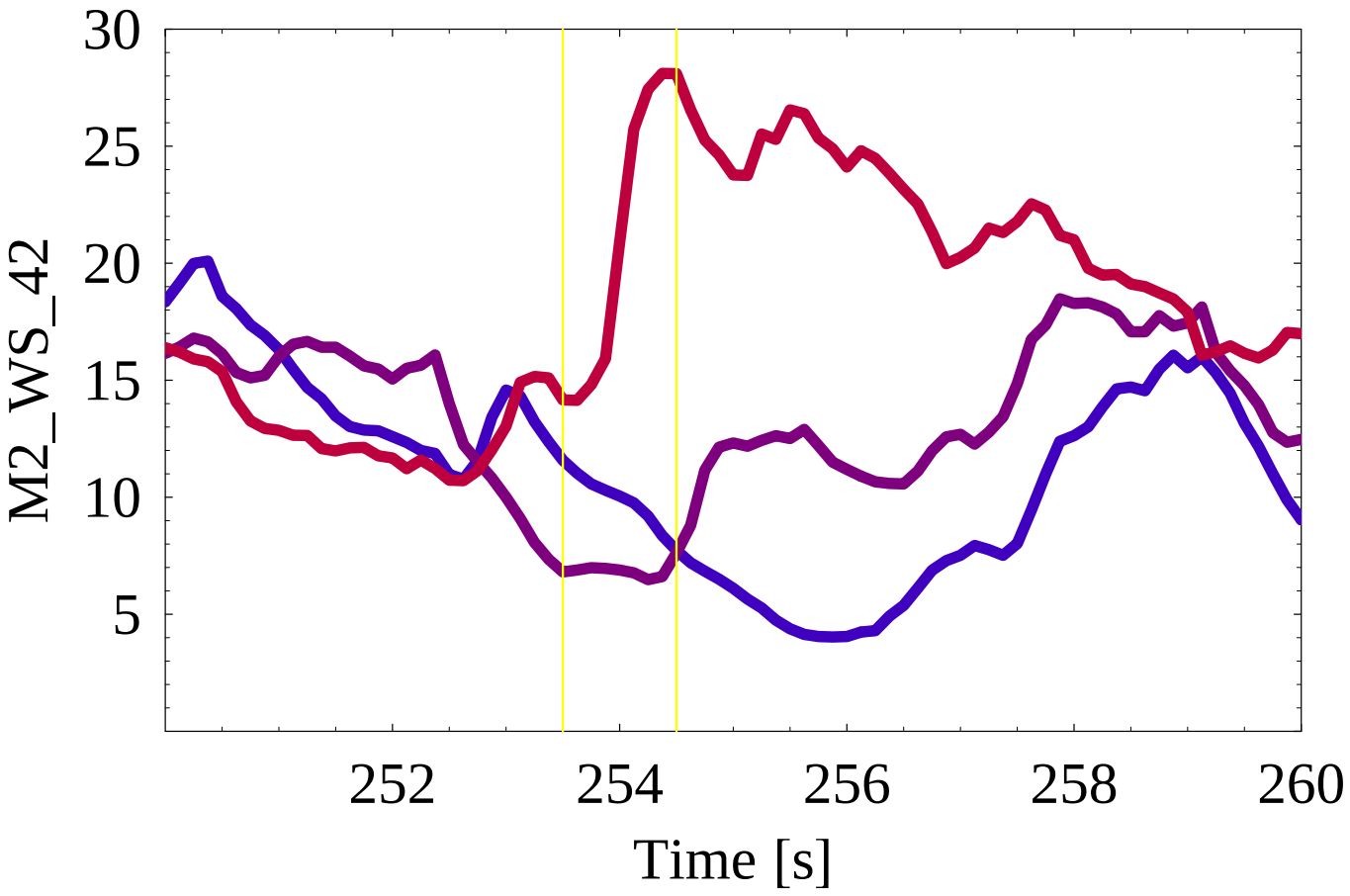
Mast 2: Ratio of wind speed at lower tip and hub height



Mast 2: Diff. in wind direction at hub and lower tip height



Instantaneous shears



Mast measurements at z =19, 31, and 42 m (hub). Direction diff. up to 90°.

Outline of Part II: Turbulence simulation

4. Recapitulation of Part I

Outline of Part II: Turbulence simulation

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Wind statistics over flat terrain

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- Stationarity often OK
- Homogeneity often OK, at least in the horizontal. $dU/dz = u_*/\kappa z$
- Taylor's hypothesis OK for most practical purposes

Gauss OK $\Rightarrow$ 2. order statistics is everything

Taylor's frozen turbulence hypothesis

$$\tilde{\mathbf{u}}(x, y, z, t) = \tilde{\mathbf{u}}(x - Ut, y, z, 0)$$

Need to know: covariance tensor $R_{ij}(\mathbf{r}) = \langle u_i(\mathbf{x}) u_j(\mathbf{x} + \mathbf{r}) \rangle$, the spectral tensor $\Phi_{ij}(\mathbf{k})$

or

One-dimensional spectrum $F_i(k_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{ij}(x, 0, 0) \mathrm{e}^{-\mathrm{i}k_1 x} \mathrm{d}x$, and cross-spectra $\chi_{ij}(k_1, \Delta y, \Delta z)$ or the coherence $\mathrm{coh}_{ij}(k_1, \Delta y, \Delta x)$

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- Large scale anisotropy, $\sigma_u > \sigma_v > \sigma_w$, $\langle uw \rangle < 0$, $L_u > L_v > L_w$
- Notice: Only *three* parameters: L, $\alpha\varepsilon^{2/3}$, and Γ.

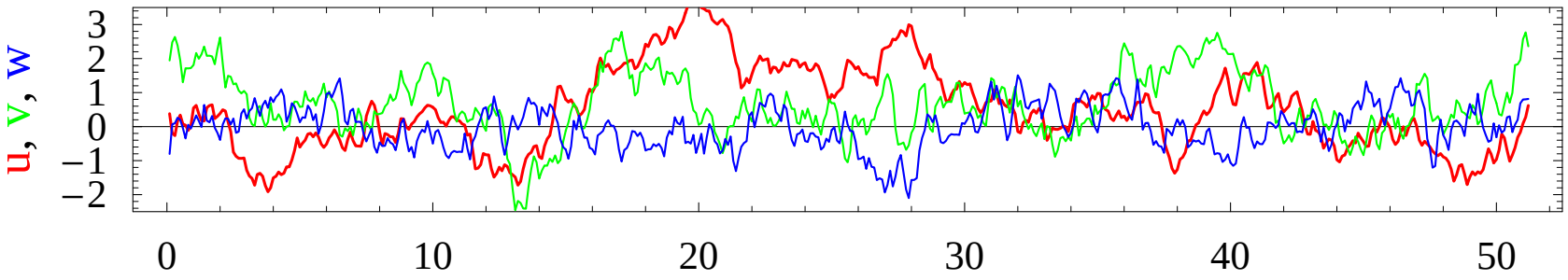
Fourier simulation

$$\mathbf{u}(\mathbf{x}) = \sum \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\mathbf{u}(\mathbf{k})$$

where

$$\left\langle u_i(\mathbf{k}) u_j^*(\mathbf{k}') \right\rangle \propto \Phi_{ij}(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}')$$

Wind simulation



u v w

Constrained simulation

Simulate $\mathbf{v} = \{v_1, v_2, ...\}$ provided that $\sum_{i=1}^{n} \varphi_i v_i = \boldsymbol{\varphi} \cdot \mathbf{v} = f_c$.

Examples:

1. Gust in the middle of the time series: $\varphi_i = 1$ for $i = n/2$ and zero elsewhere. I.e. the condition is $v_{n/2} = f_c$.

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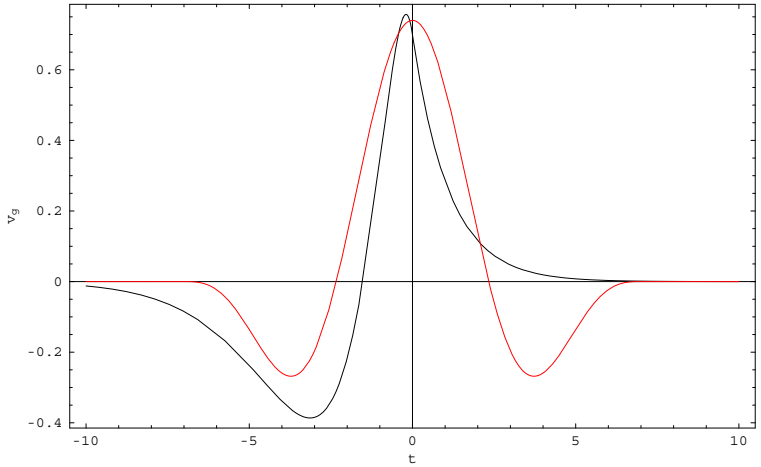
3. Choose $\boldsymbol{\varphi}$ to match time scales in the control system.



Constrained simulation

Most probable $\mathbf{v}$:

$$\begin{aligned} \text{Minimize} \quad & \frac{1}{2}\mathbf{v}\cdot\mathbf{Q}\mathbf{v} \\ \text{subject to} \quad & \varphi\cdot\mathbf{v} = f_c \end{aligned}$$

The solution is the "average gust" $\mathbf{v}_{ave}$:

$$\mathbf{v}_{ave} = \frac{f_c\mathbf{R}\varphi}{\varphi\cdot\mathbf{R}\varphi}$$

Ex: In the continuum limit suppose $\varphi(t) = \delta(t)$. Then

$$v_{ave}(t) = \langle v(t)|v(0) = f_c\rangle = \frac{f_c}{\sigma^2}R(t)$$

Constrained Gaussian simulation

To simulate $\mathbf{v}$ under the constraint $\varphi \cdot \mathbf{v} = f_c$:

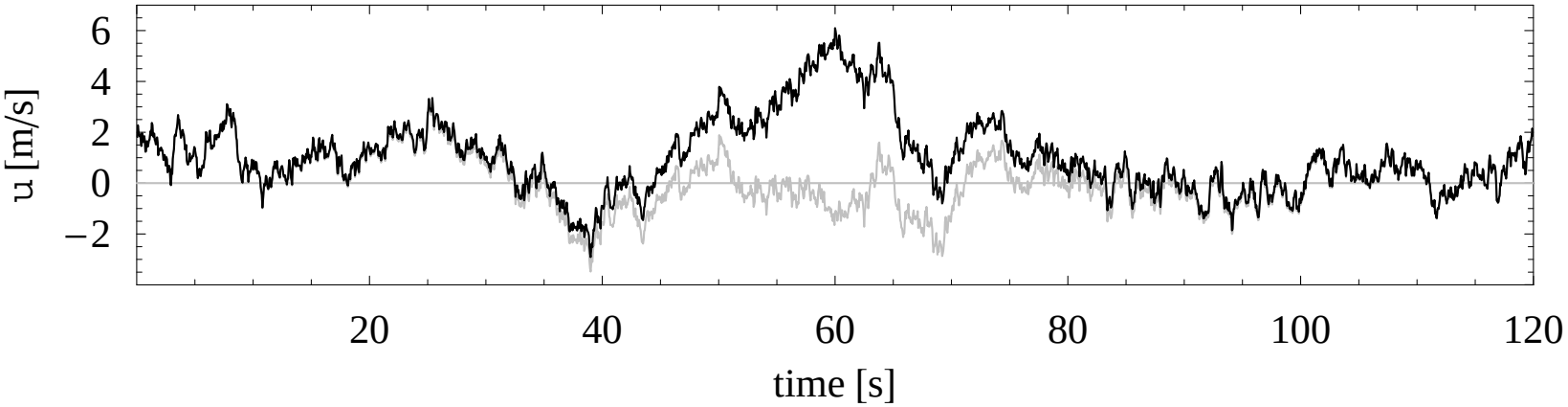
1. Simulate stationary $\mathbf{v}_s = \{v_{s1}, v_{s2}, ...\}$ with no constraints.
2. calculate

$$\mathbf{v} = \mathbf{v}_s + \frac{f_c - \varphi \cdot \mathbf{v}_s}{\varphi \cdot \mathbf{R}\varphi} \mathbf{R}\varphi$$

$\mathbf{v}$ fulfils the constraint $\varphi \cdot \mathbf{v} = f_c$ and is generated at the right rate.

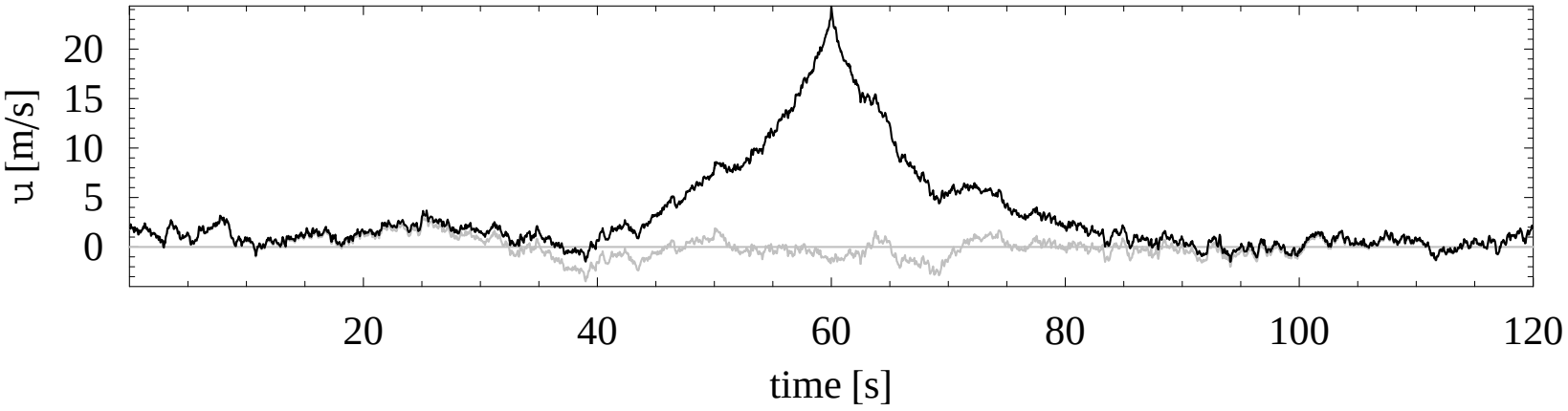
Example: "Velocity gust"

The stationary gaussian turbulence $\mathbf{v}_s$ is simulated assuming a von Kármán spectrum. The correlation $\mathbf{R}$ is derived from this spectrum.



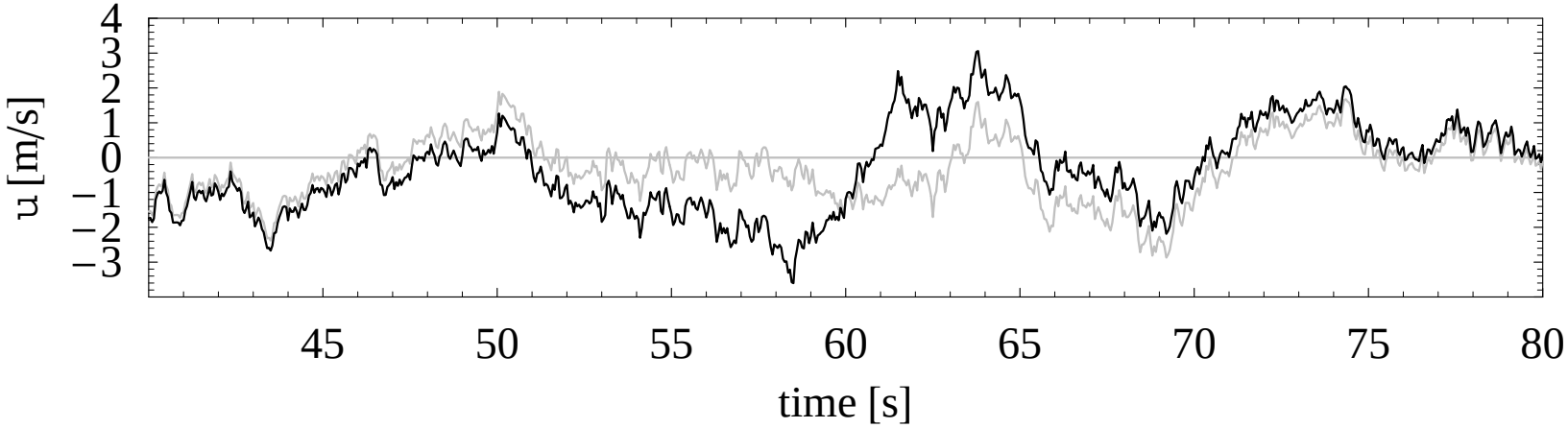
$\varphi(t) = \delta(t - 60\mathrm{s})$. f_c is 5 times the standard deviation.

Larger velocity gust



$\varphi(t) = \delta(t - 60\mathrm{s})$. f_c is 20 times the standard deviation.

Example: "Velocity jump"



jump of size $\Delta u(t) \equiv u(t + \Delta t/2) - u(t - \Delta t/2) = 5\sigma_u$ and with $\Delta t = 3$ s

Generalizations of constrained simulation

1. More than one component of the velocity.

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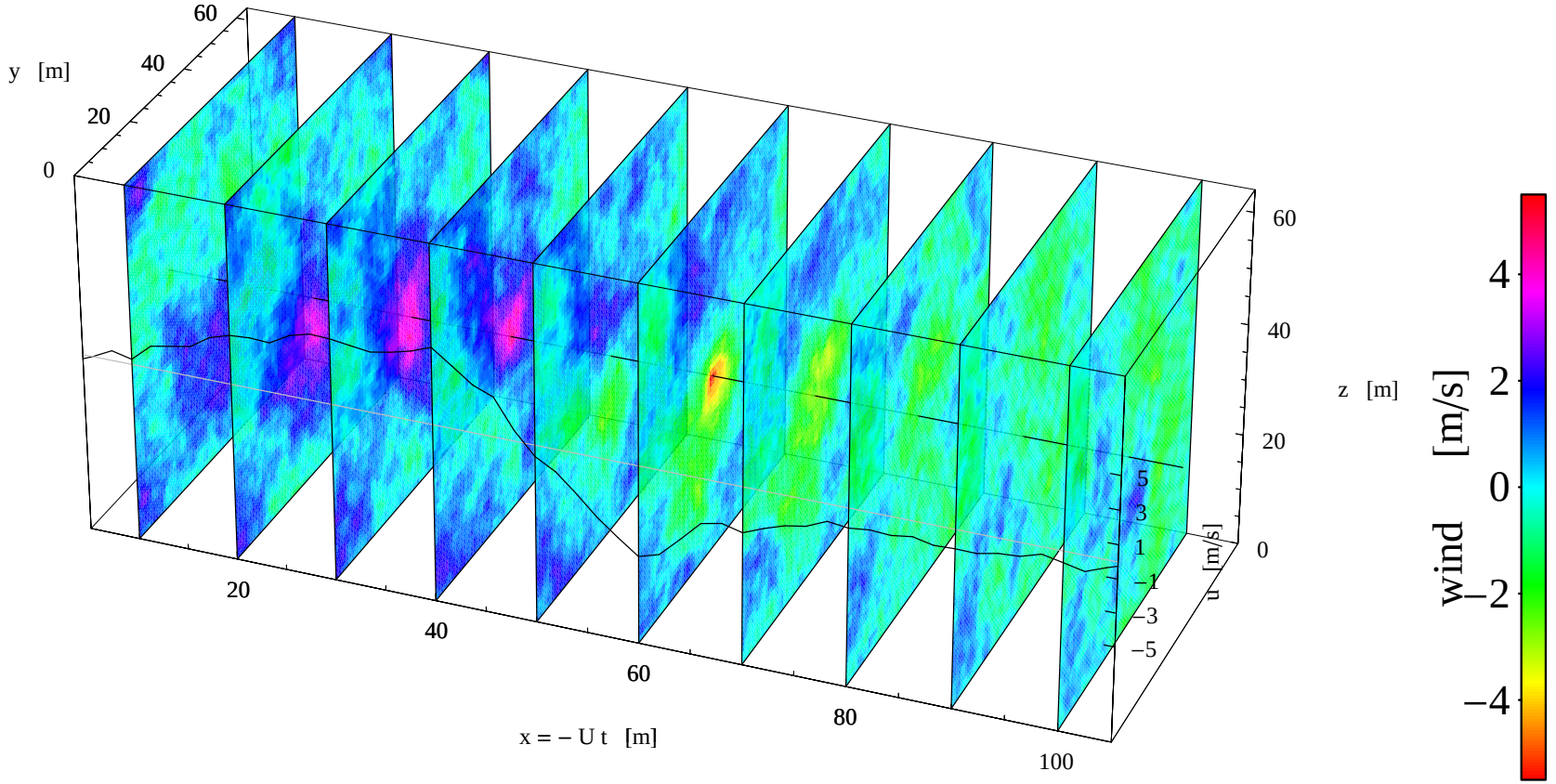
3. Time series $\longrightarrow$ spatial fields

Example: "Anisotropic 3D jump" (1:2)

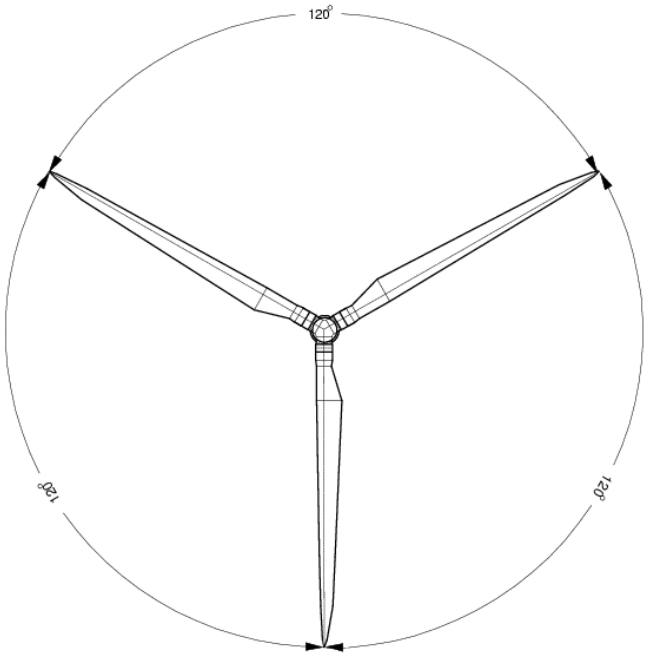


u-turbulence simulation of velocity jump based on the spectral tensor by Mann

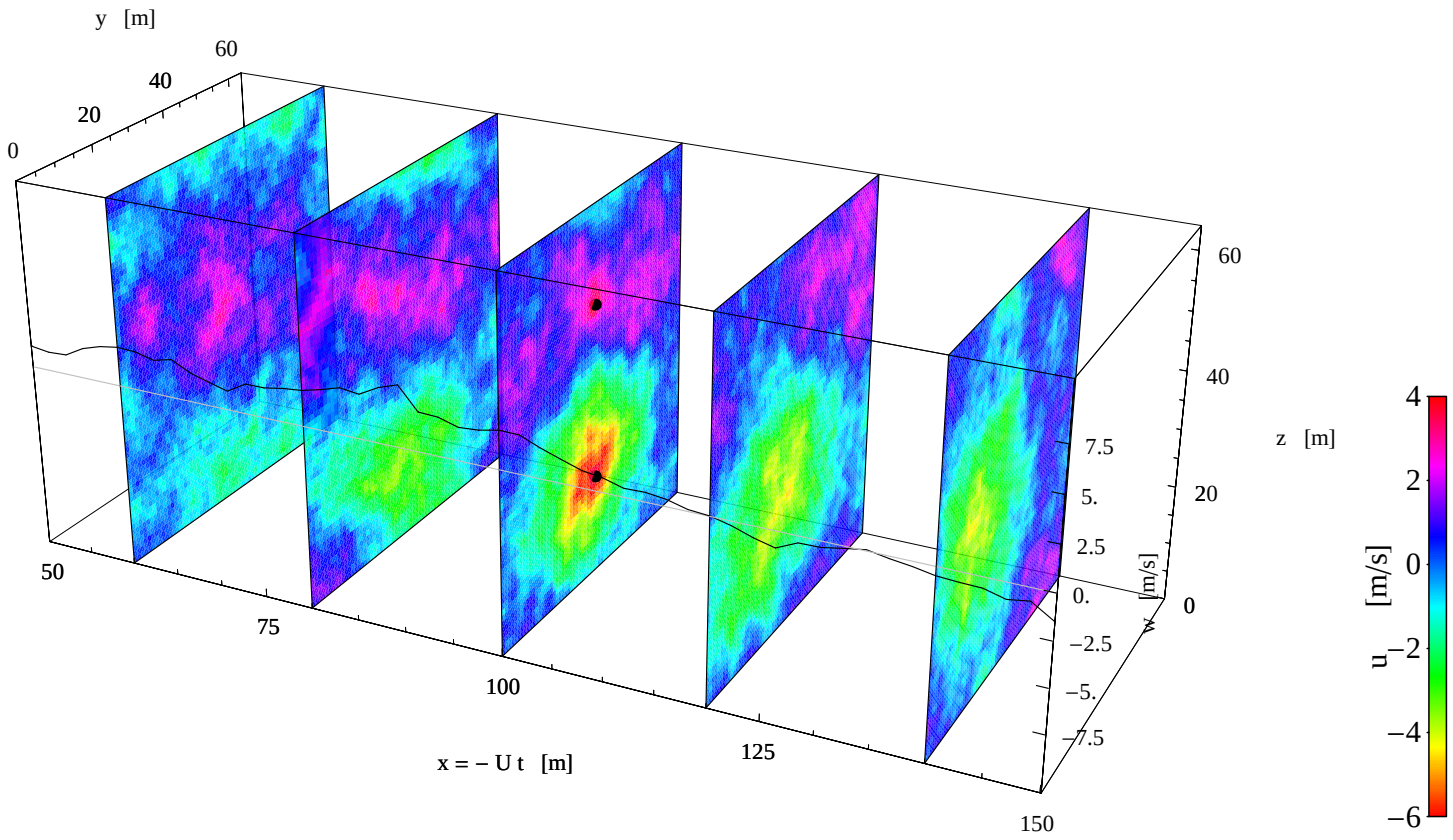
Example: "Strong velocity shear" (1:2)

Shear increases

- Tilt moment on rotor
- Dynamic loads on blades

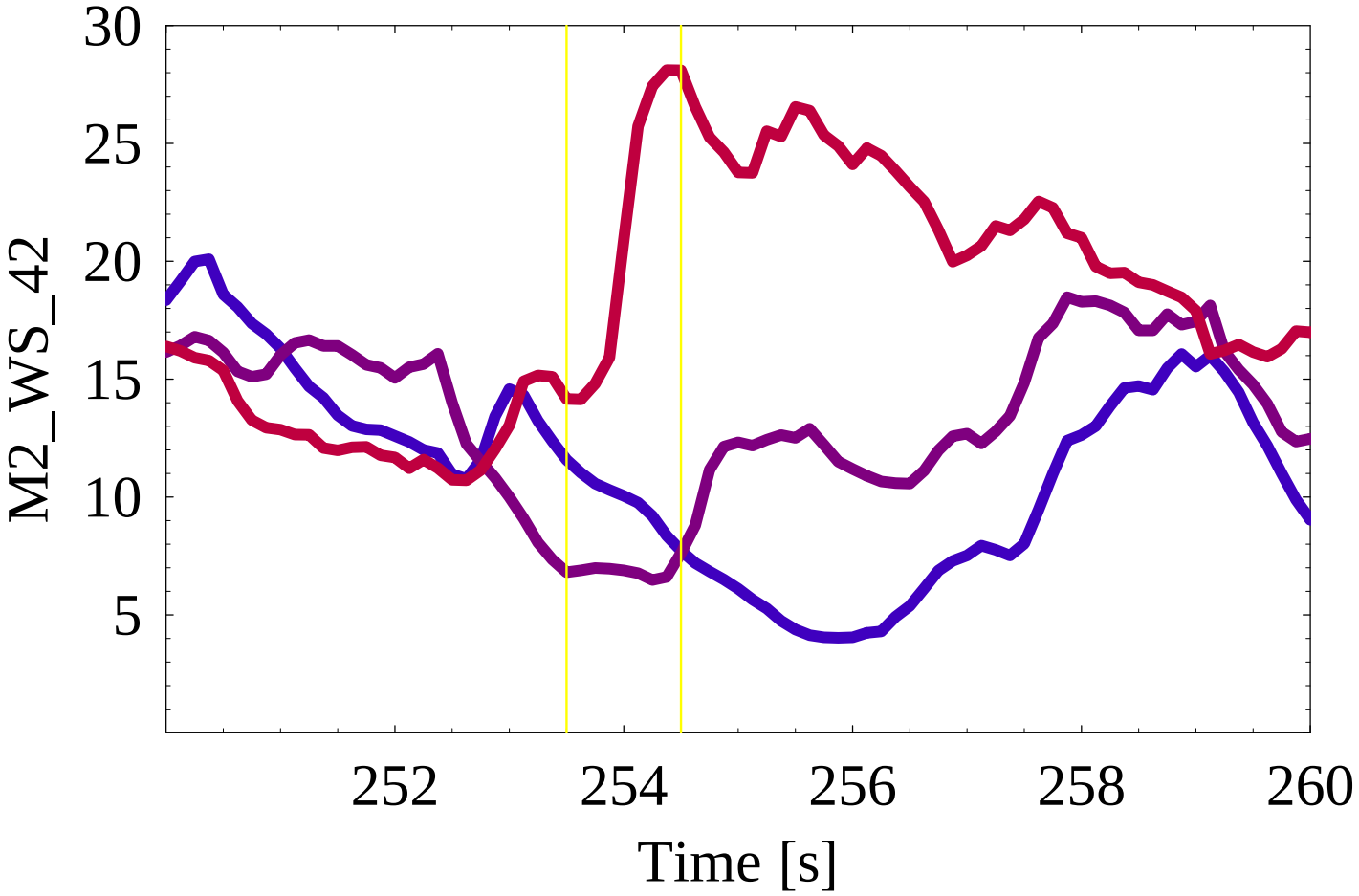


Example: "Strong velocity shear" (2:2)



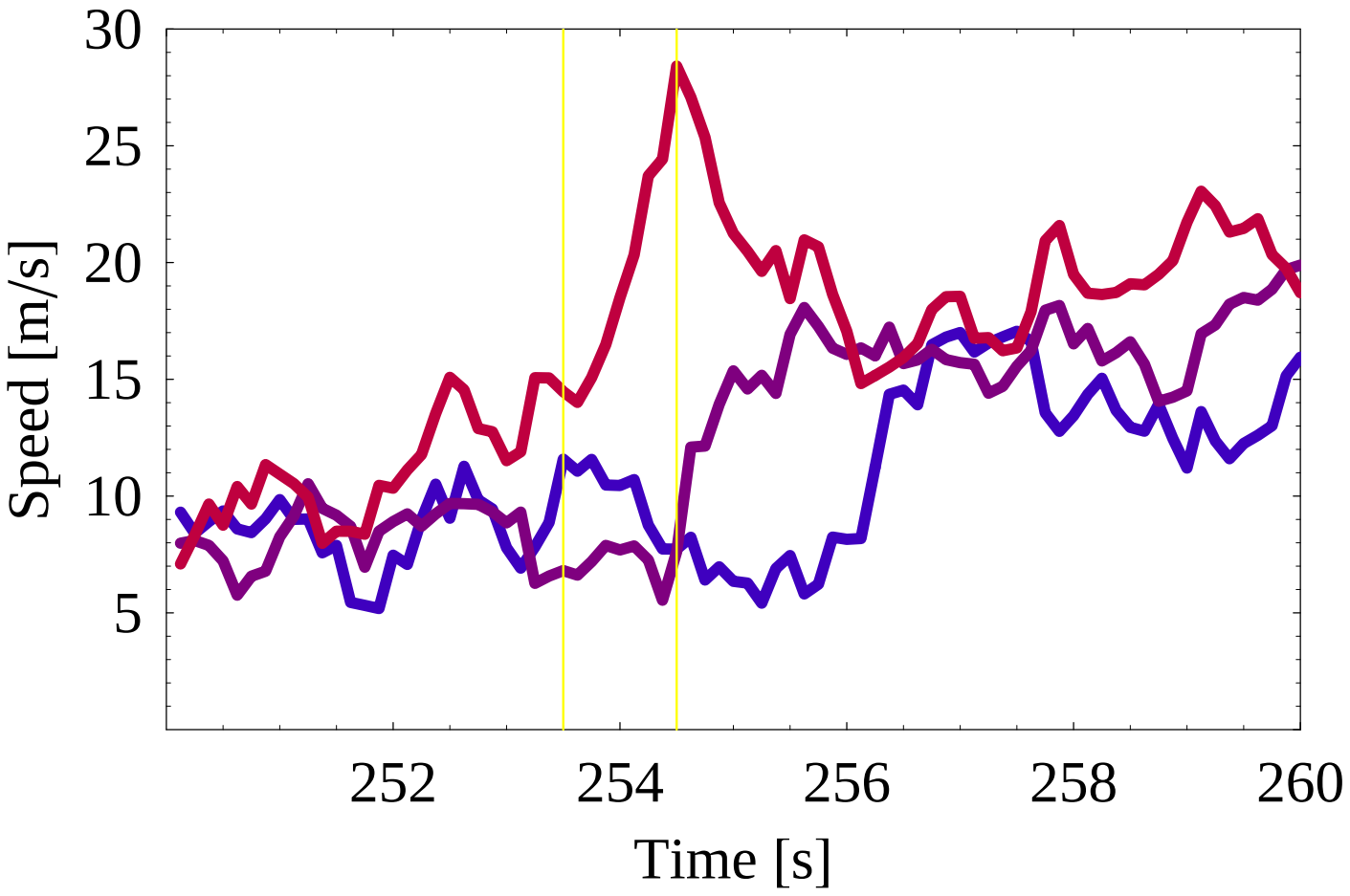
The difference in u at the two black point is 10 m/s.

Example: "Measured time series" (1:3)



Mast measurements at z =19, 31, and 42 m (hub).

Example: "Measured time series" (2:3)



Result of the constrained simulation at z =19, 31, and 42 m,...

Example: "Measured time series" (3:3)

... and the entire u-field.

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- There is a myriad of other ways to simulate extreme events...
- for example Rosales & Meneveau (2006, 2008)

Conclusion

- The spatial structure is important for loads on turbines
- Turbulence is quite well described over flat homogeneous terrain/sea under neutral atmospheric stratification.
- Turbulence is very dependent on stability and on the terrain
- There is no simple connection between the terrain and the turbulence.
- Constrained simulation can incorporate extreme events in stochastic fields

Exercise

1. Download and install the *IEC turbulence simulator* http://www.wasp.dk/Products/weng/IECturbulenceSimulator.htm
2. Simulate and display a turbulence field
3. Investigate the meaning of the many parameters