LES and Actuator Line Model

by

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Actuator disc/line models

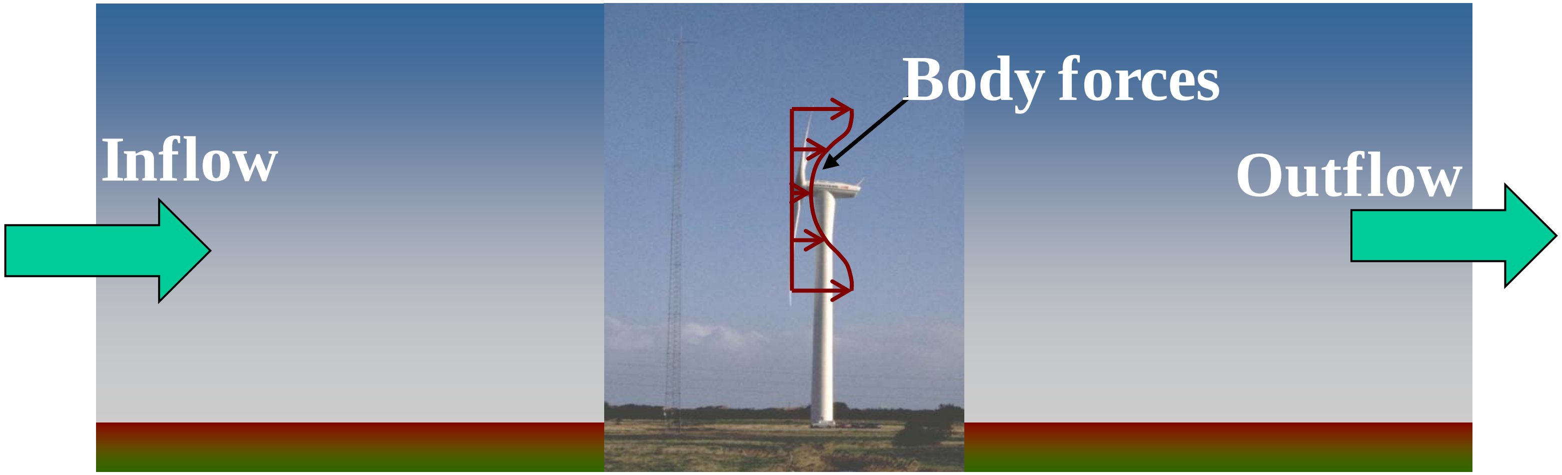
Basic elements of the model:

- **Flow governed by Navier-Stokes equations**
- **Influence of rotor introduced through body forces**
- **Body forces determined either from axial drag force or from blade element theory using tabulated airfoil data**
- **Needs to be combined with LES**
- **No limiting scale restrictions:**
  - **Large scales created by body forces**
  - **Small scales feeded by larger scales**
  - **Small scales die out through dissipatation**

The actuator line technique

Basic idea:

- Replace rotor blades by body forces
- Determine body forces from aerofoil data
- Simulate flow domain using DNS or LES



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Mixed scale SGS model

Mixed scale model:

$$k_{sgs} = \frac{1}{2}\overline{u_i'}\,\overline{u_i'}$$

$$\overline{u_i'} = \overline{u}_i'$$

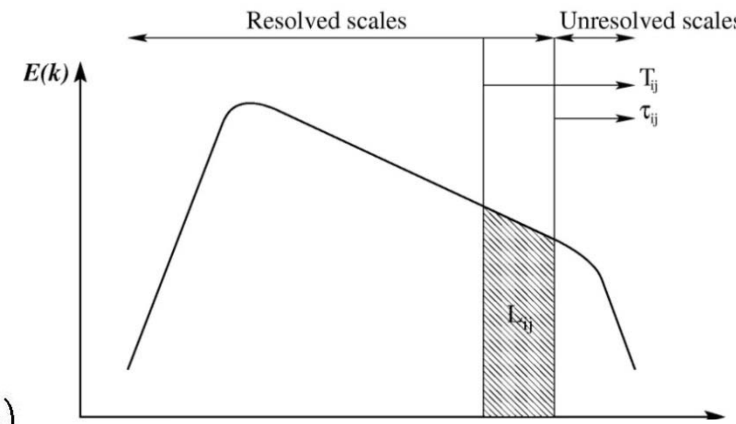
$$k_{sgs} = \frac{1}{2}\overline{u_i'}\,\overline{u_i'} = (\bar{u}_i - \hat{\bar{u}}_i)(\bar{u}_i - \hat{\bar{u}}_i)$$

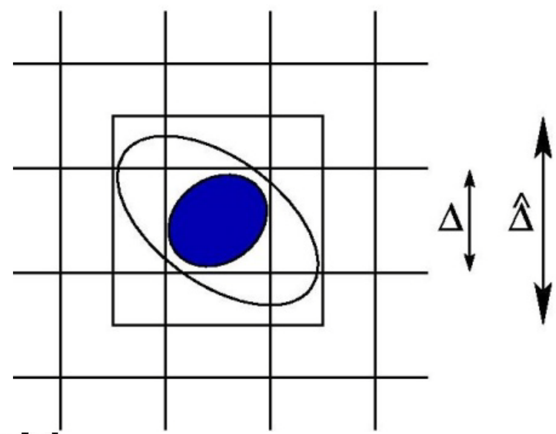
$$\hat{\bar{u}}_i = \frac{1}{4}(\bar{u}_{i-1} + 2\bar{u}_i + \bar{u}_{i+1})$$

$$\nu_{sgs} = C_m \Delta^{1+\alpha} k_{sgs}^{\frac{1+\alpha}{2}} \left|\bar{S}_{ij}\right|^{\alpha}$$

$$C_m = C_q^{1-\alpha} C_s^{2\alpha},\ C_q \approx 0.20,\ 0 \leq \alpha \leq 1$$

$\alpha = 1$ equation transforms to equation from Smagorinsky model.





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Scale requirements in wind energy

Turbulent scales:

	Length scale (m)	Velocity scale (m/s)	Time scale (s)
Airfoil boundary layer	10^{-3}	10^{2}	10^{-5}
Airfoil	1	10^{2}	10^{-2}
Rotor	10^{2}	10	10
Cluster	10^{3}	10	10^{2}
Wind farm	10^{4}	10	10^{3}

Aerodynamic Computations

Requirements for Direct Navier-Stokes computations:

Smallest turbulent length scale: ℓ
Largest geometrical length scale: L

Estimate based on scales:

L/ ℓ ≈ Re**(3/4)

Reynolds Number: Re=L·U/ν ,
where
L: Length of object
U: Typical velocity
ν: Kinematic viscosity

Number of mesh points: N ≈ (L/ℓ)**3 = Re**(9/4)

Typically Re = O(10**6) - O(10**7), thus N = O(10**15)

Computing performance is generally 10-doubled every 5 years, hence full DNS simulations can be anticipated in about (15-8) x 5 = 35 years.

Aerodynamic Computations

Wakes and clusters:

	No. Mesh points	CPU-time	Comments
PNS	O(10**6)	< 1h	Both axisymm. and 3D
AD-NS	O(10**5)	< 1h	Polar coordinates
RANS	O(10**7)	< 10h	Steady
DES	O(10**8)	< 1 week	3D-Unsteady
AL-LES	O(10**8)	< 1 week	Airfoil data required
LES	O(10**12)	N/A	3D-unsteady

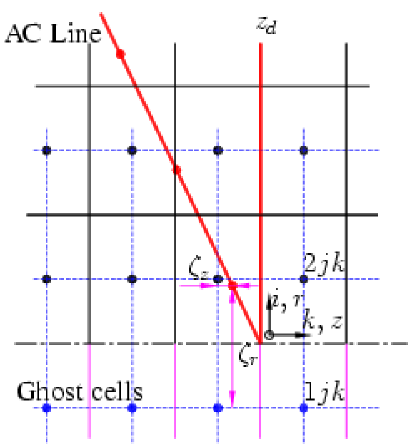
Actuator disc/line model

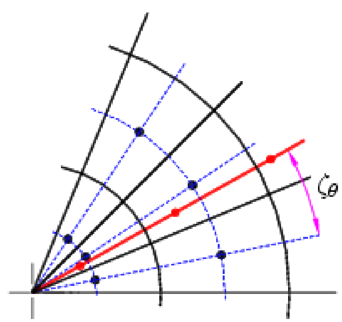
Extensions of the model:

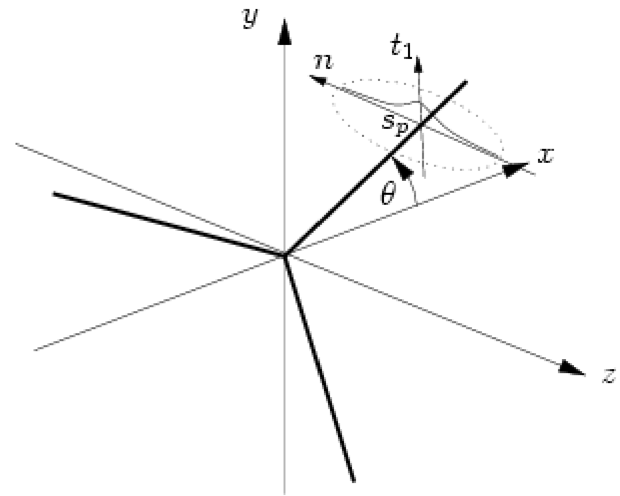
- **Shear and veer can be introduced through body forces (analogous to immersed boundary conditions)**
- **Ambient turbulence introduced as a numerical turbulence lattice through body forces**
- **The model is can easily be coupled to an aeroelastic code, e.g. Flex5, enabling detailed aeroelastic computations**
- **Atmospheric boundary layer stability can be introduced through the energy equation**

The Actuator Line Technique

- Regular 3D Grid
- $\mathbf{V}_{r\theta z}$ – Linear interpolation
- No tip correction is applied







Forces smearing

$$\mathbf{f}(\mathbf{x}) = \sum_{i=1}^{B} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathbf{F}^{i}(s)\eta\,(p^{i})\,dn\,dt_1$$

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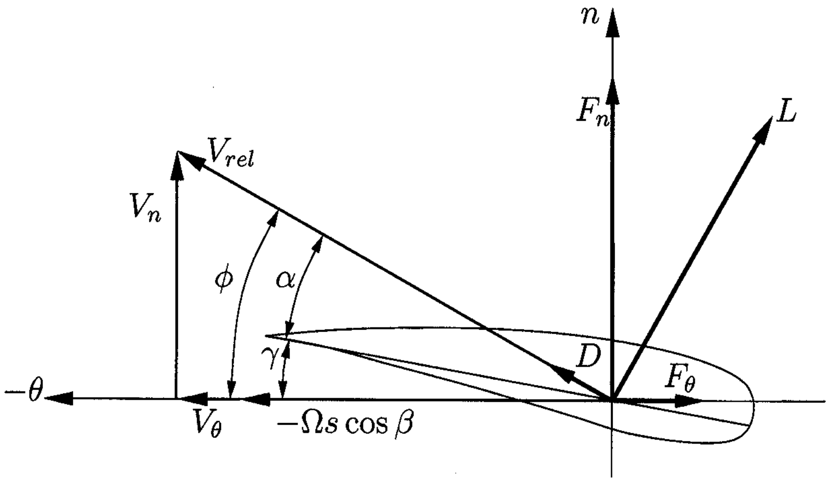
Aerodynamic Forces

Evaluation of velocities

$$\phi = \tan^{-1}\left(\frac{V_n}{\Omega r - V_\theta}\right), \quad \alpha = \phi - \gamma$$

$$V_{rel}^2 = V_n^{\ 2} + (\Omega r - V_\theta)^2 , \quad V_n = V_z$$



Evaluation of aerodynamic forces

$$\begin{Bmatrix}\mathbf{L}\\ \mathbf{D}\end{Bmatrix} = \tfrac{1}{2}\rho V_{rel}^2 cB \begin{Bmatrix} C_L(\alpha,\mathrm{Re})\mathbf{e}_L \\ C_D(\alpha,\mathrm{Re})\mathbf{e}_D\end{Bmatrix},$$

$$F_z = L\cos\phi + D\sin\phi, \quad F_\theta = L\sin\phi - D\cos\phi$$

Smearing of forces

$$\mathbf{f}_\varepsilon = \mathbf{f} \otimes \eta_\varepsilon, \quad \eta_\varepsilon(p) = \frac{1}{\varepsilon\sqrt{\pi}} e^{-\left(\frac{p}{\varepsilon}\right)^2} \Rightarrow \mathbf{f}_\varepsilon(r,z) = \int_{-\infty}^{+\infty} \frac{\mathbf{F}(r,z)}{\varepsilon\sqrt{\pi}} e^{-\left(\frac{p}{\varepsilon}\right)^2} dz, \quad p = z_d - z$$

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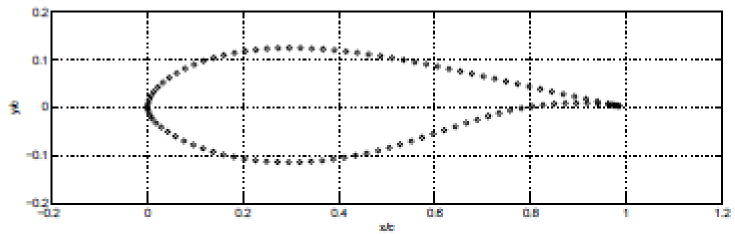
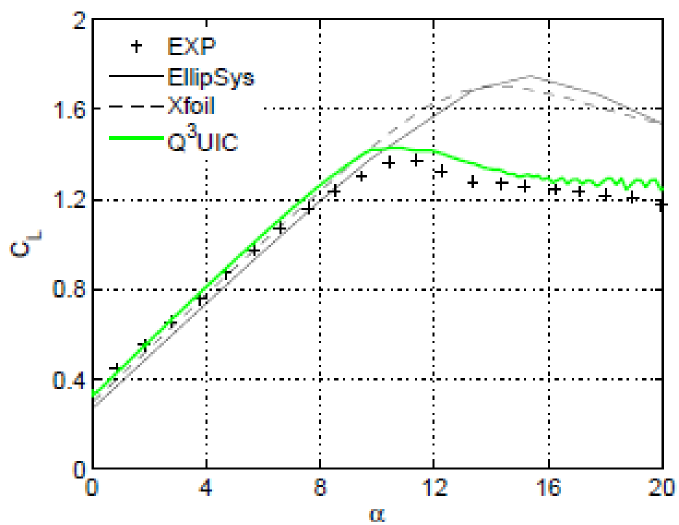
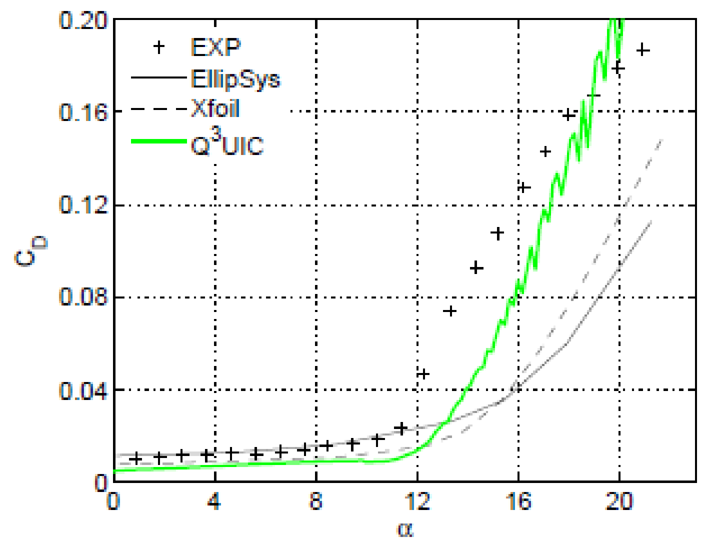
Actuator Disc/Line Model

2D Airfoil data

FFA-W3-241







Actuator Disc/Line Model

Suggested 3D corrections for airfoil data

Correction for rotational effects (AoA < 20 degrees):

$$C_{l,3D} = C_{l,2D} + a(c/r)^b \left[C_{l,inv} - C_{l,2D} \right]$$

Correction for finite rotor size (AoA > 45 degrees):

$$C_n = C_l \cos\phi + C_d \sin\phi = C_d(\alpha = 90^0)$$

$$C_t = C_l \sin\phi - C_d \cos\phi > 0$$

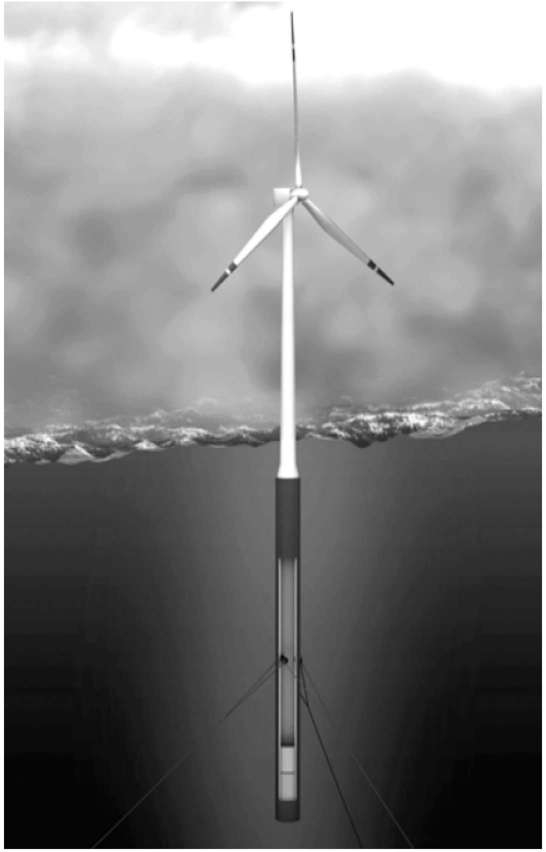
Linear interpolation for 20 degrees < AoA < 45 degrees

Actuator Disc/Line Model

Dynamic stall

Dynamic stall is caused by:

- **Wind shear**
- **Atmospheric turbulence**
- **Tower shadow**
- **Rotors operating in yaw or tilt**
- **Dynamically deflected blades**
- **Turbine placed on floating structure**

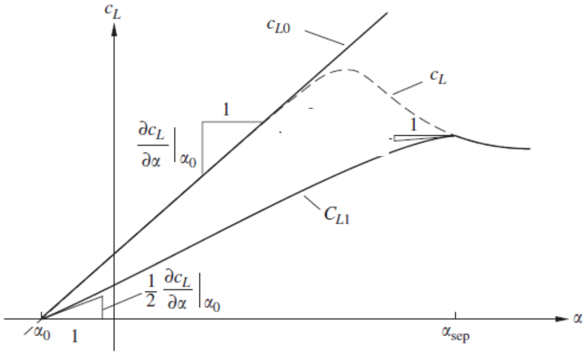


Actuator Disc/Line Model

Dynamic stall model (Øye)

$f = 0$: **Fully separated**

$f = 1$: **Fully attached**



Linear interpolation:

$$C_L(f) = 2\pi(\alpha - \alpha_0)\cdot f + C_{L,sep}\cdot(1-f) \Rightarrow$$

$$f_{static} = (C_{L,static} - C_{L,sep})/(2\pi(\alpha - \alpha_0) - C_{L,sep})$$

Dynamic approach:

$$\frac{df}{dt} = \frac{f_{static} - f}{\tau}$$

Final algorithm:

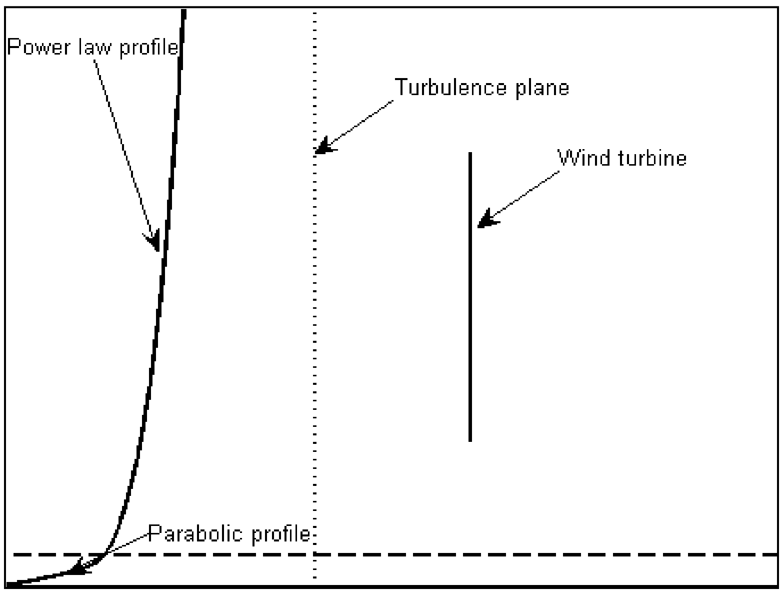
$$f_i = f_{i-1} + (f_{i-1} - f_{static})\cdot \exp\left(-\frac{\Delta t}{\tau}\right)$$

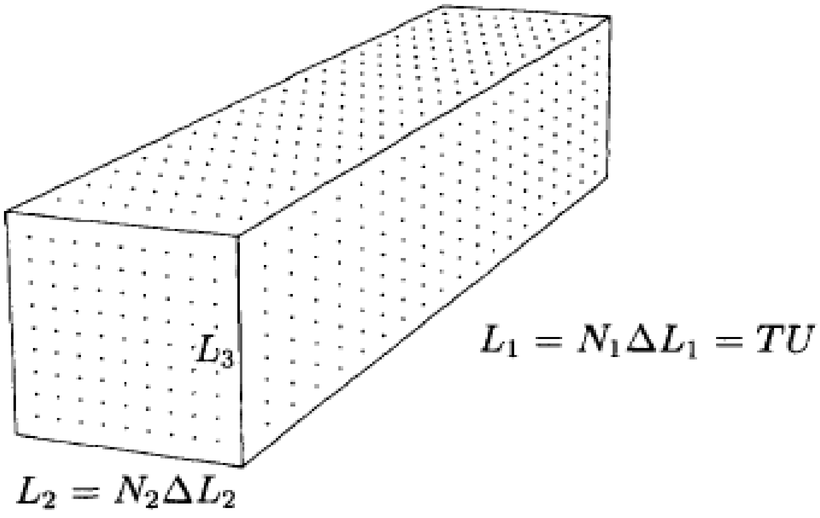
Wind Shear and Turbulence

$$w(y)=\begin{cases} w_0\cdot(c_2y^2+c_1y) & y\le\Delta \\ w_0\cdot\left(\dfrac{y}{h_{hub}}\right)^{\alpha} & y>\Delta \end{cases}$$

$$\mathbf{f}=\dot{m}\mathbf{u}+\rho\varepsilon\frac{\partial\mathbf{u}}{\partial t}$$

$$\mathbf{f}_\varepsilon=\mathbf{f}\otimes\eta_\varepsilon,\quad \eta_\varepsilon(d)=\frac{1}{\varepsilon\sqrt{\pi}}\exp(-\left(\frac{d}{\varepsilon}\right)^2)$$





Power law wind shear profile

Model of wind turbulence

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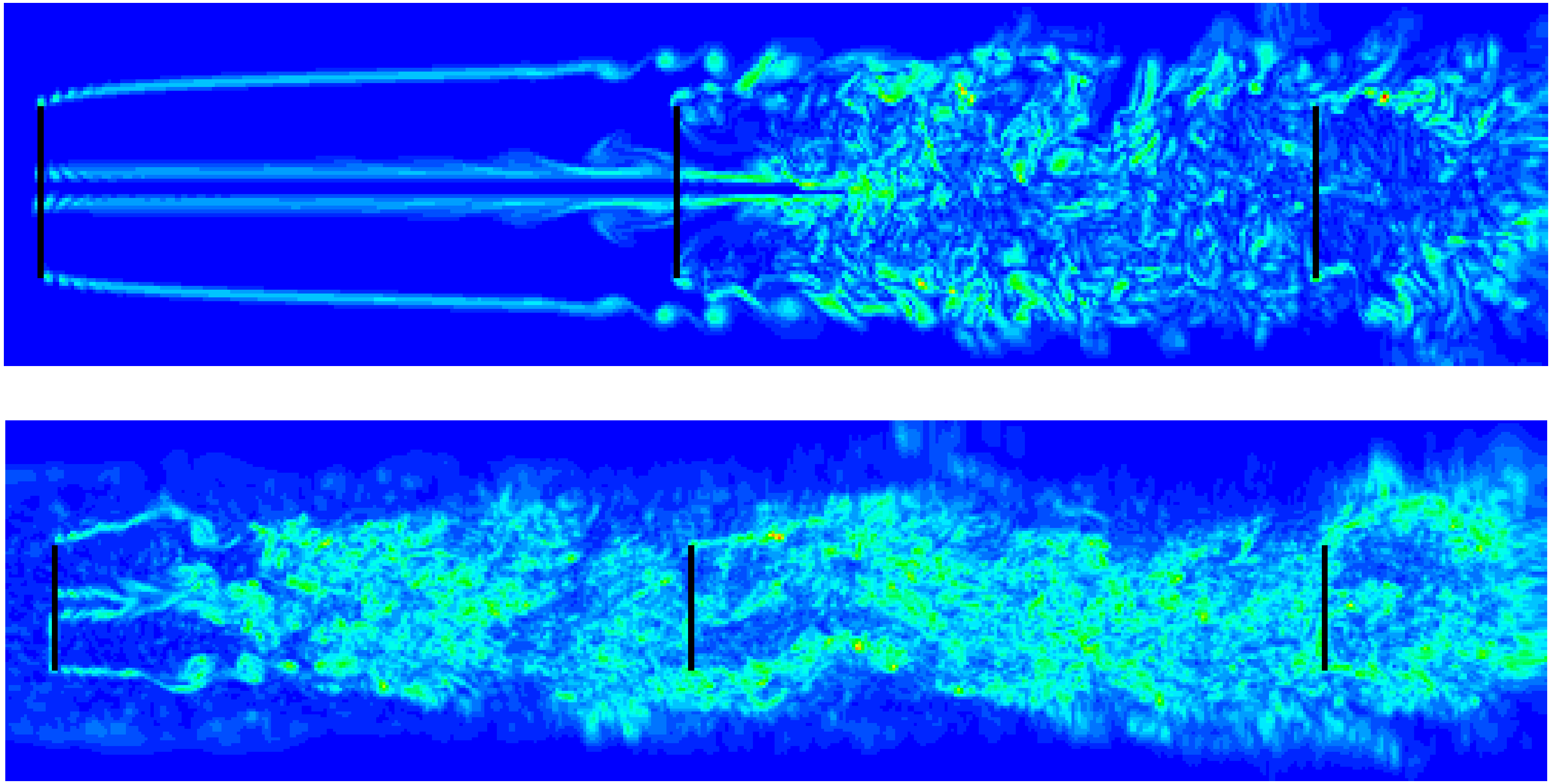
Vortex structures in the wake of a row of rotors



Development of wake behind three rotors in a row at W_0 = 10 m/s; Turbine spacing 6 rotor radii. A) Constant inflow; B) Turbulent inflow.

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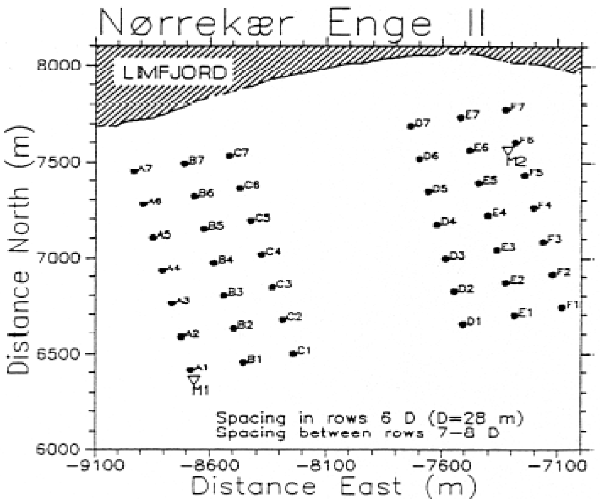
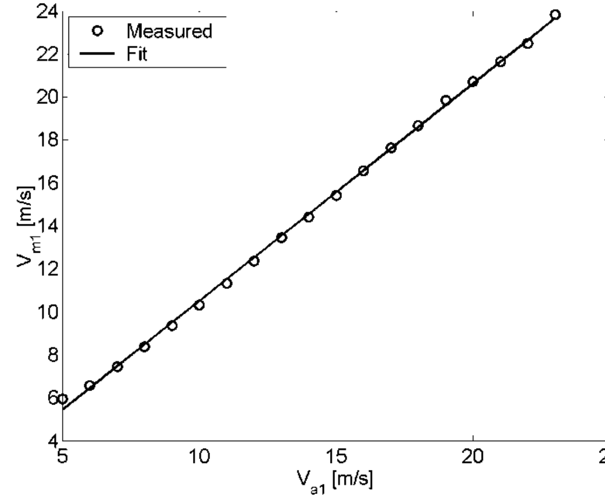
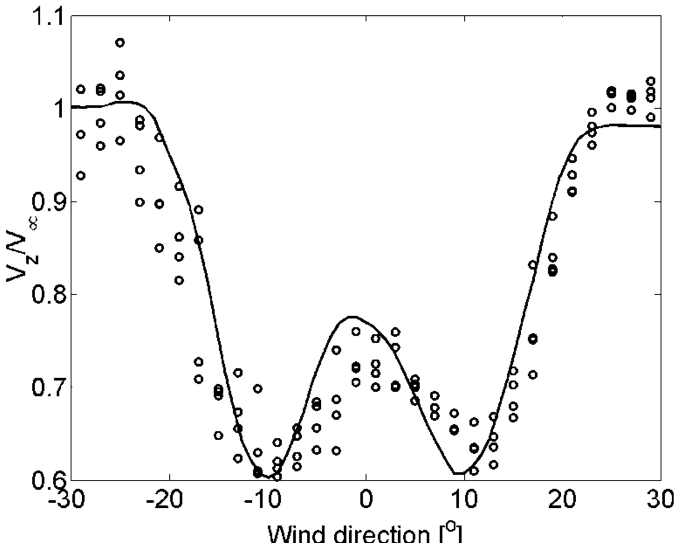
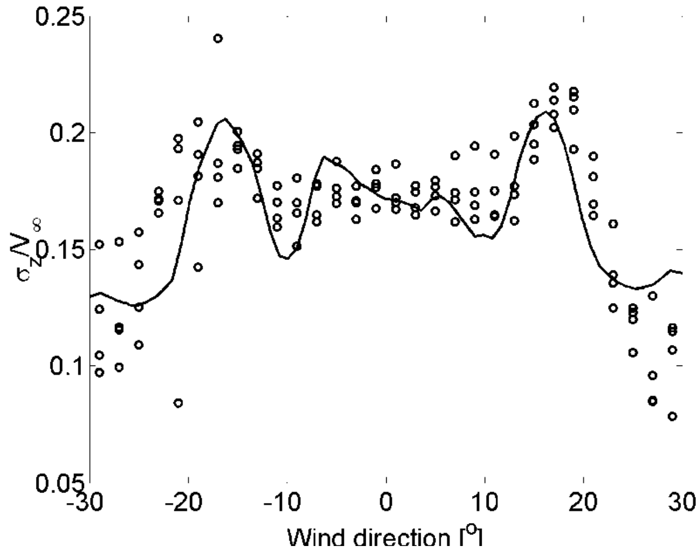
Validation: Nørrekær Enge II









Turbulence intensity

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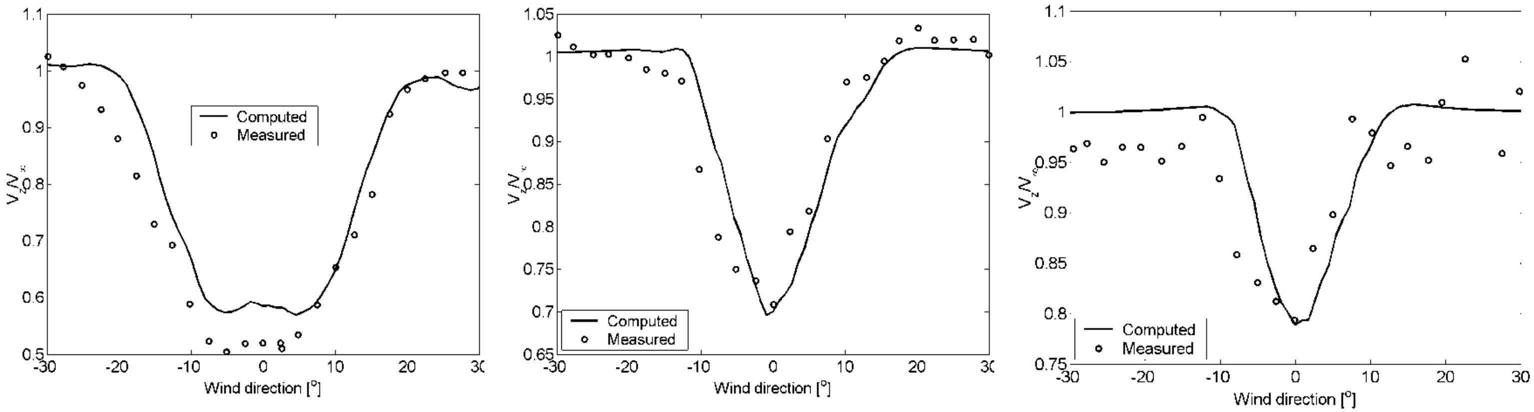
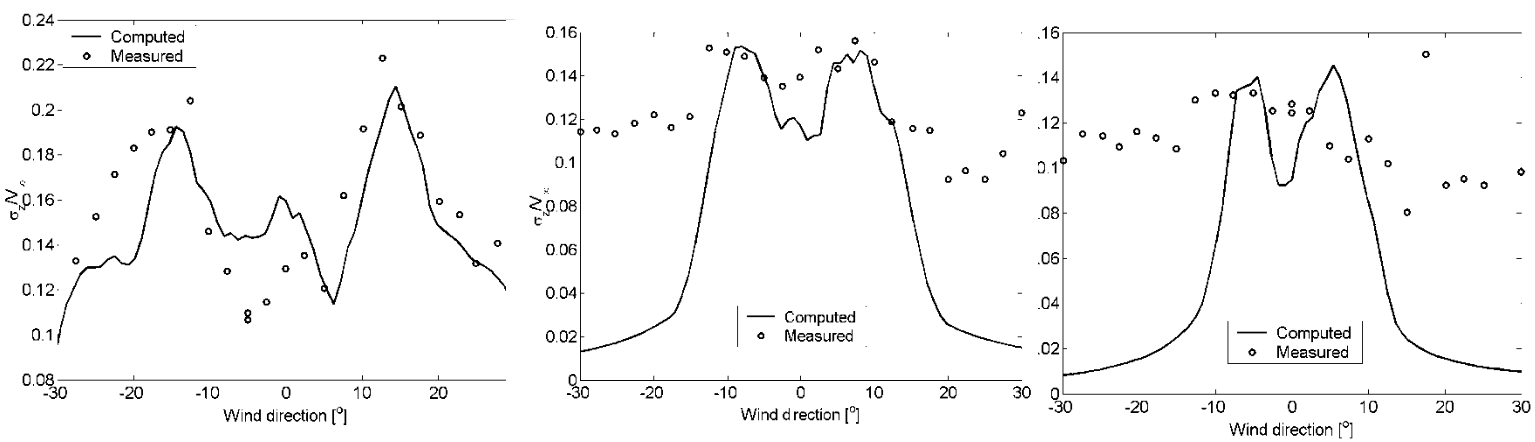
Sexbierum



Mean axial velocity



Turbulence intensity of axial velocity component

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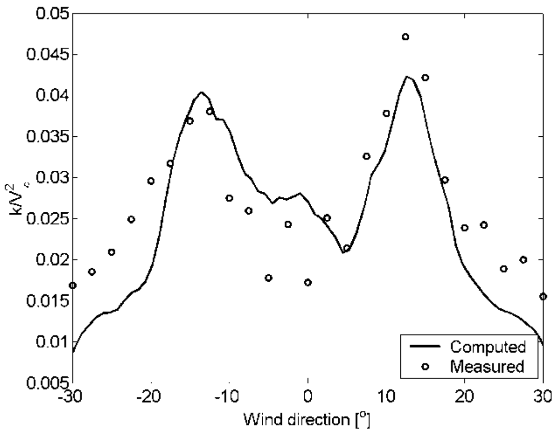
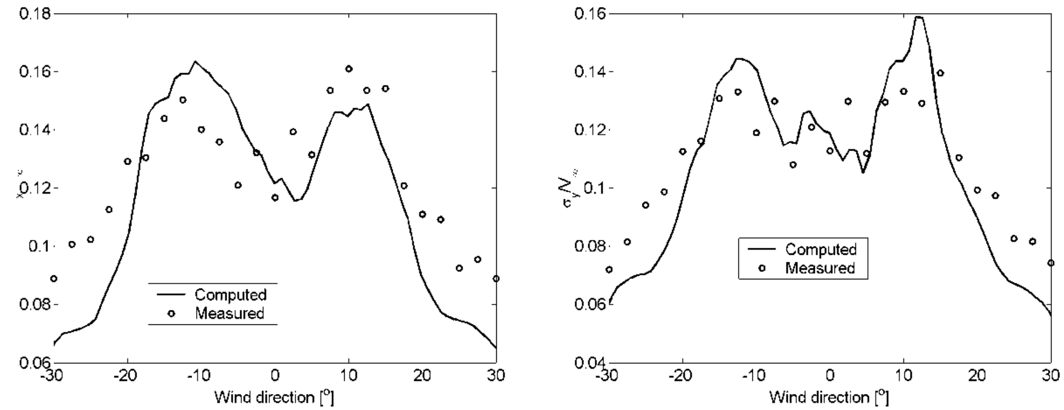
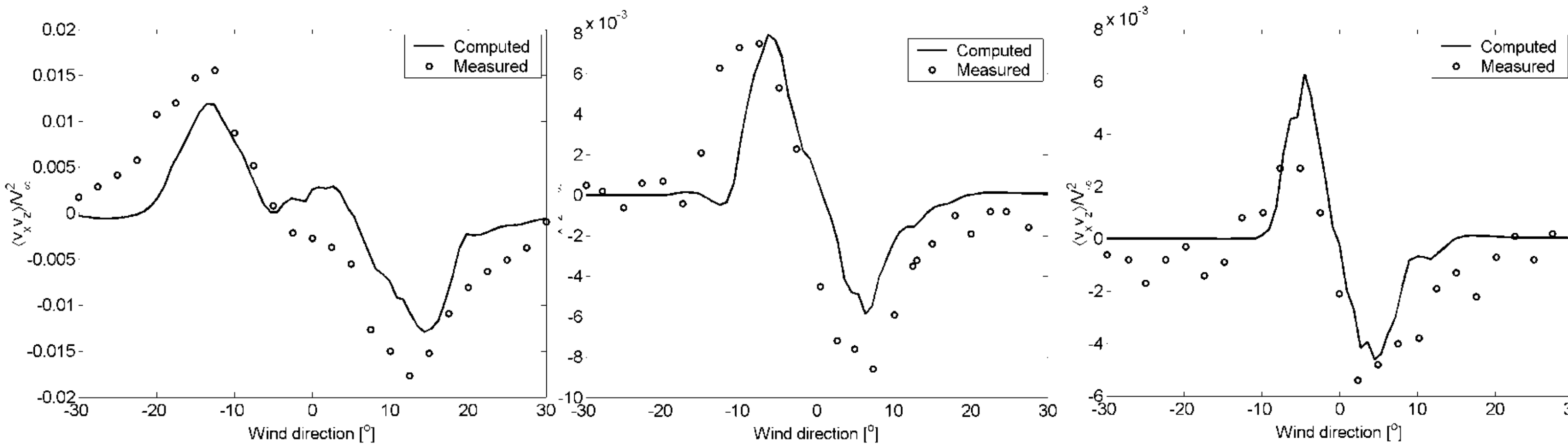


Turbulent kinetic energy

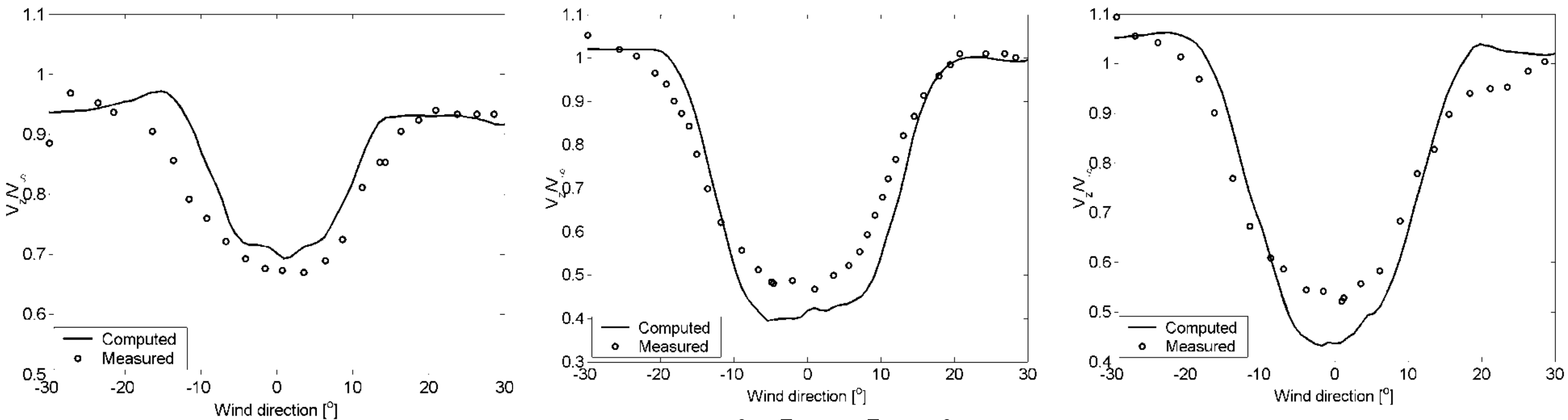


Turbulence intensity of lateral and vertical components

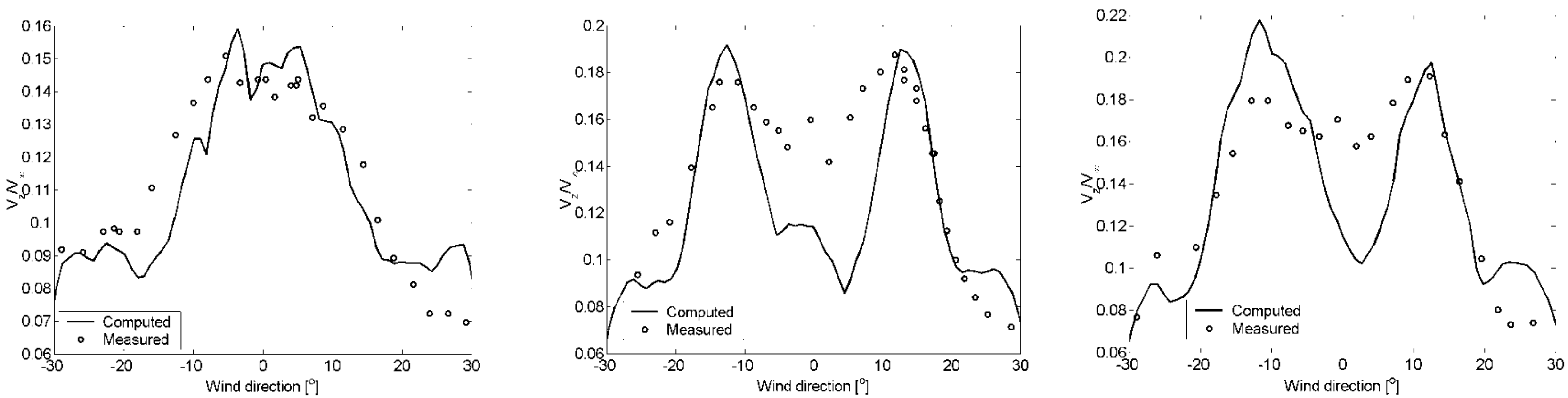


Reynolds-stresses

Nibe



Mean axial velocity

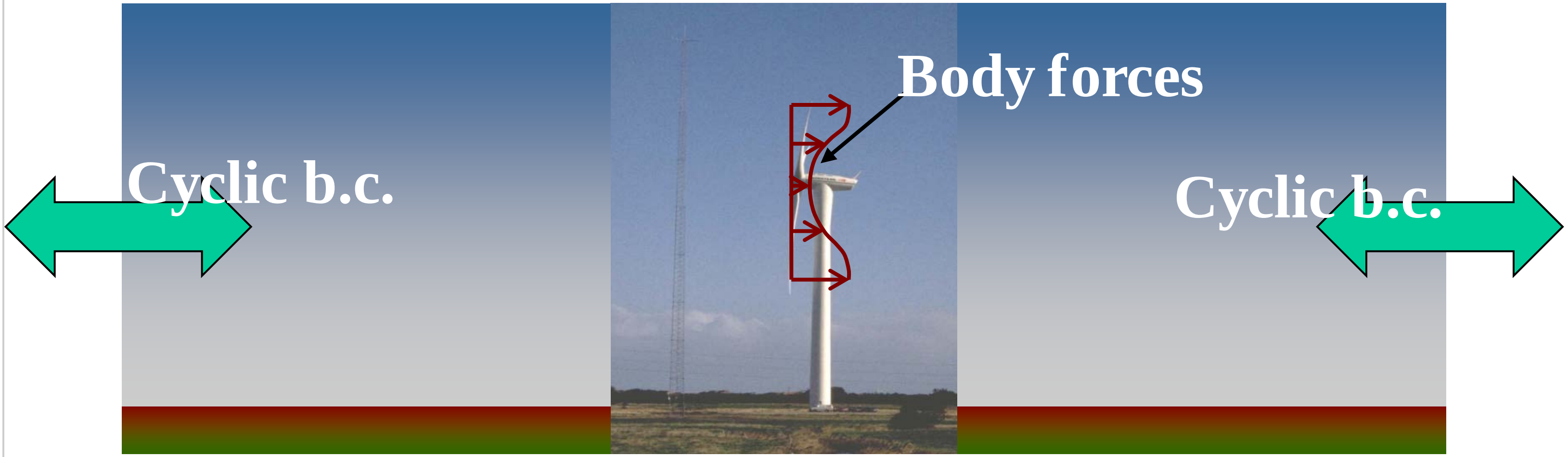


Turbulence intensity

Simulation of turbulence inside wind farm

Basic idea:

- **Replace rotor blades by body forces**
- **Determine body forces from aerofoil data**
- **Simulate an 'infinite' row of turbines using cyclic boundary conditions**



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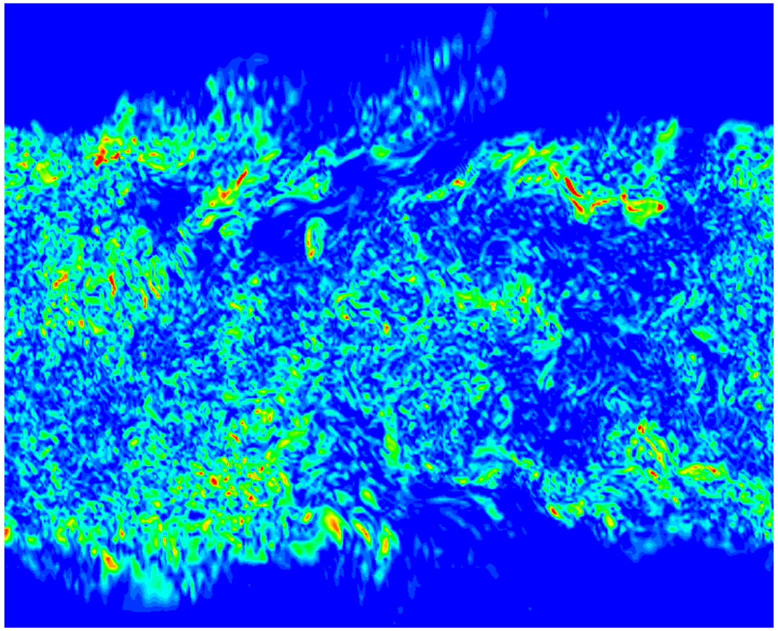
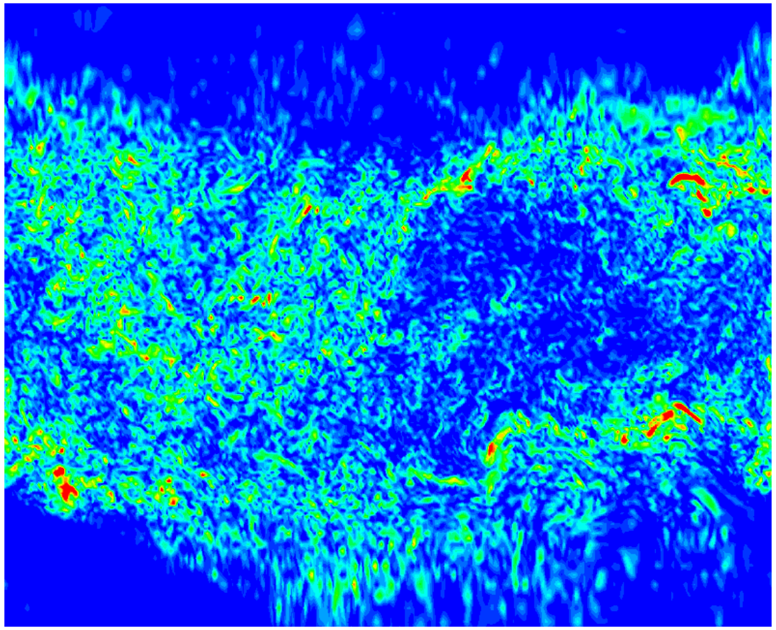
Simulation of turbulence inside wind farm

Cross sectional turbulent flow fields:





Iso-vorticity contours in the final stage