# Nonlinear wave loads on fixed offshore turbines

LES in marine hydrodynamics and offshore wind power, part II, Lyngby, 24-26 August 2011

John Grue, Mechanics Division, Department of Mathematics

University of Oslo

Nonlinear forces on vertical columns exposed to waves, how and why?

There 3 reasons/effects:

- pressure is nonlinear

- free surface condition is nonlinear

- elevation along the column is finite

Eq. of motion

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p - g\mathbf{k}$$

Pressure

$$-\frac{p}{\rho} = \phi_t + \tfrac{1}{2} \nabla \phi \cdot \nabla \phi + gy$$

Elevation

$$\eta_t = \phi_y - \nabla \phi \cdot \nabla \eta \quad \text{on} \quad y = \eta$$

Free surface condition, dynamic

$$\phi_t + \tfrac{1}{2}\nabla\phi \cdot \nabla\phi + g\eta = 0 \quad \text{on} \quad y = \eta$$

Time derivative of dynamic b.c.

$$\phi_{tt} + \nabla\phi_t \cdot \nabla\phi + g\eta_t = 0 \quad \text{on} \quad y = \eta$$

Free surface condition, kinematic

$$g\eta_t = g\phi_y + \nabla\phi_t \cdot \nabla\phi + \tfrac{1}{2}\nabla\phi \cdot \nabla|\nabla\phi|^2 \quad \text{on} \quad y = \eta$$

Perturbations: $\phi = \phi^{(1)} + \phi^{(2)} + \phi^{(3)} + ...$ where
- first term is proportional to the wave amplitude $A$,
- 2nd term is proportional to $A^2$,
- third term is proportional to $A^3$, etc.

$$\phi_t^{(1)} + g\eta^{(1)} = 0, \quad \eta_t^{(1)} = \phi_y^{(1)} \quad \text{on} \quad y = 0$$

$$\phi_t^{(2)} + g\eta^{(2)} = -\eta^{(1)}\phi_y^{(1)} - \tfrac{1}{2}\nabla\phi^{(1)} \cdot \nabla\phi^{(1)} \quad \text{on} \quad y = 0$$

$$\eta_t^{(2)} = \phi_y^{(2)} - \nabla\phi^{(1)} \cdot \nabla\eta^{(1)} + \phi_{yy}^{(1)}\eta^{(1)} \quad \text{on} \quad y = 0$$

etc.

Harmonics:

Incoming wave of frequency $\omega$ results in a response with frequency $\omega$.

Quadratic products introduce sum frequency $\omega + \omega = 2\omega$ and difference frequency $\omega - \omega = 0\,\omega$

Cubic products introduce sum/difference frequencies
- $\omega + \omega + \omega = 3\omega$, $2\omega + \omega = 3\omega$, and
- $\omega + \omega - \omega = \omega$, $2\omega - \omega = \omega$

Quartic products introduce sum/difference frequencies
- $4\omega$, $2\omega$, $0\,\omega$,

etc.

Incoming Stokes wave in deep water with frequency $\omega$ has a velocity field that is a pure sine (up to third order), given by the potential

$$\phi = \mathrm{Re}\left[\frac{\mathrm{i}gA}{\omega}e^{ky}e^{-\mathrm{i}kx+\mathrm{i}\omega t}\right] + O((kA)^4),$$

This velocity field implies a force on the column, in the $x$-direction, composed by harmonics, by

$$F(t) = \mathrm{Re}\Big[F_1 e^{\mathrm{i}\omega t} + F_2 e^{2\mathrm{i}\omega t} + F_3 e^{3\mathrm{i}\omega t} + F_4 e^{4\mathrm{i}\omega t} + ... + \Big]$$

where, asymptotically, when $A << 1$, $F_1 \sim A$, $F_2 \sim A^2$, $F_3 \sim A^3$, $F_4 \sim A^4$, etc.

Faltinsen, Newman and Vinje, JFM 289 (1995); slender body wave analysis ($kR << 1$) including nonlinearity up to cubic order ($kA << 1$; $A \sim R$). Periodic waves. Forces:

$$F_1 = 2\pi\rho g R^2 A \cos\omega t$$

$$\tilde{F}_x = \pi\rho g K R^2 A^2 \sin 2\omega t + \pi\rho g K^2 R^2 A^3 \cos\omega t - 2\pi\rho g K^2 R^2 A^3 \cos 3\omega t$$

$$|F_1/F_3| = (KA)^2$$

$$|F_2/\rho g R A^2| = \pi K R \qquad |F_3/\rho g A^3| = 2\pi (KR)^2$$

Malenica and Molin, JFM 302 (1995); Perturbation expansion in the wave slope $\epsilon = Ak$; analysis valid asymptotically for $Ak << 1$, with $kR$ arbitrary:

$$\Phi = \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \epsilon^3\phi^{(3)} + O(\epsilon^4)$$

$$\epsilon\phi^{(1)} = \mathrm{Re}[\varphi^{(1)}e^{-\mathrm{i}\omega t}]$$

$$\epsilon^2\phi^{(2)} = \bar{\varphi}^{(2)} + \mathrm{Re}[\varphi^{(2)}e^{-2\mathrm{i}\omega t}]$$

$$\epsilon^3\phi^{(3)} = \mathrm{Re}[\bar{\varphi}^{(3)}e^{-\mathrm{i}\omega t} + \varphi^{(3)}e^{-3\mathrm{i}\omega t}]$$

Perturbation expansions; free surface condition; $\varphi^{(3)}$

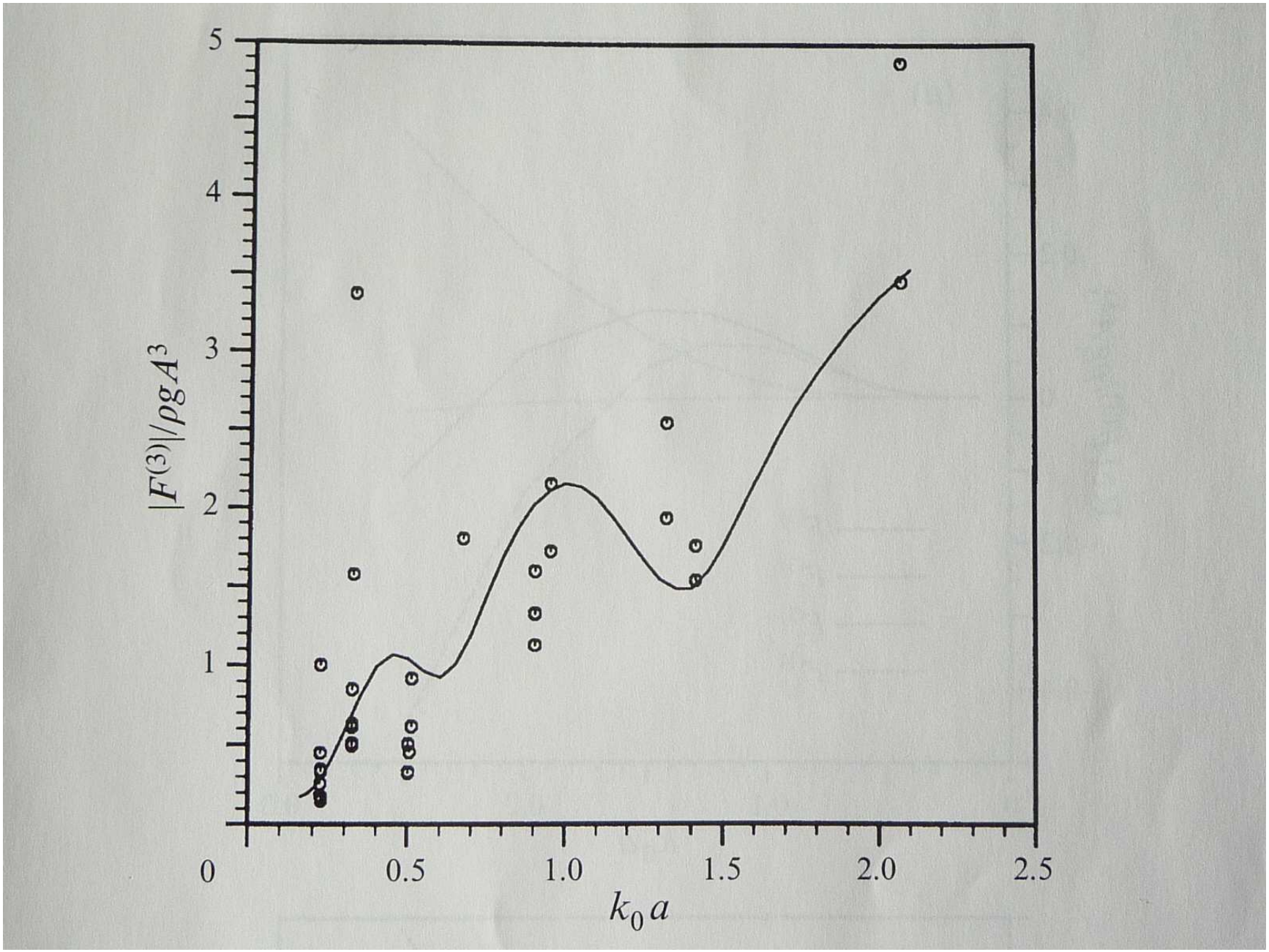
$$
\begin{aligned}
&-9\nu\varphi^{(3)} + \frac{\partial\varphi^{(3)}}{\partial y} = \frac{3\mathrm{i}\omega}{g}\nabla\varphi^{(2)}\cdot\nabla\varphi^{(1)} \\
&-\frac{\mathrm{i}\omega}{2g}\left[\varphi^{(1)}\left(\frac{\partial^2\varphi^{(2)}}{\partial y^2} - 4\nu\frac{\partial\varphi^{(2)}}{\partial y}\right) + 2\varphi^{(2)}\left(\frac{\partial^2\varphi^{(1)}}{\partial y^2} - \nu\frac{\partial\varphi^{(1)}}{\partial y}\right)\right] \\
&-\frac{1}{8g}\nabla\varphi^{(1)}\cdot\nabla(\nabla\varphi^{(1)}\cdot\nabla\varphi^{(1)}) - \frac{\nu}{g}\varphi^{(1)}\nabla\varphi^{(1)}\cdot\nabla\frac{\partial\varphi^{(1)}}{\partial y} \\
&+\frac{1}{4g}\left(\nu\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial y} + \tfrac{1}{2}\nabla\varphi^{(1)}\cdot\nabla\varphi^{(1)}\right)\left(\tfrac{\partial^2\varphi^{(1)}}{\partial y^2} - \nu\tfrac{\partial\varphi^{(1)}}{\partial y}\right)
\end{aligned}
$$

($\nabla$- horizontal gradient; $\nu = \omega^2/g$)

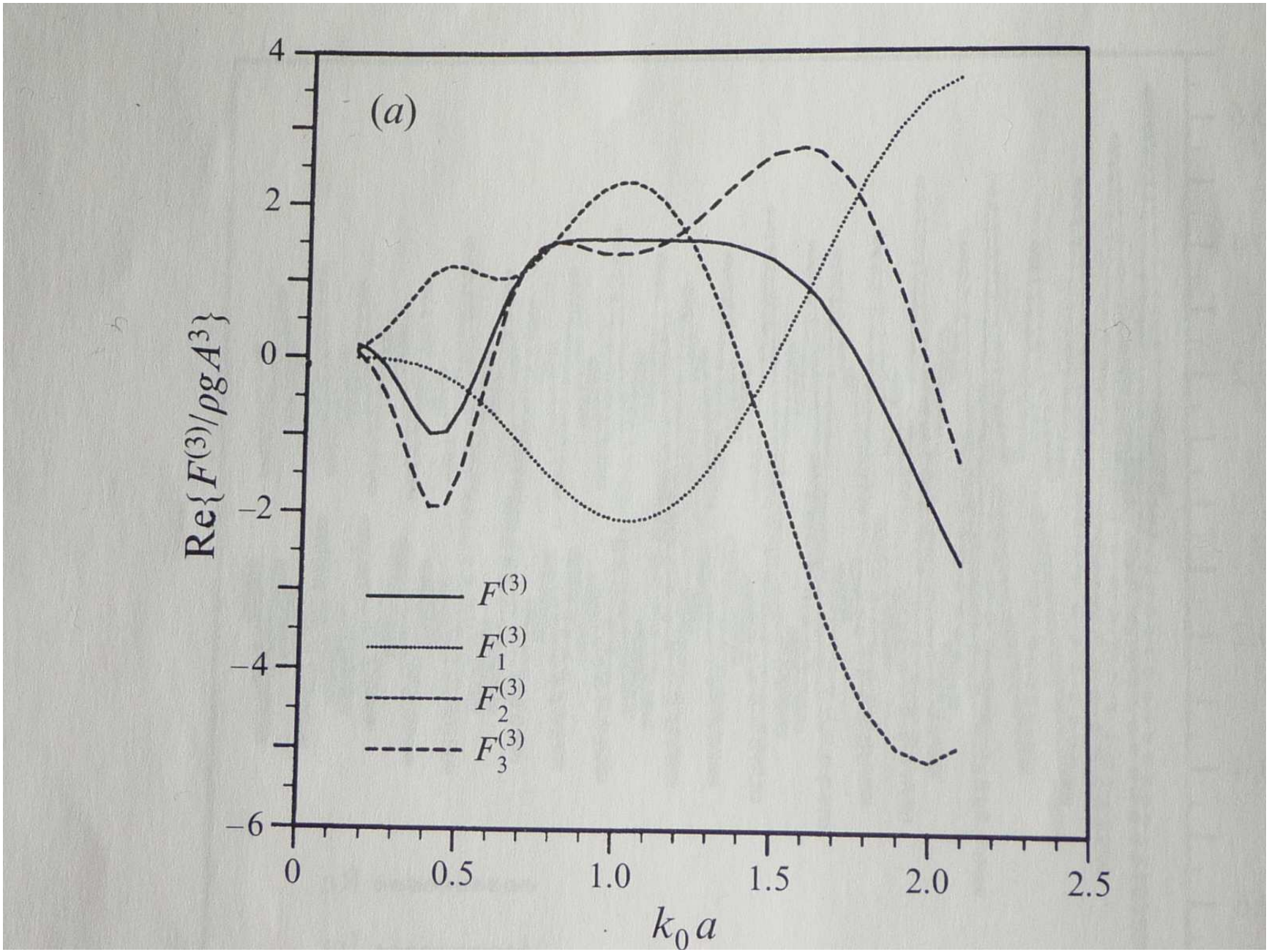
Forces:

$$
\begin{aligned}
F &= \epsilon F^{(1)} + \epsilon^2 F^{(2)} + \epsilon^3 F^{(3)} + O(\epsilon^4) \\
&= \mathsf{Re}[F^{(1)} e^{-\mathsf{i}\omega t}] \\
&+ \bar{F}^{(2)} + \mathsf{Re}[F^{(2)} e^{-2\mathsf{i}\omega t}] \\
&+ \mathsf{Re}[\bar{F}^{(3)} e^{-\mathsf{i}\omega t} + F^{(3)} e^{-3\mathsf{i}\omega t}] + O(\epsilon^4)
\end{aligned}
$$

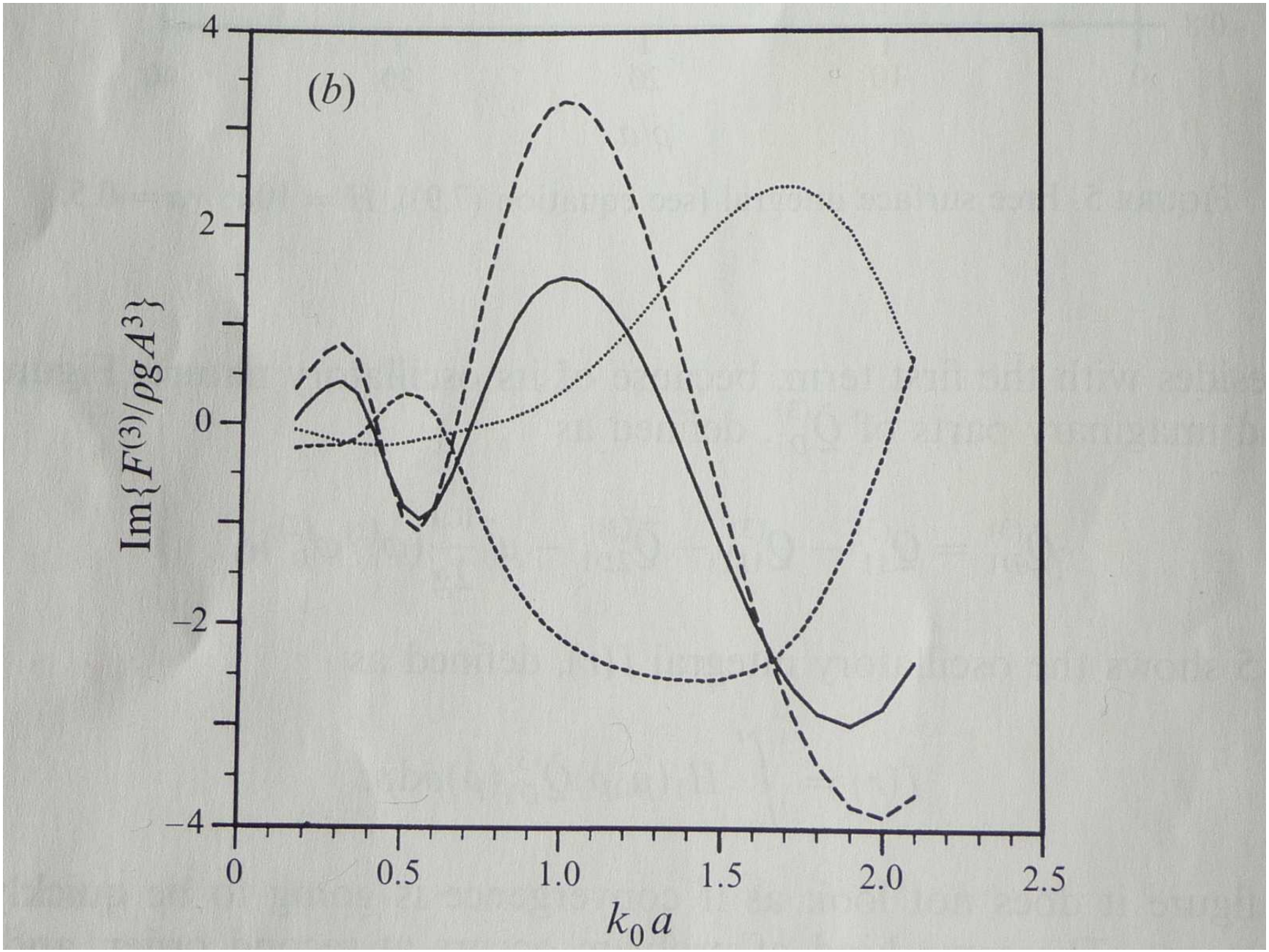
$$
\begin{aligned}
F^{(3)} = \int_{SB0} \left( 3\mathsf{i}\omega\rho\varphi^{(3)} - \tfrac{1}{2}\rho\nabla\varphi^{(1)} \cdot \nabla\varphi^{(2)} \right) \mathbf{n} dS \\
+ \tfrac{1}{2}\rho g \int_{CB0} \eta^{(1)} (\eta^{(2)} - \tfrac{1}{4}\eta^{(1)}\eta^{(1)}) \mathbf{n} dC
\end{aligned}
$$



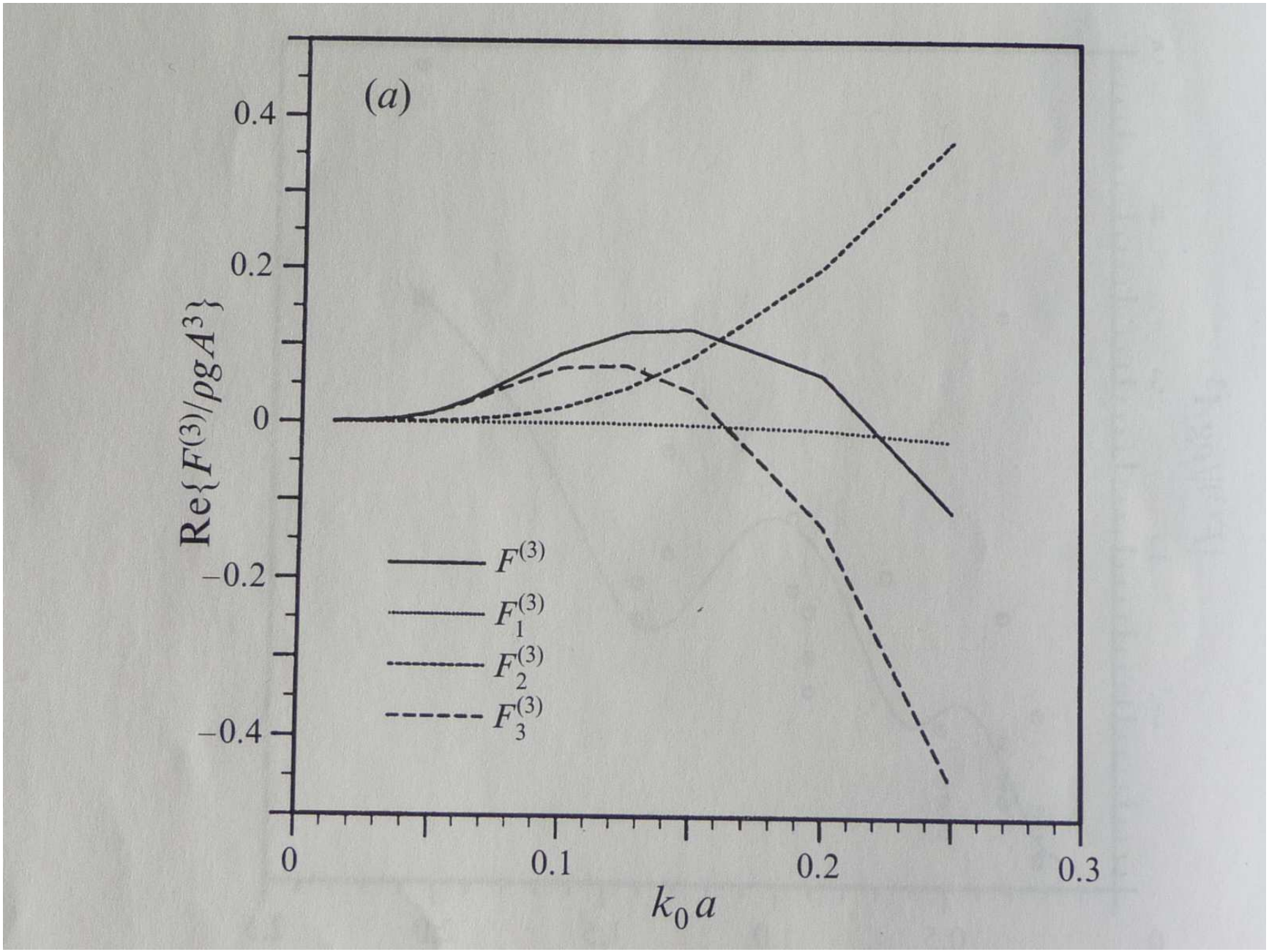

Malenica & Molin, JFM 302 (1995)



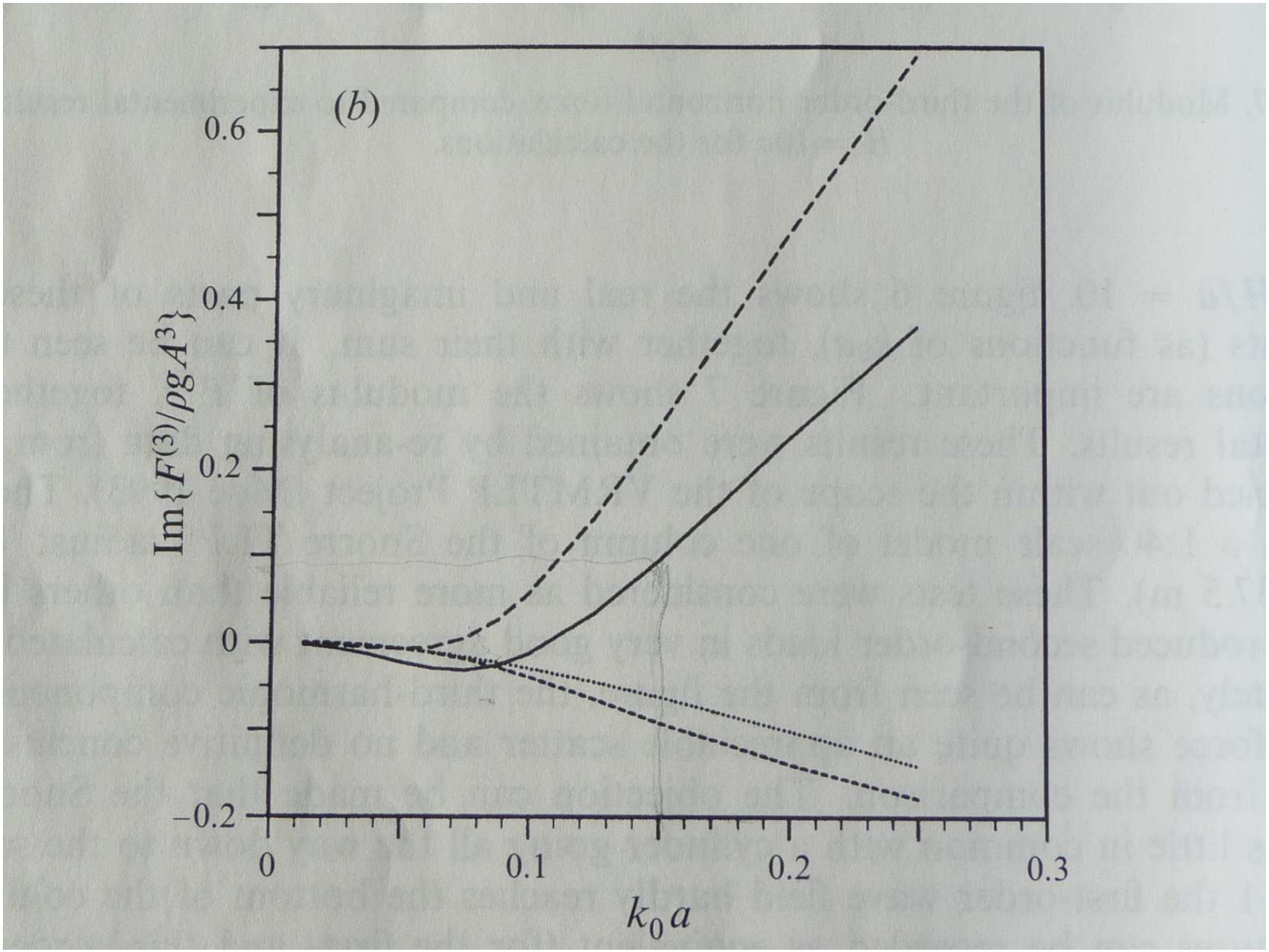

Malenica & Molin, JFM 302 (1995)




Malenica & Molin, JFM 302 (1995)




Malenica & Molin, JFM 302 (1995)




Malenica & Molin, JFM 302 (1995)

Note, for example for $kR = 0.11$, we find from M&M

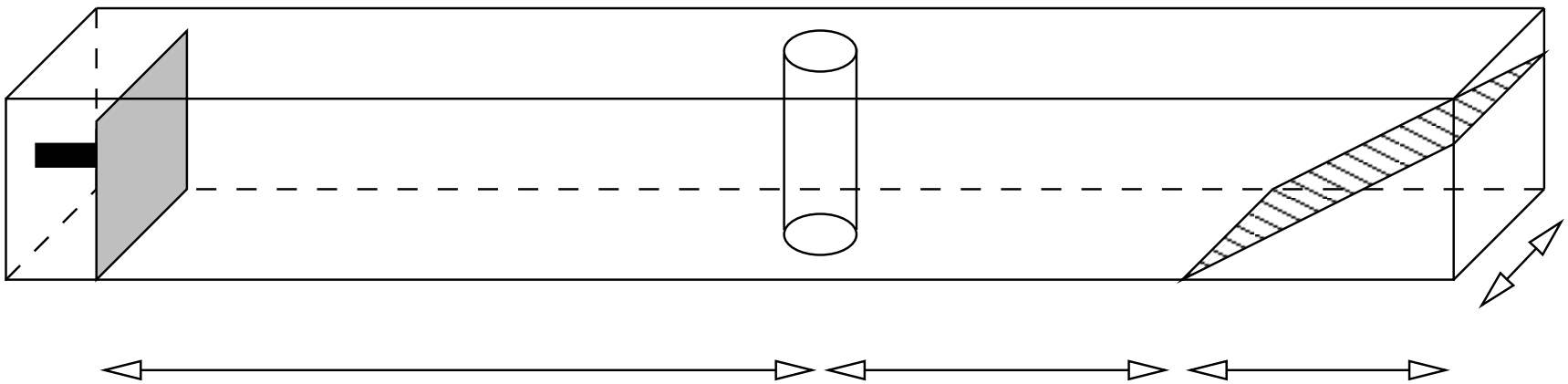
$$\frac{F_3}{\rho g A^3} \simeq \mathrm{Re}[(0.1, 0.0)e^{-3\mathrm{i}\omega t}] = 0.1 \cos 3\omega t$$

while FNV gives

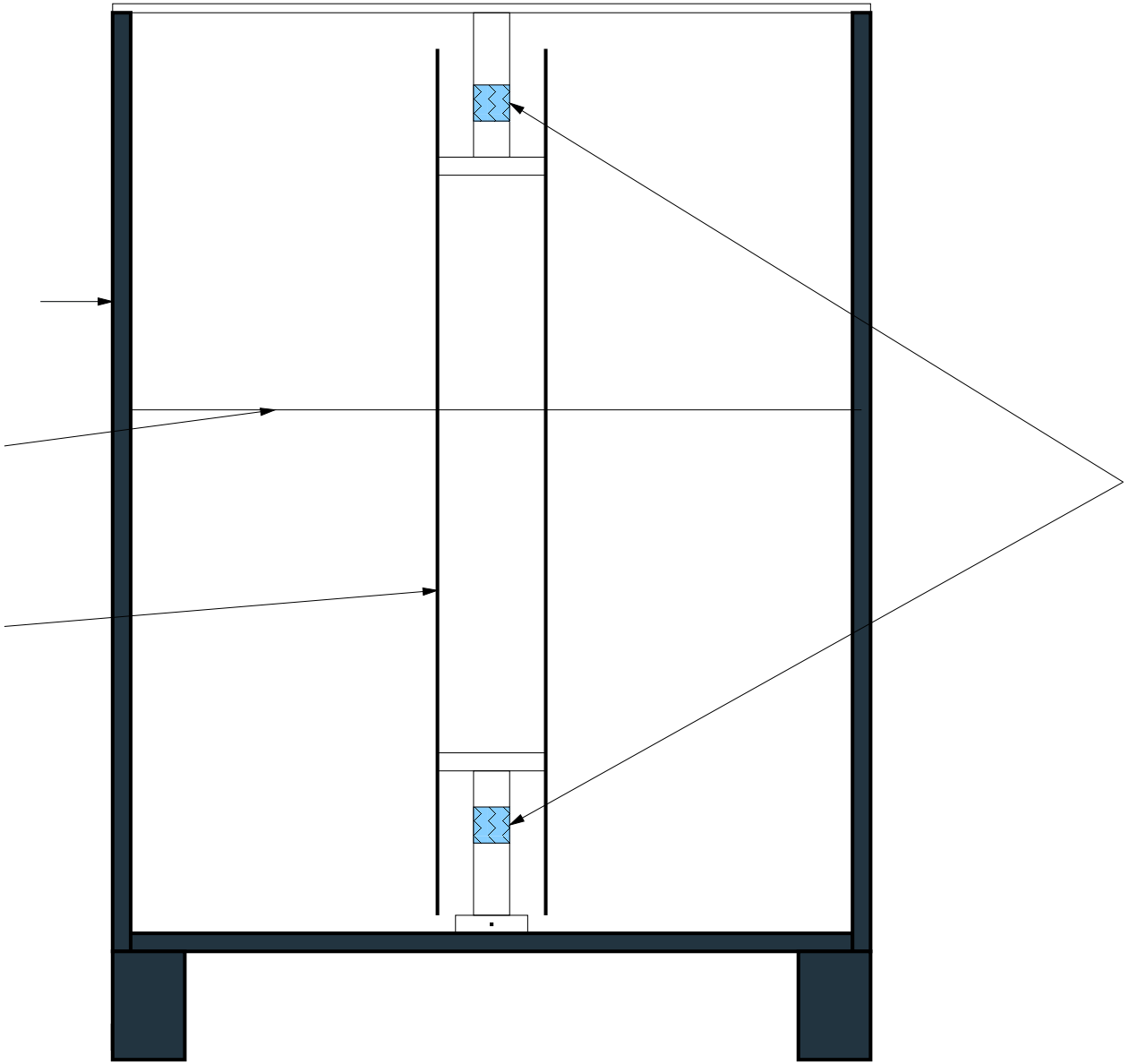
$$\frac{F_3}{\rho g A^3} = -2\pi (KR)^2 \cos 3\omega t \simeq -0.076 \cos 3\omega t$$

where the most important difference is that the latter is 180 degrees out of phase with the former.

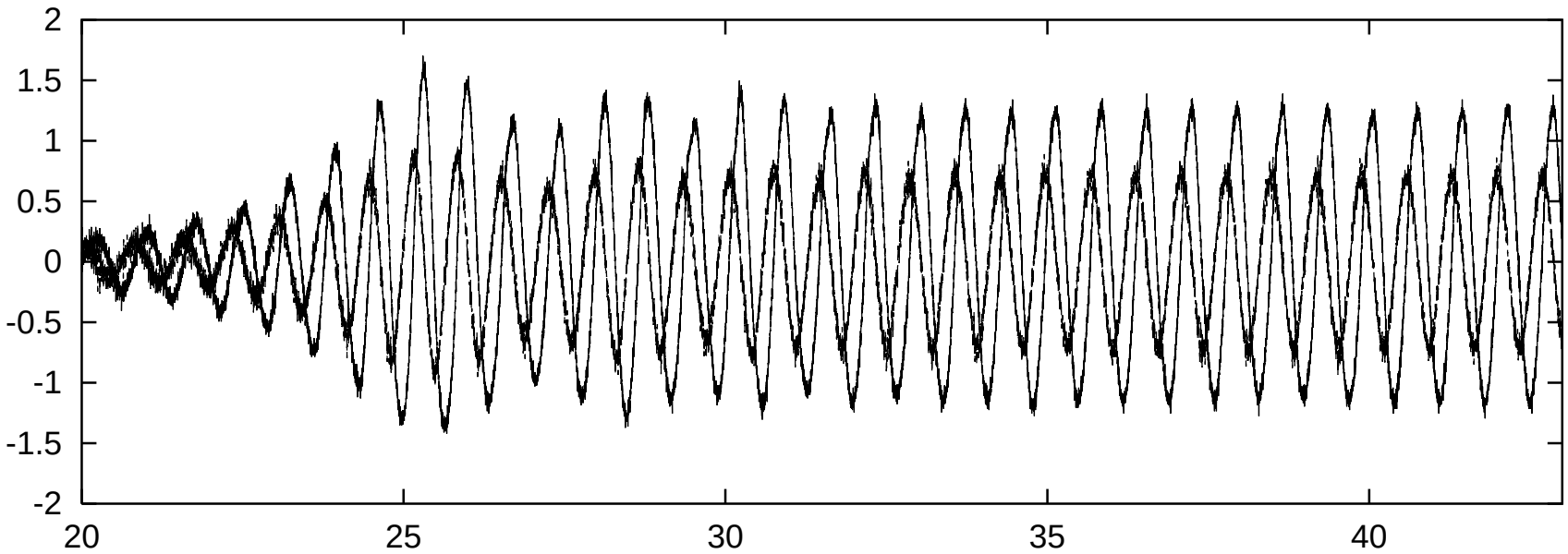
M&M argued that FNV is valid only for very small $KR$, i.e. up to about 0.05. However, the FNV-analysis is a beautiful exercise in matched asymptotics and application of Bessel function theory!



Wave tank experiment, sketch; Huseby & Grue, JFM 414 (2000)



Cylinder and force gauges; Huseby & Grue, JFM 414 (2000)



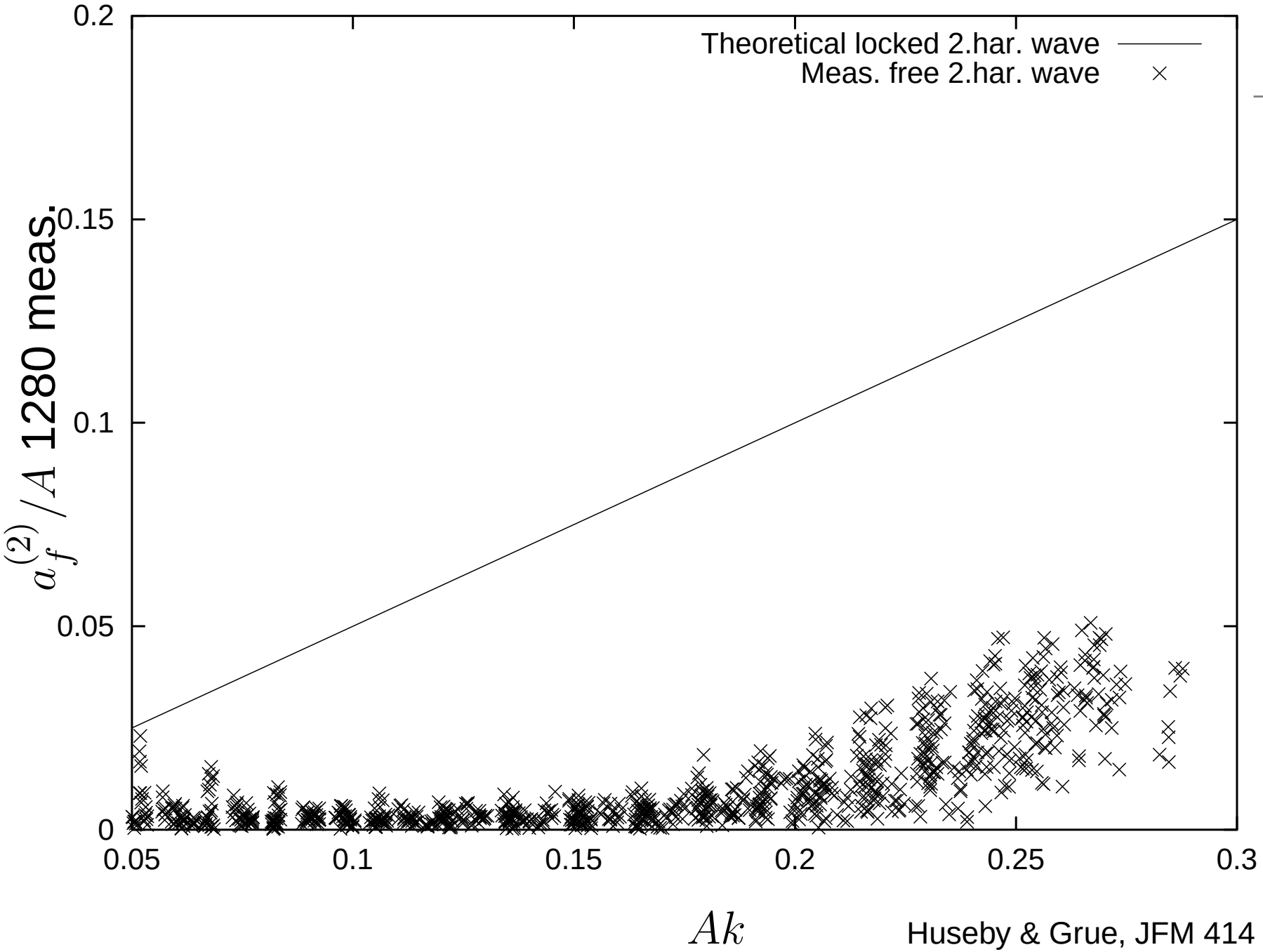

Wave and force series; wave - curve with greater excursion; Huseby & Grue, JFM 414 (2000)

## Wave elevation, Stokes waves

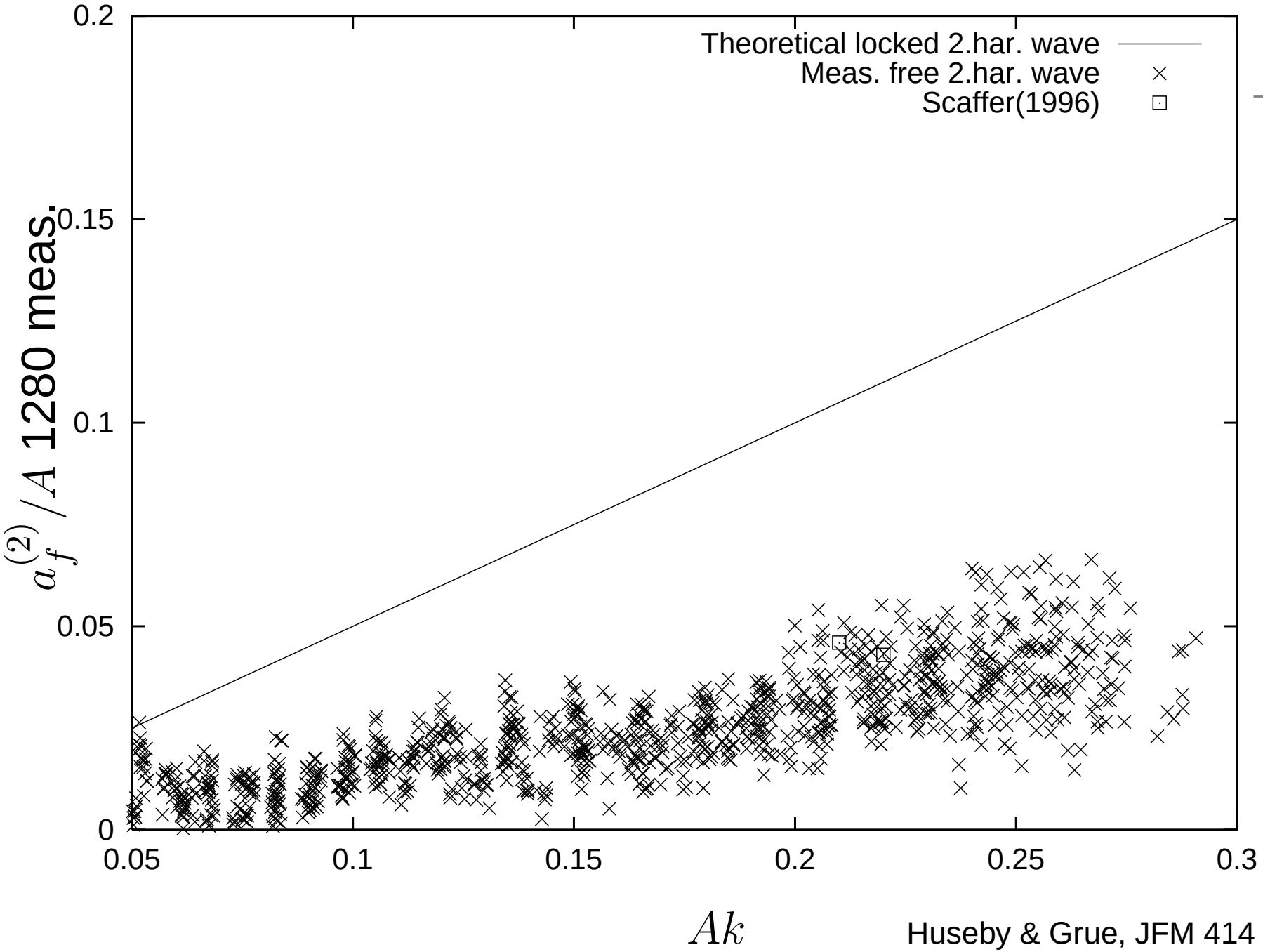
$$\begin{aligned}\eta &= A\cos(-kx+\omega t) + \tfrac{1}{2}kA^2\cos 2(-kx+\omega t)\\ &+ a_{free}\cos(-4kx+2\omega t+\delta) + ...\end{aligned}$$

The free 2nd harmonic wave is generated at the wave paddle, by nonlinear effects, and is a parasittic wave

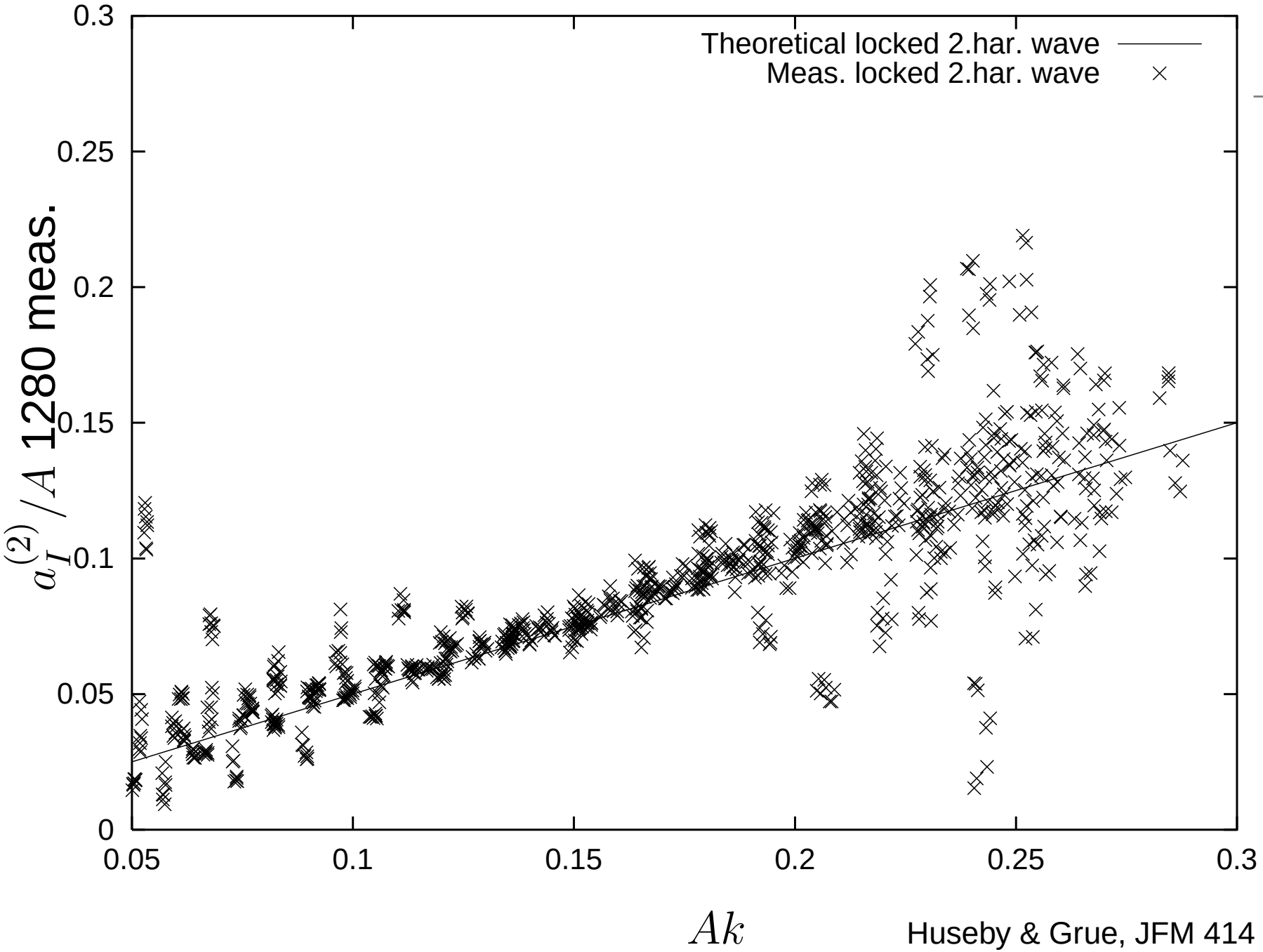
However, it moves with half the speed of the Stokes wave and therefore there is a time window without this parasittic wave.



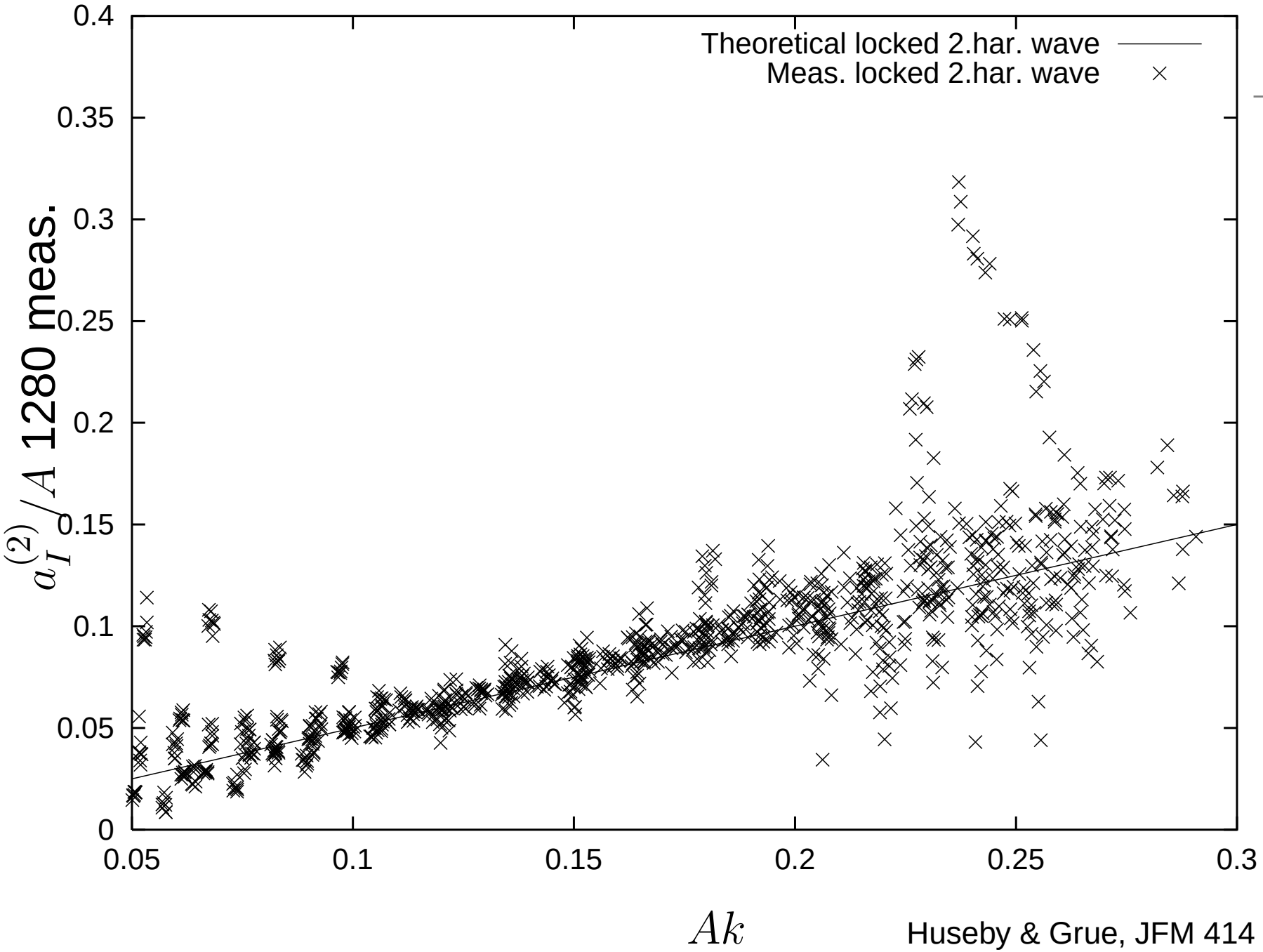

Huseby & Grue, JFM 414 (2000)




Huseby & Grue, JFM 414 (2000)




Huseby & Grue, JFM 414 (2000)

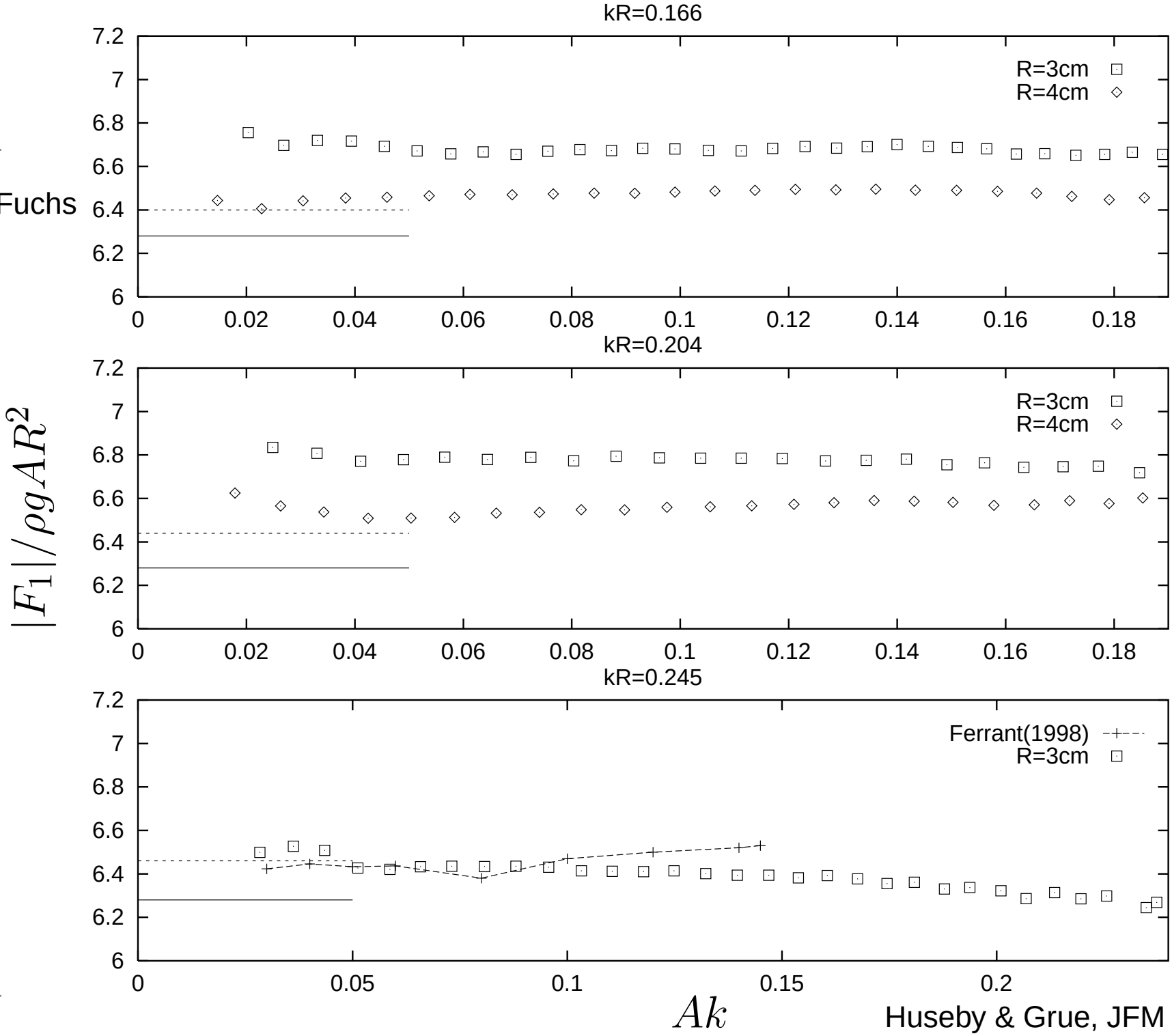



Huseby & Grue, JFM 414 (2000)

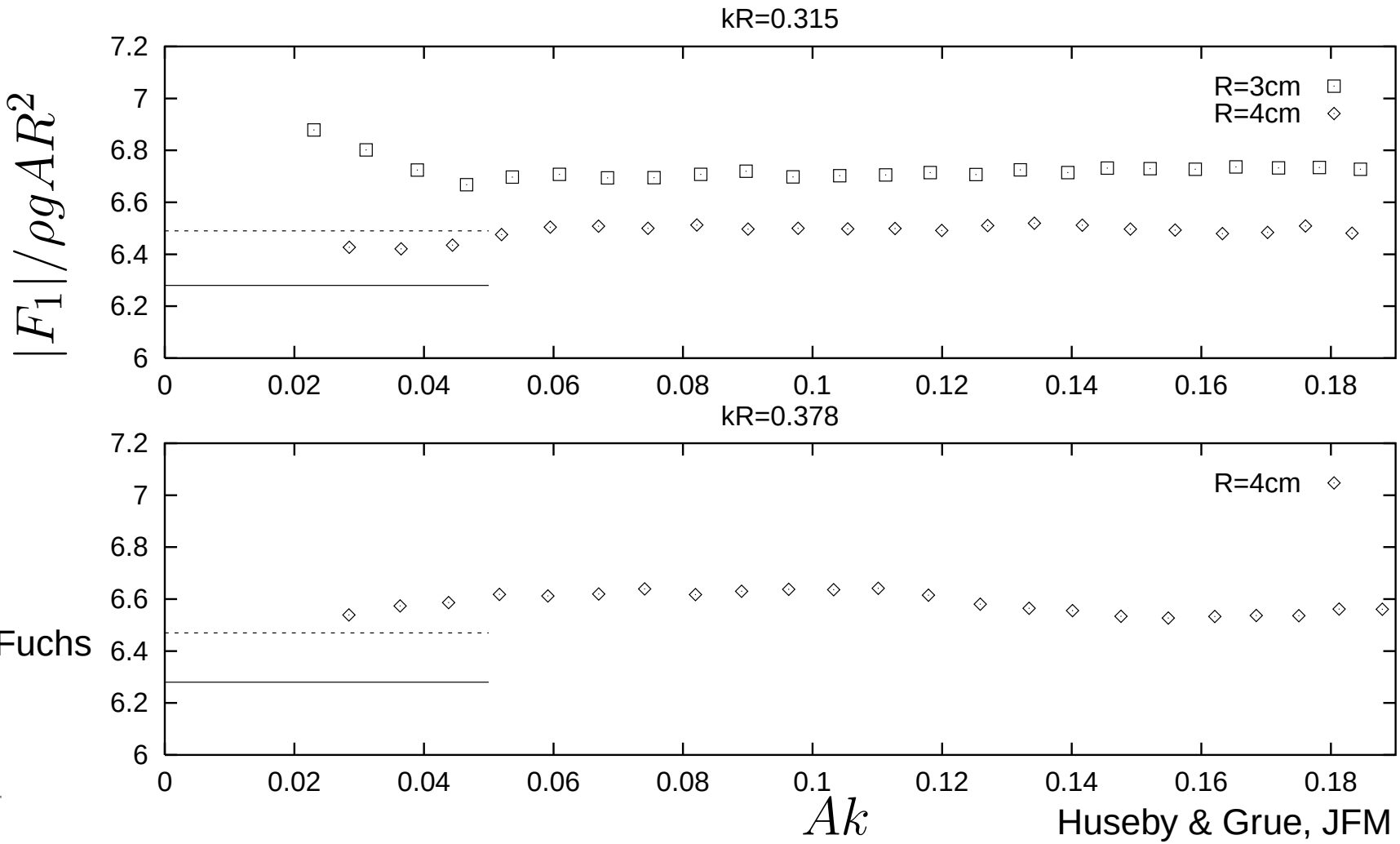
# First harmonic force

McCamy&Fuchs

$2\pi$




Huseby & Grue, JFM 414 (2000)

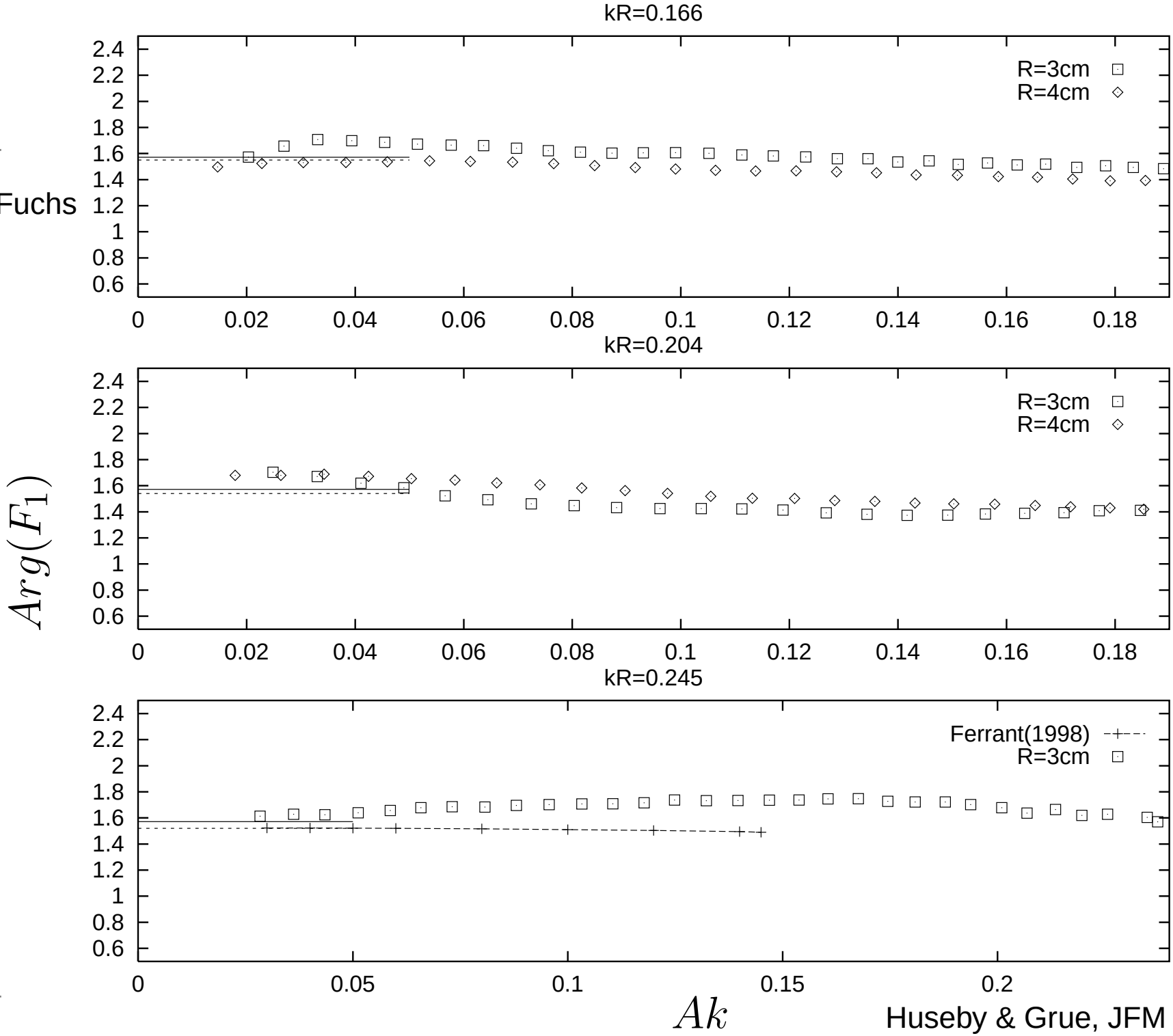



McCamy&Fuchs

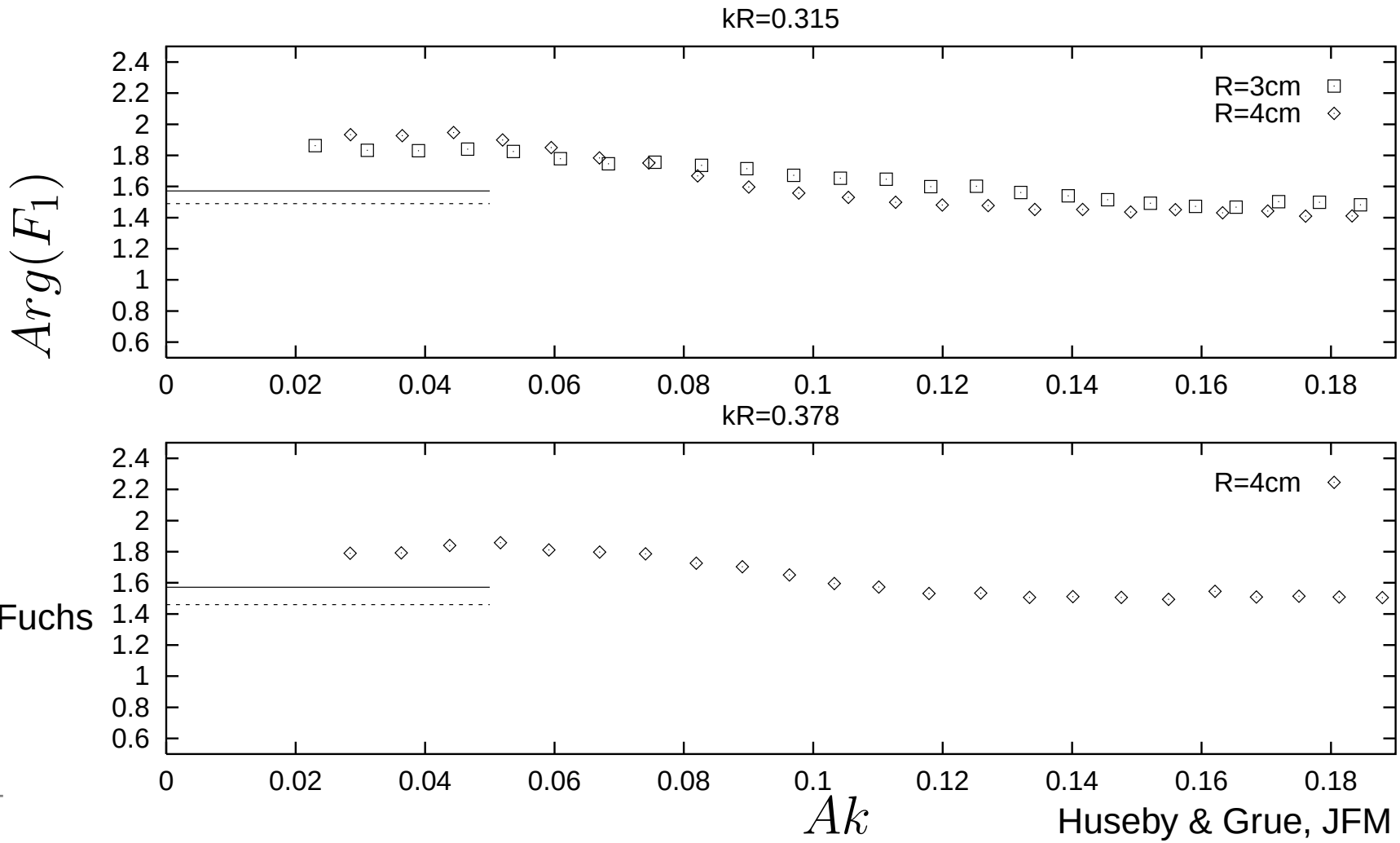
$2\pi$

Huseby & Grue, JFM 414 (2000)

$\pi/2$
McCamy&Fuchs




Huseby & Grue, JFM 414 (2000)

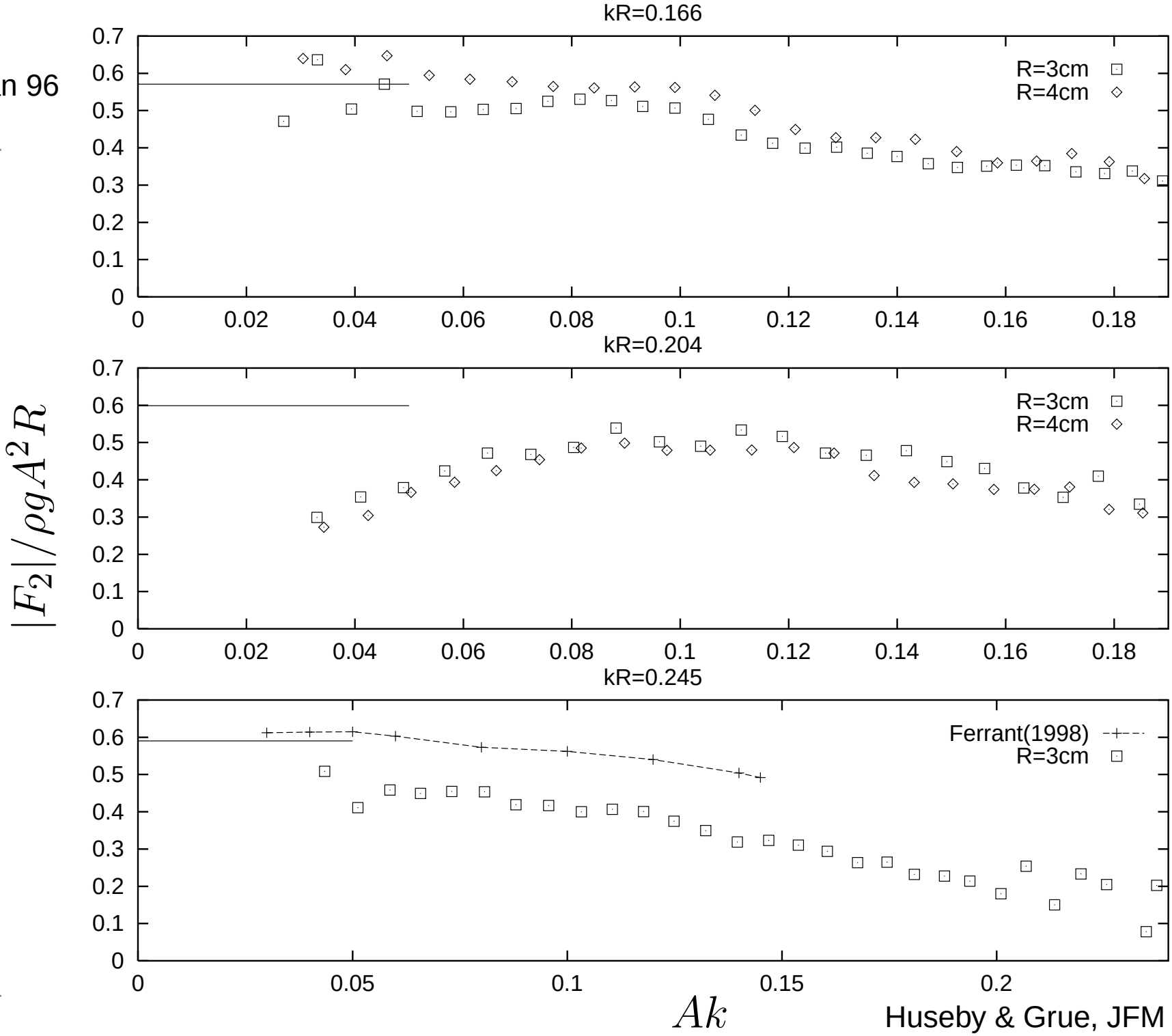



$\pi/2$
McCamy&Fuchs

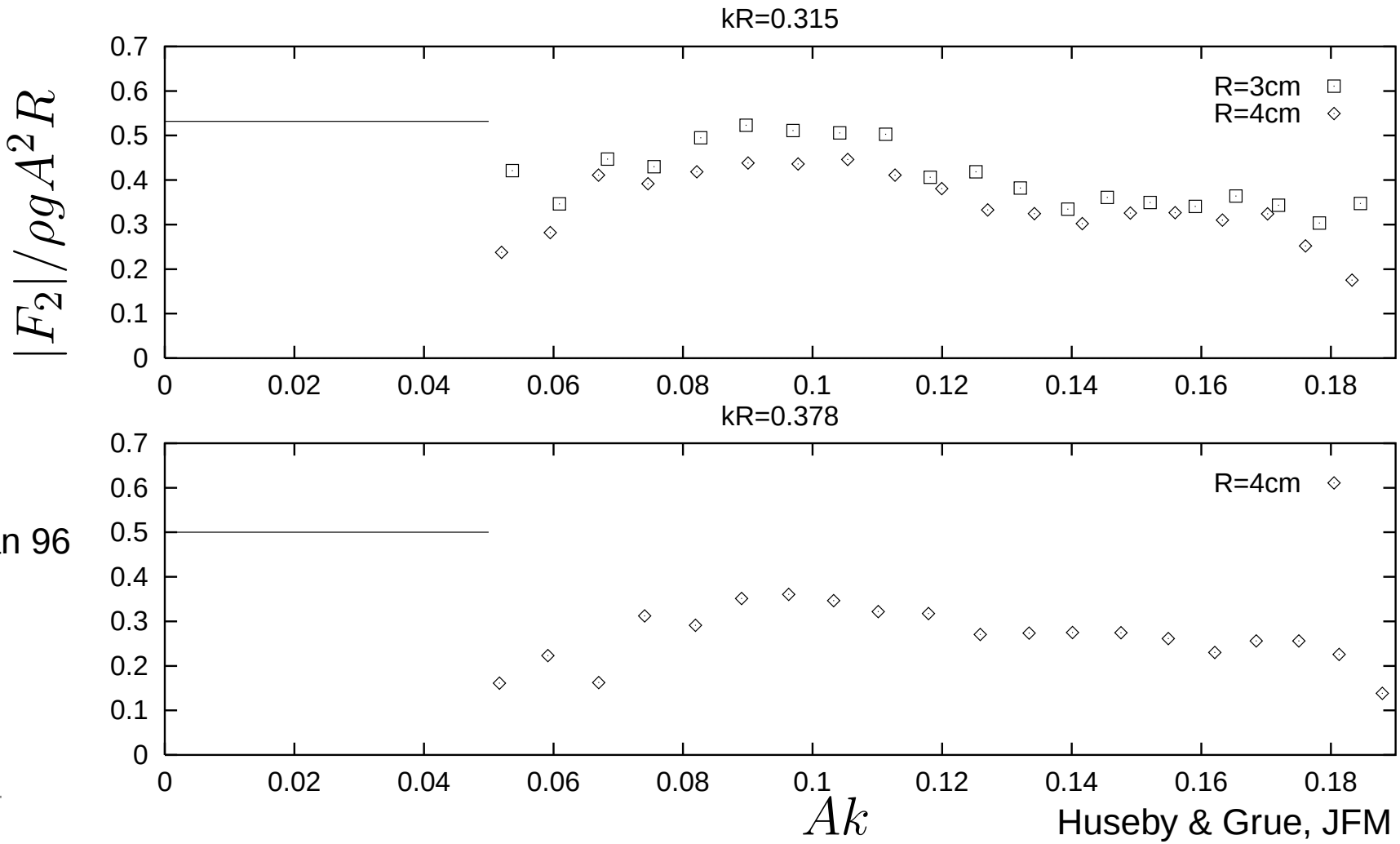
Huseby & Grue, JFM 414 (2000)

# 2nd harmonic force

Newman 96



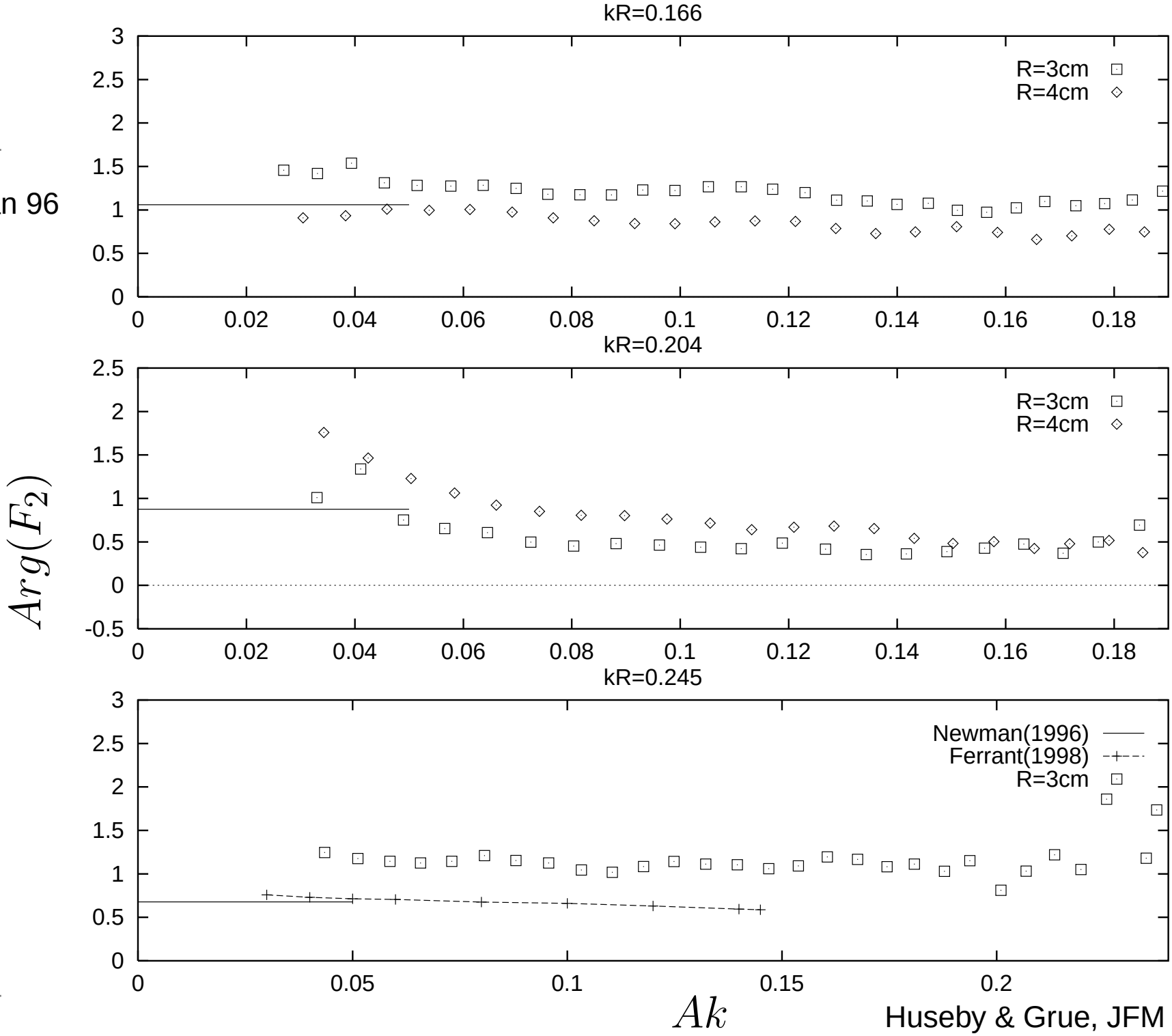

Huseby & Grue, JFM 414 (2000)



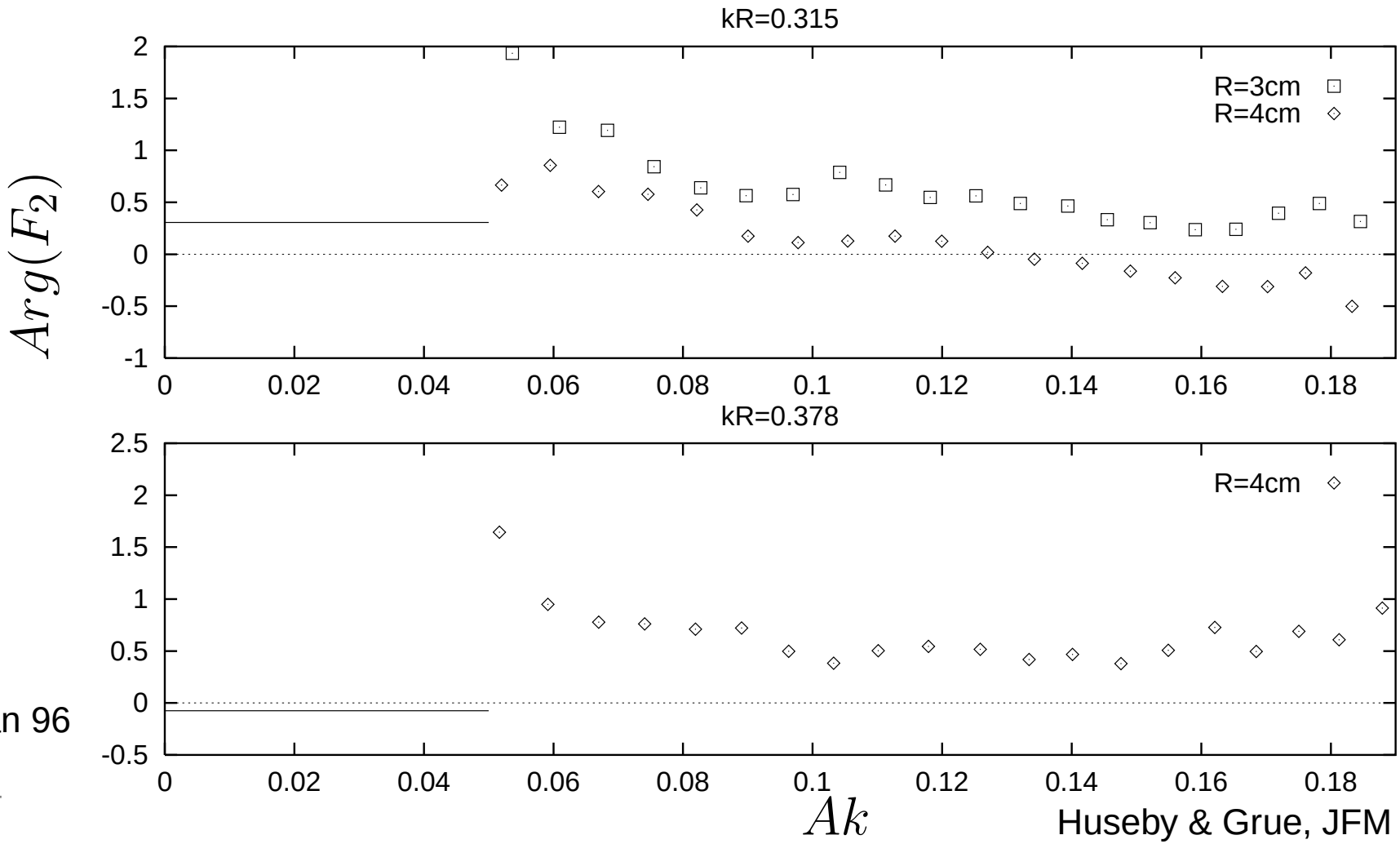

Newman 96

Huseby & Grue, JFM 414 (2000)

Newman 96

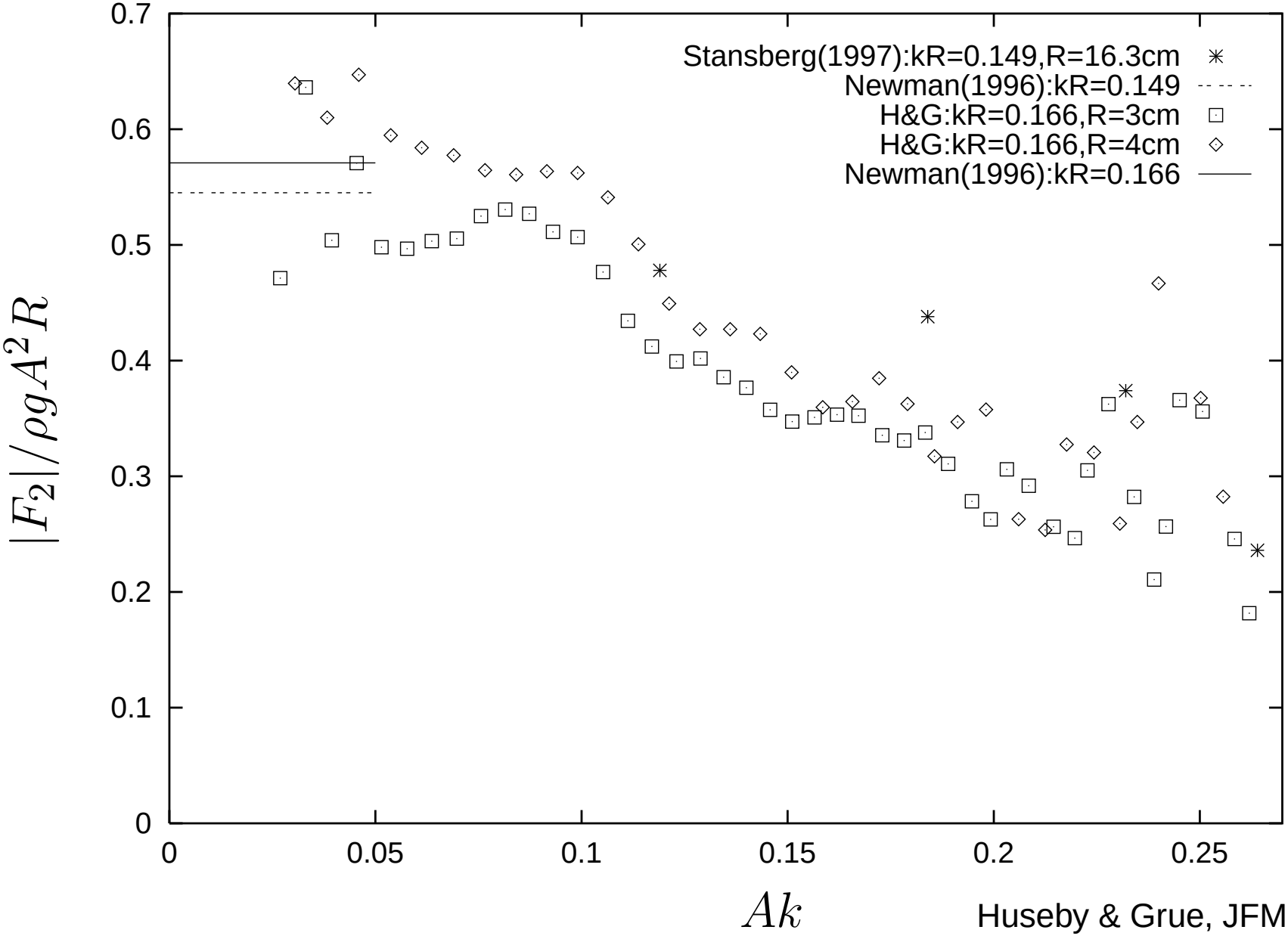



Huseby & Grue, JFM 414 (2000)

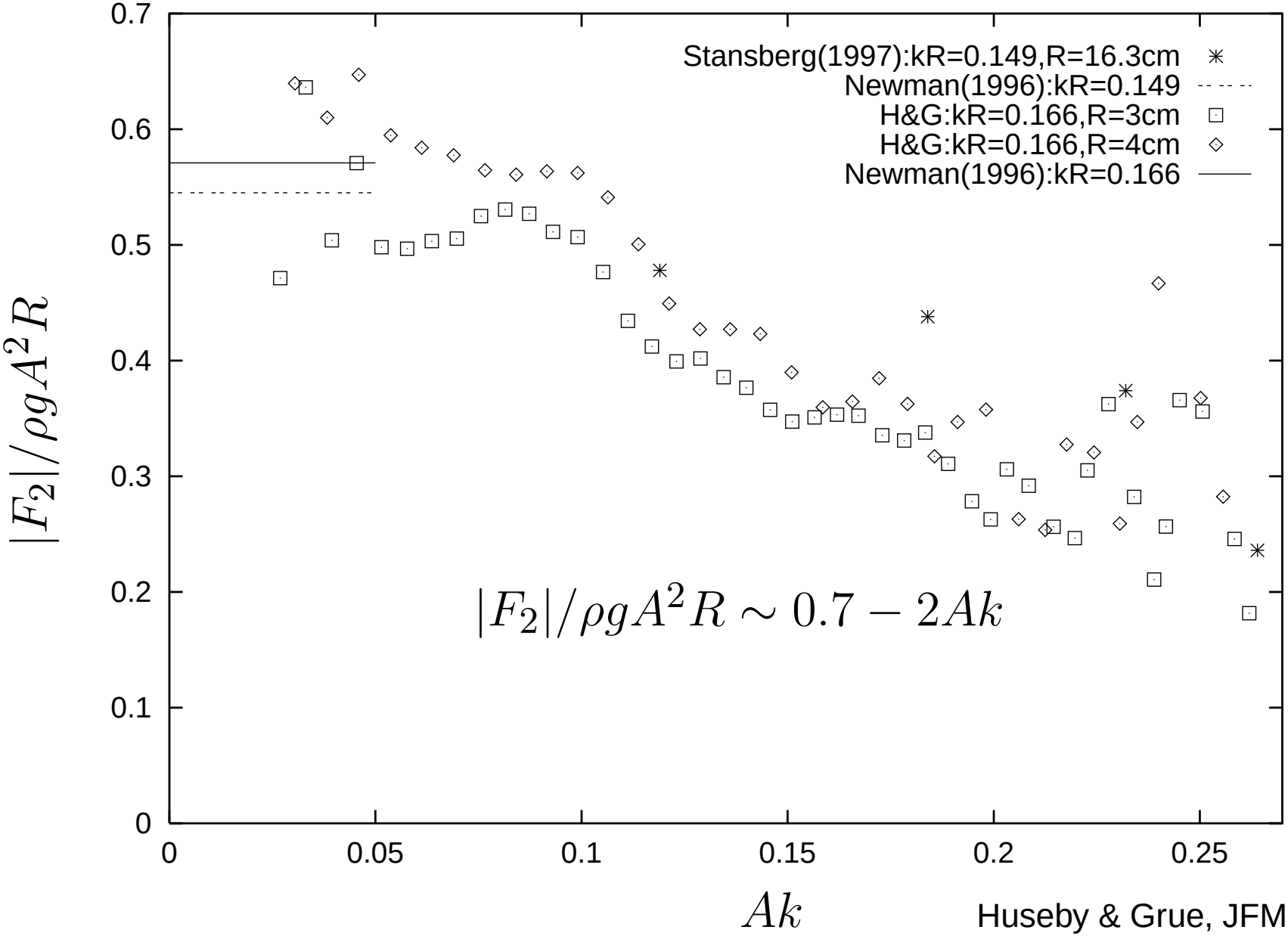



Newman 96

Huseby & Grue, JFM 414 (2000)

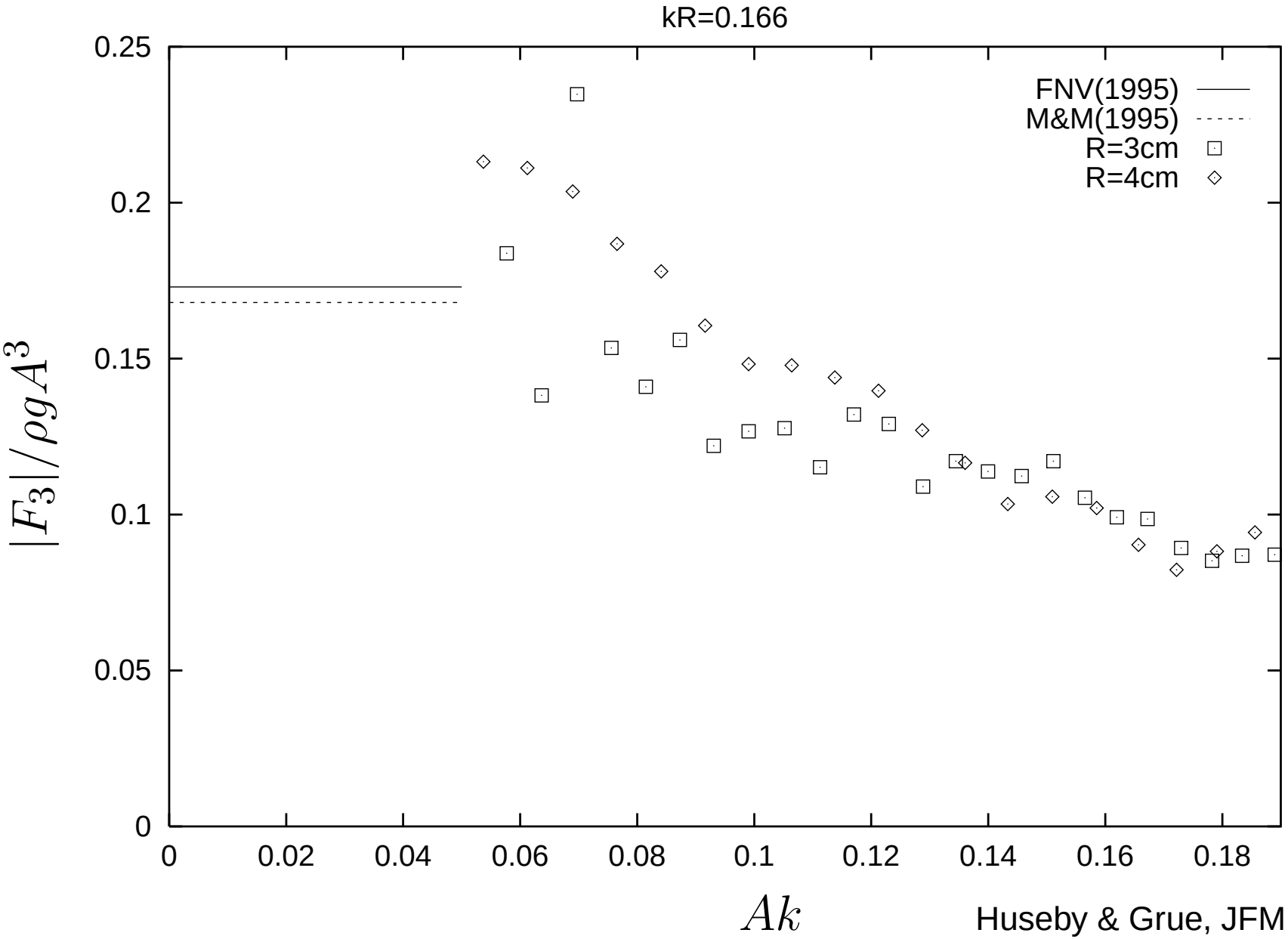



Huseby & Grue, JFM 414 (2000)

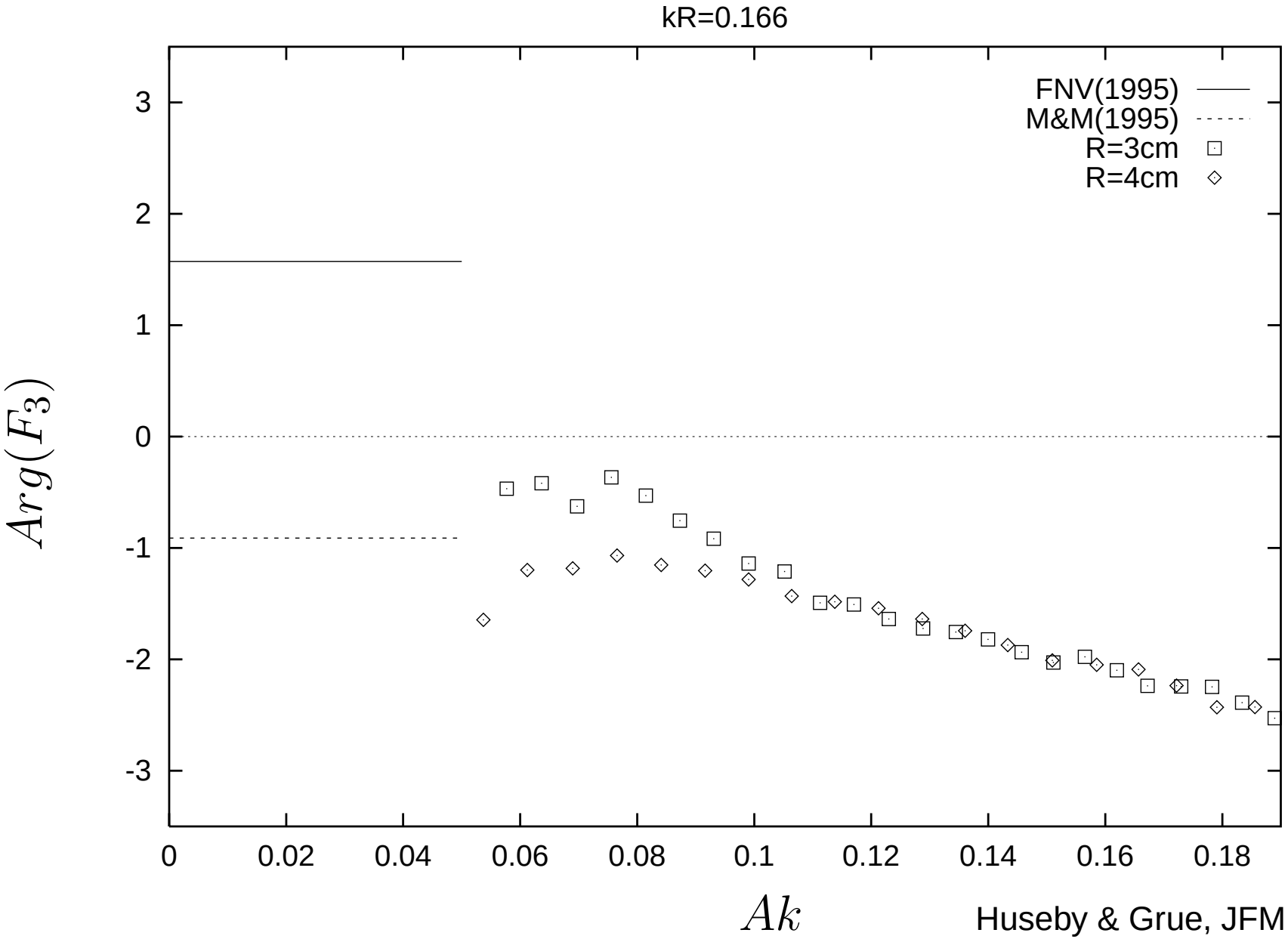



Huseby & Grue, JFM 414 (2000)

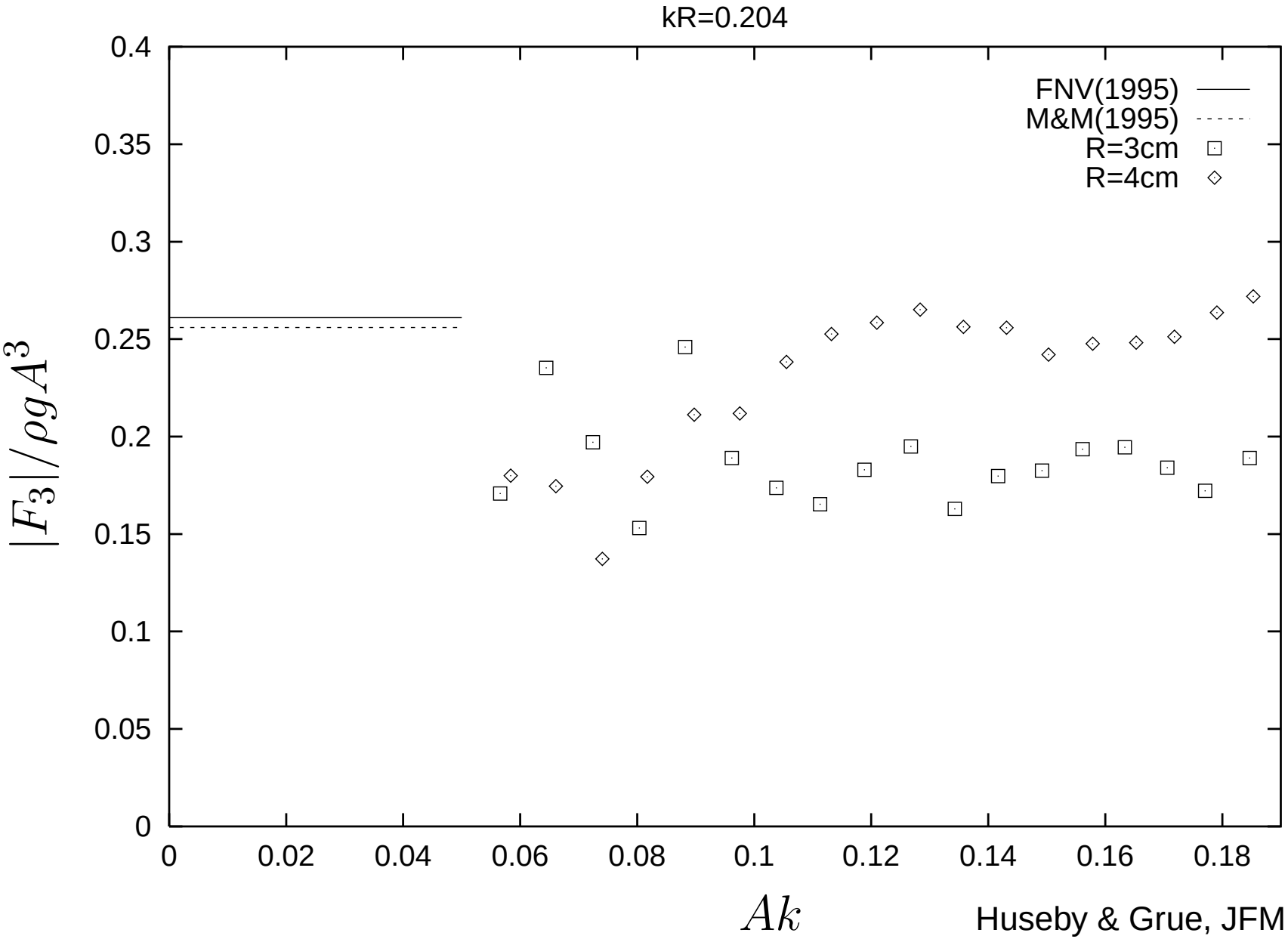
# 3rd harmonic force



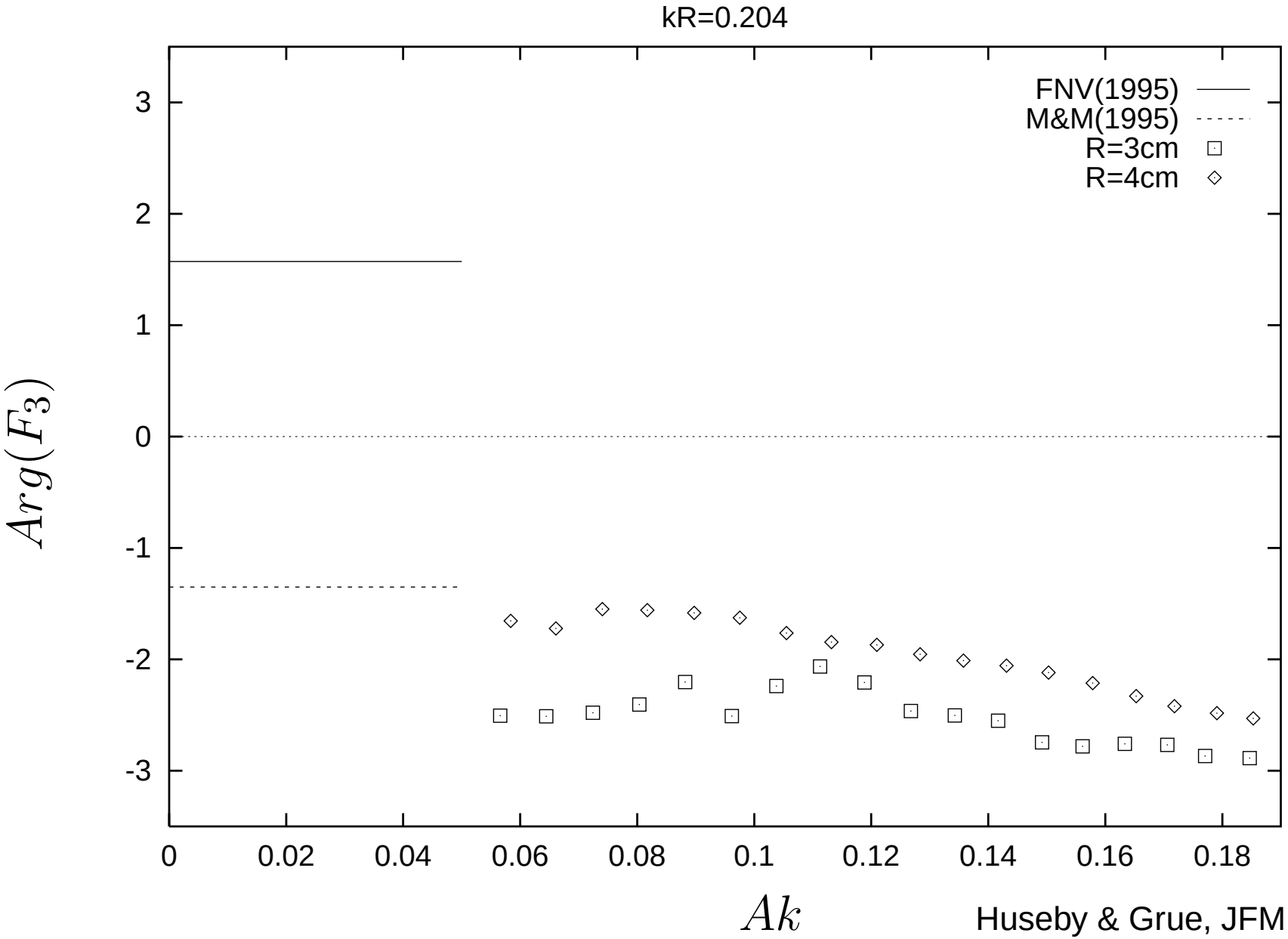

Huseby & Grue, JFM 414 (2000)



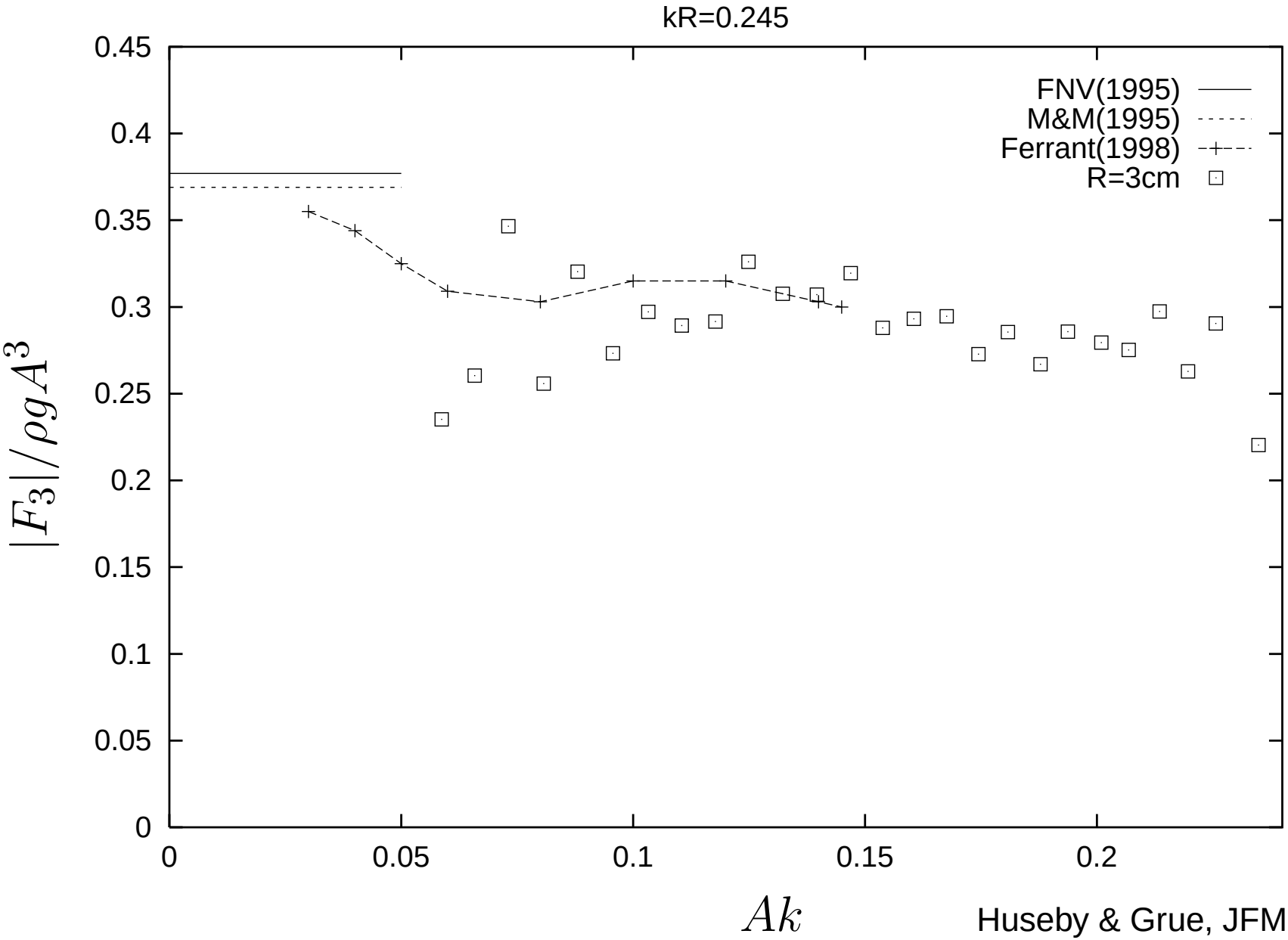

Huseby & Grue, JFM 414 (2000)



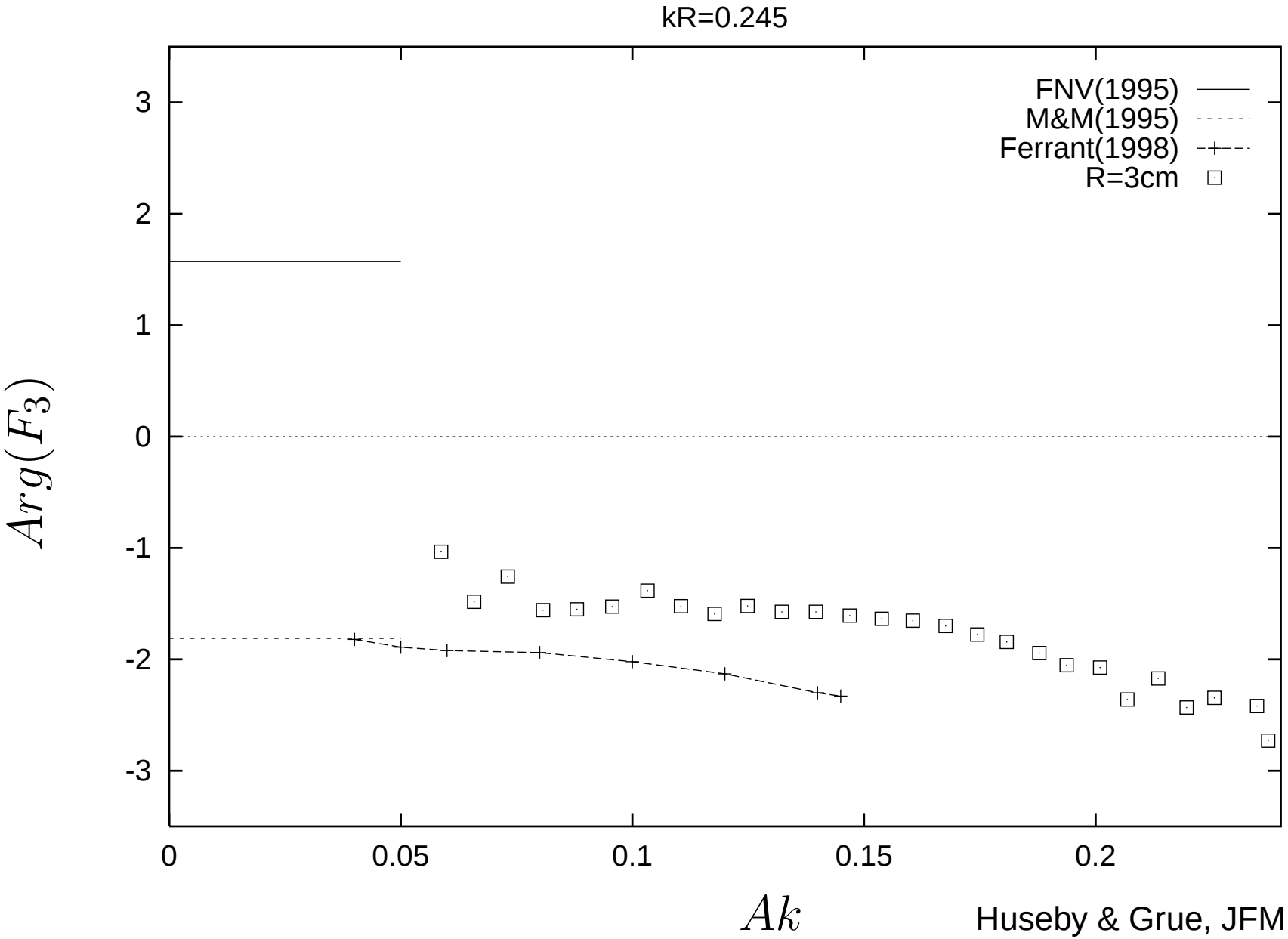

Huseby & Grue, JFM 414 (2000)



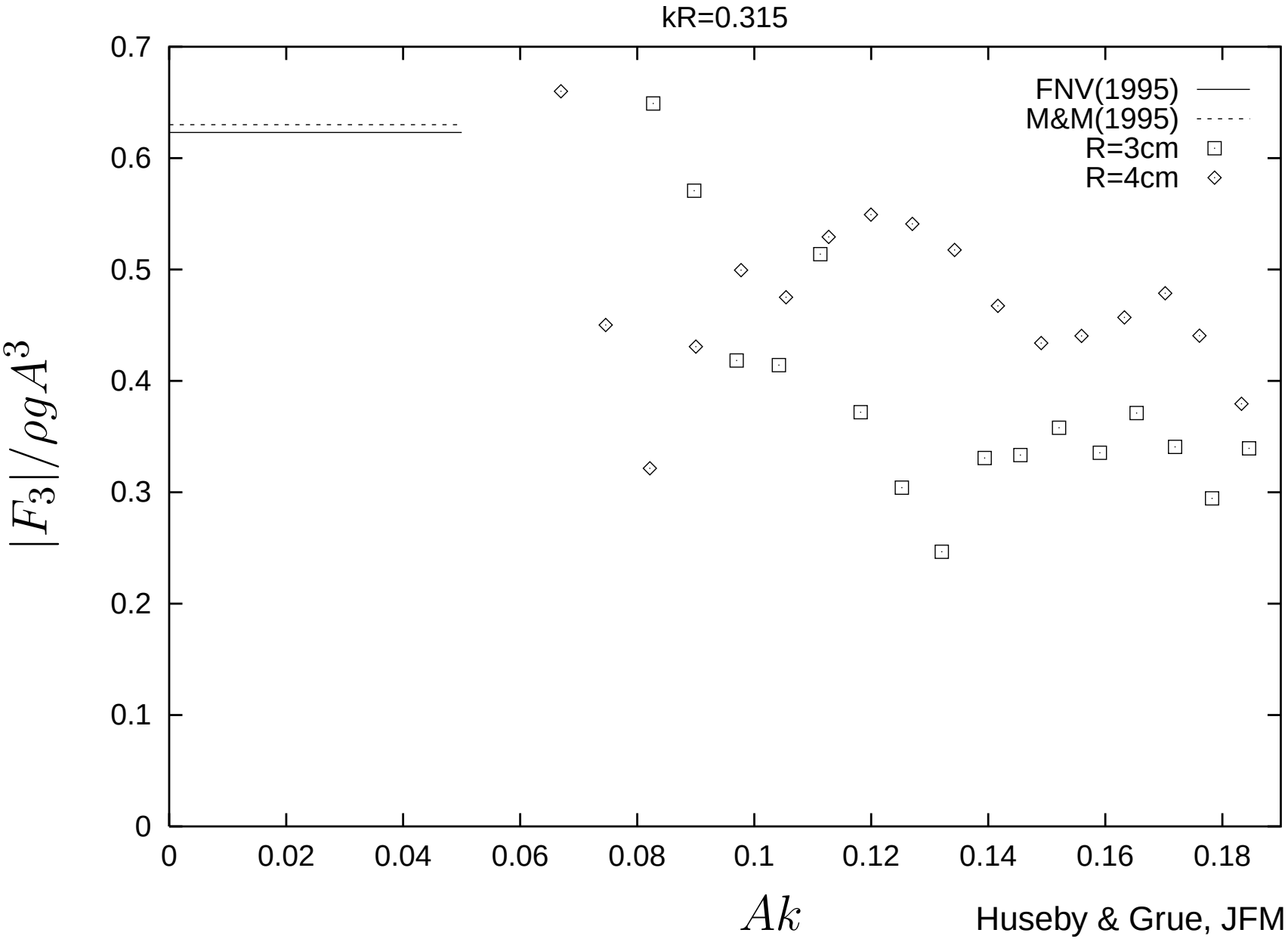

Huseby & Grue, JFM 414 (2000)



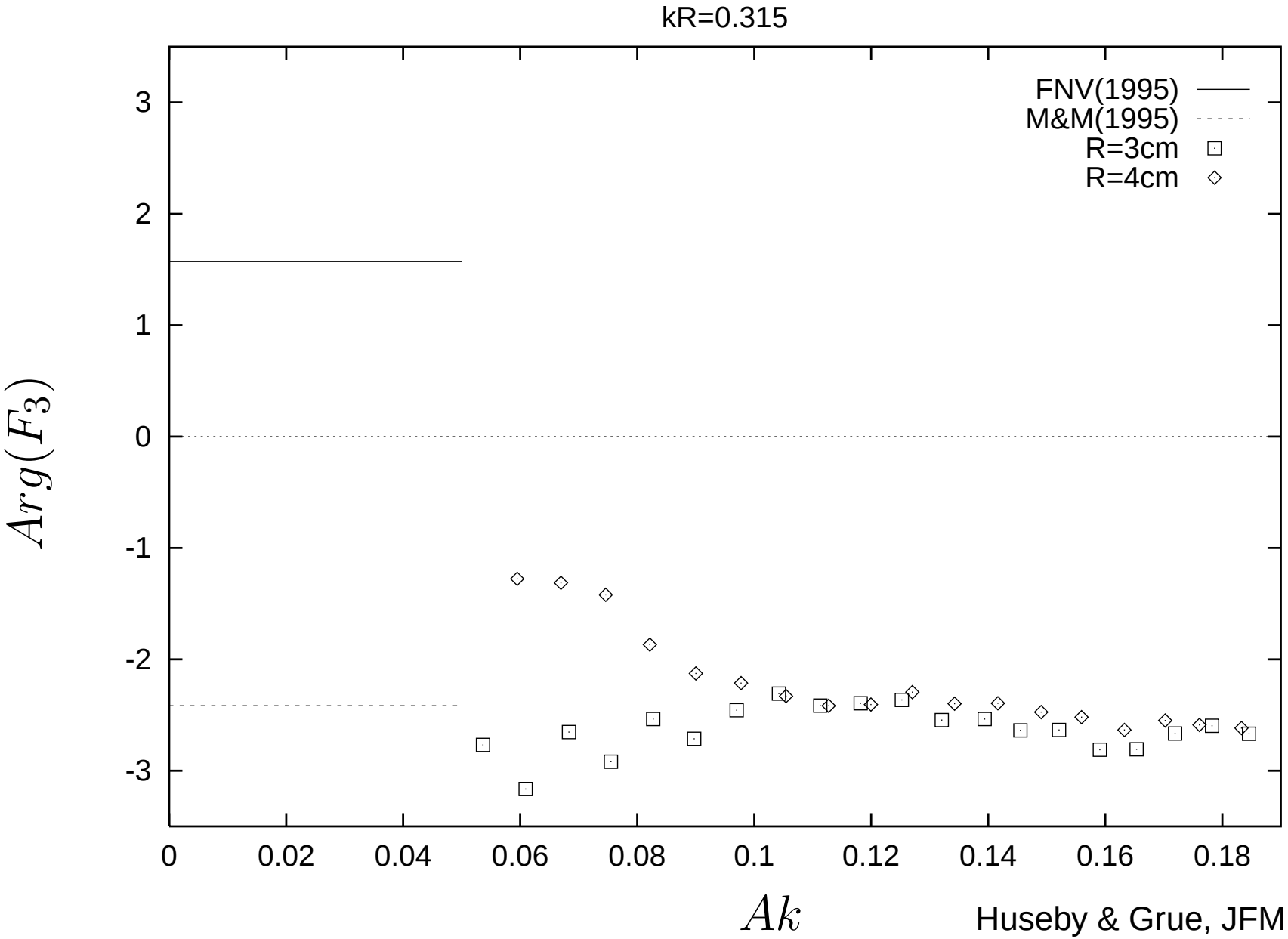

Huseby & Grue, JFM 414 (2000)



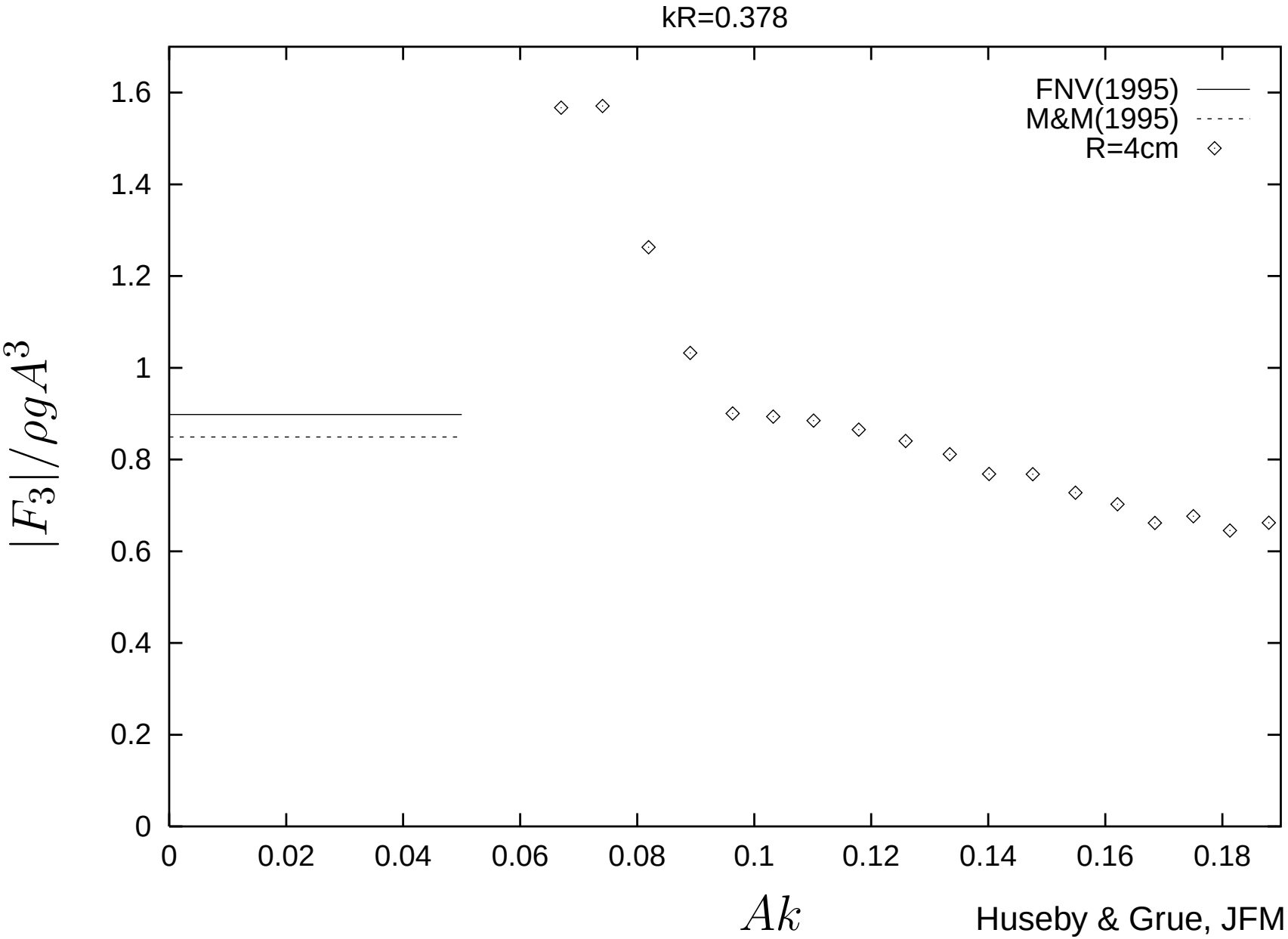

Huseby & Grue, JFM 414 (2000)



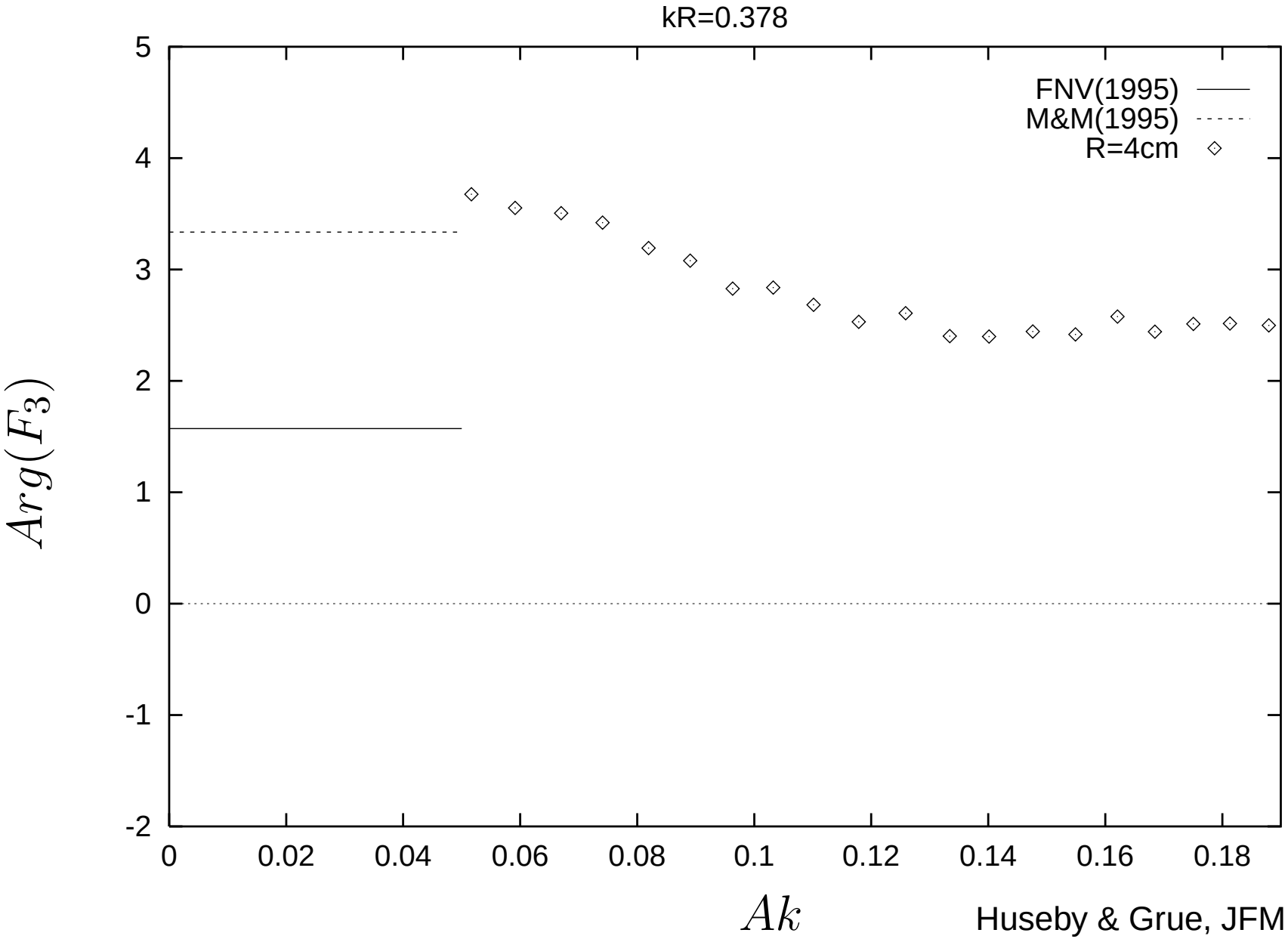

Huseby & Grue, JFM 414 (2000)



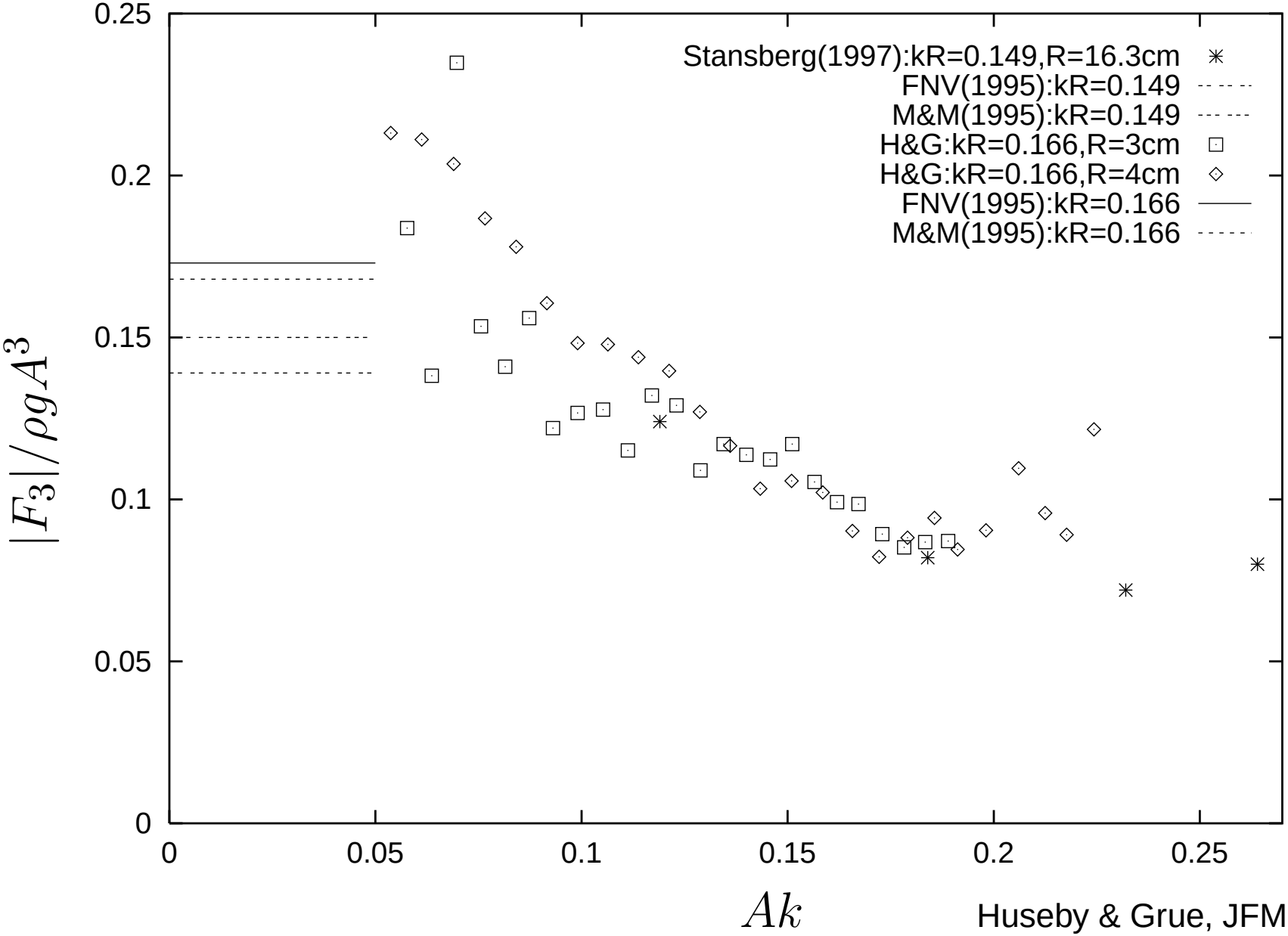

Huseby & Grue, JFM 414 (2000)



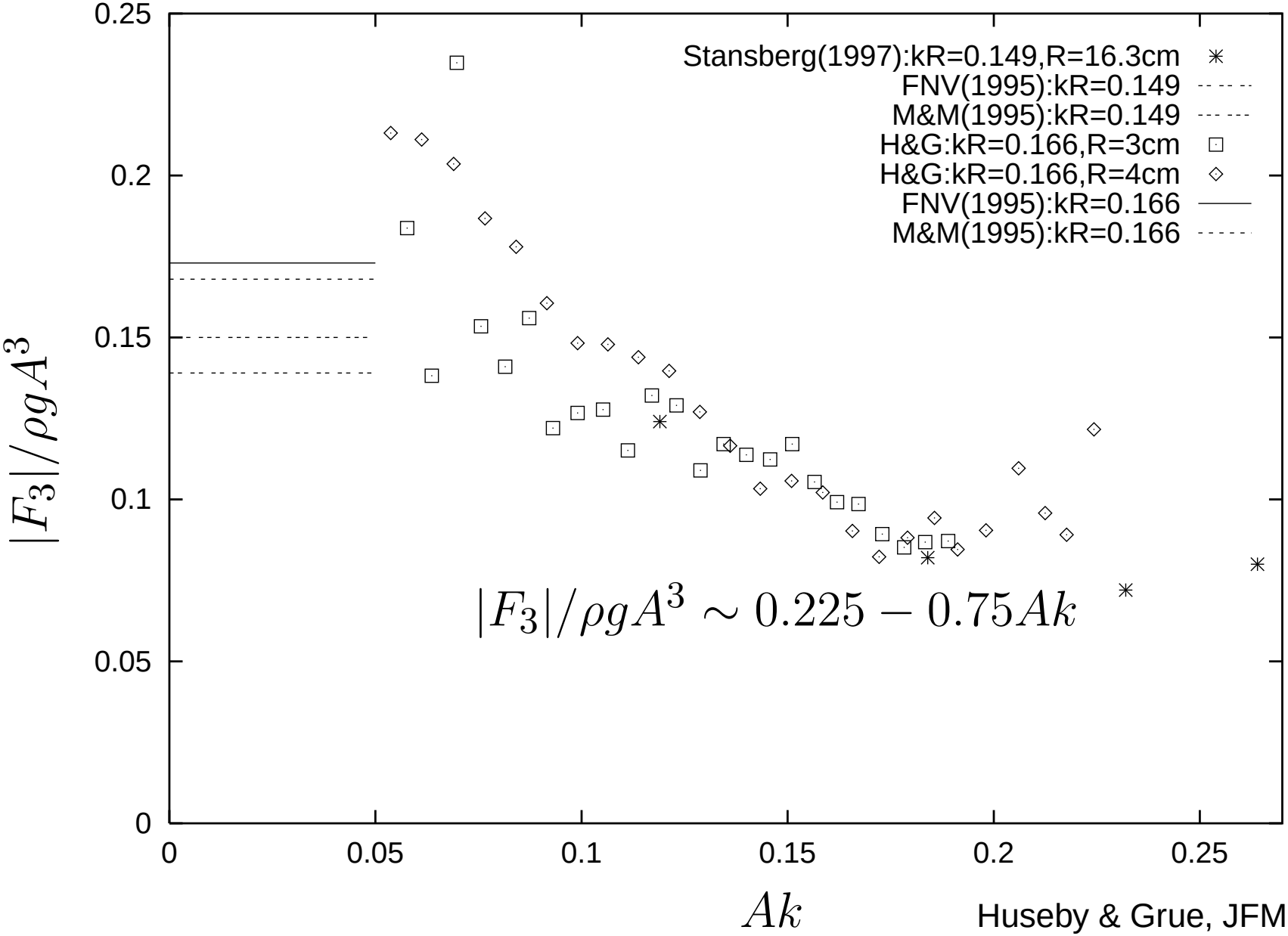

Huseby & Grue, JFM 414 (2000)




Huseby & Grue, JFM 414 (2000)

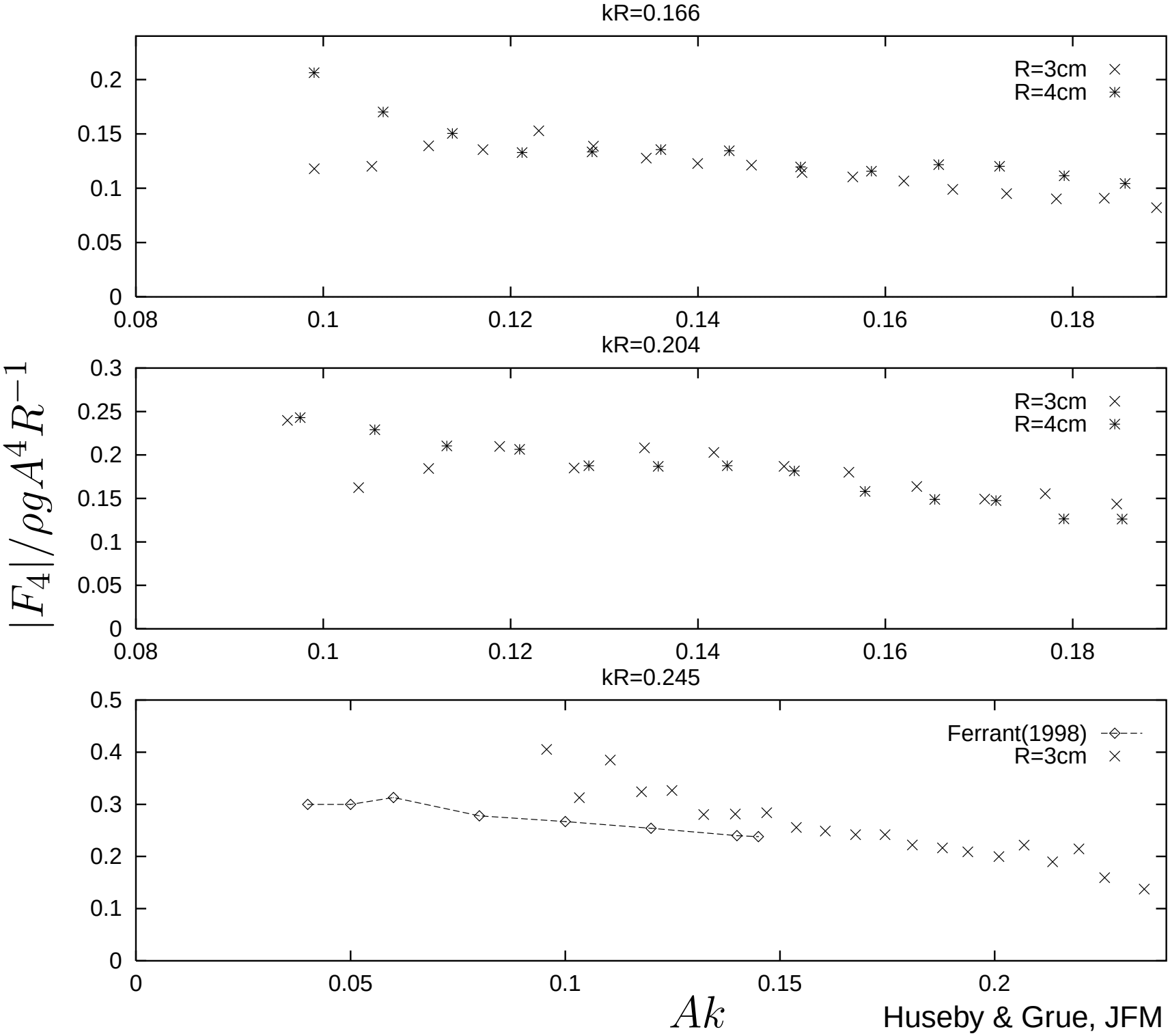



Huseby & Grue, JFM 414 (2000)

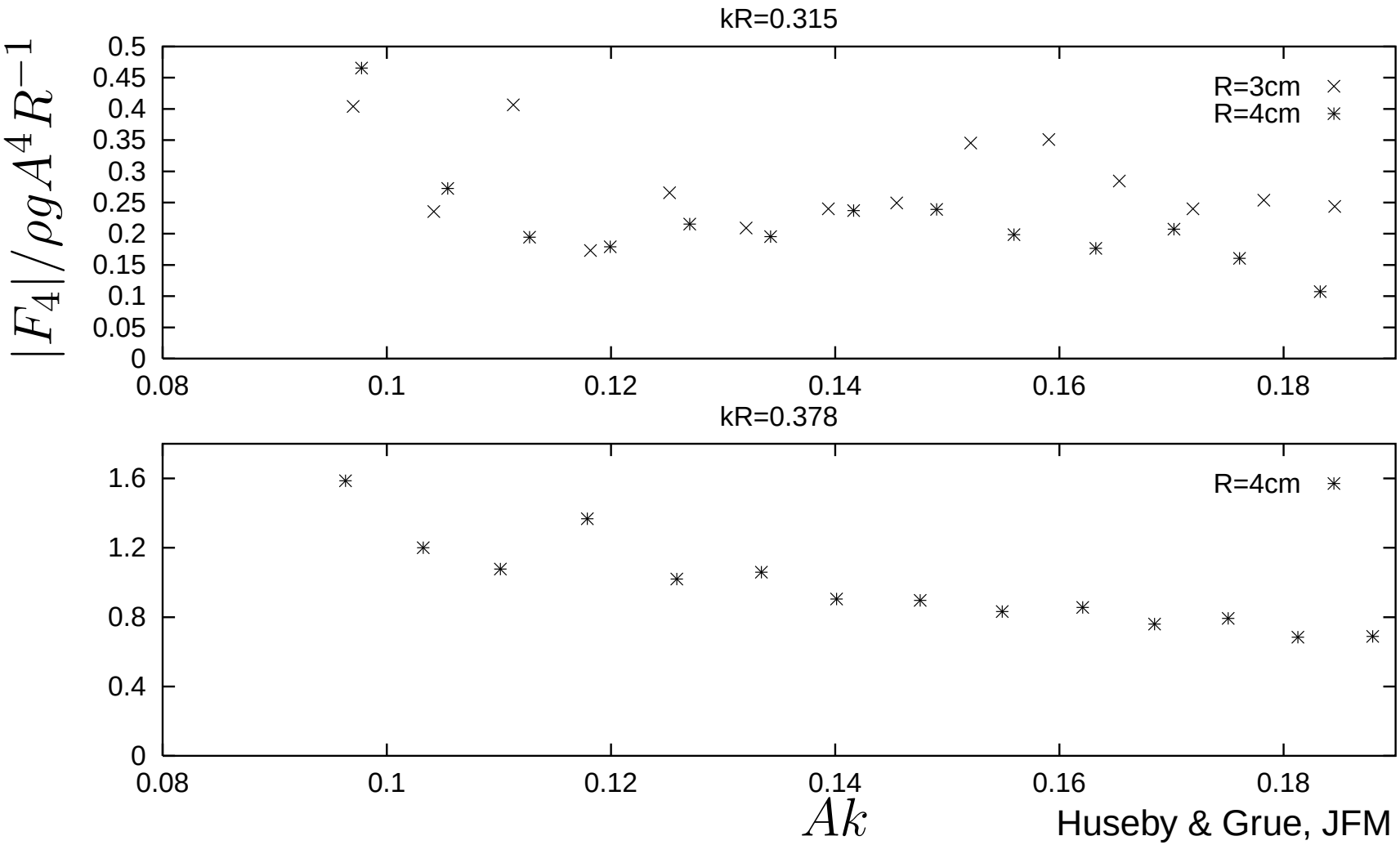



Huseby & Grue, JFM 414 (2000)

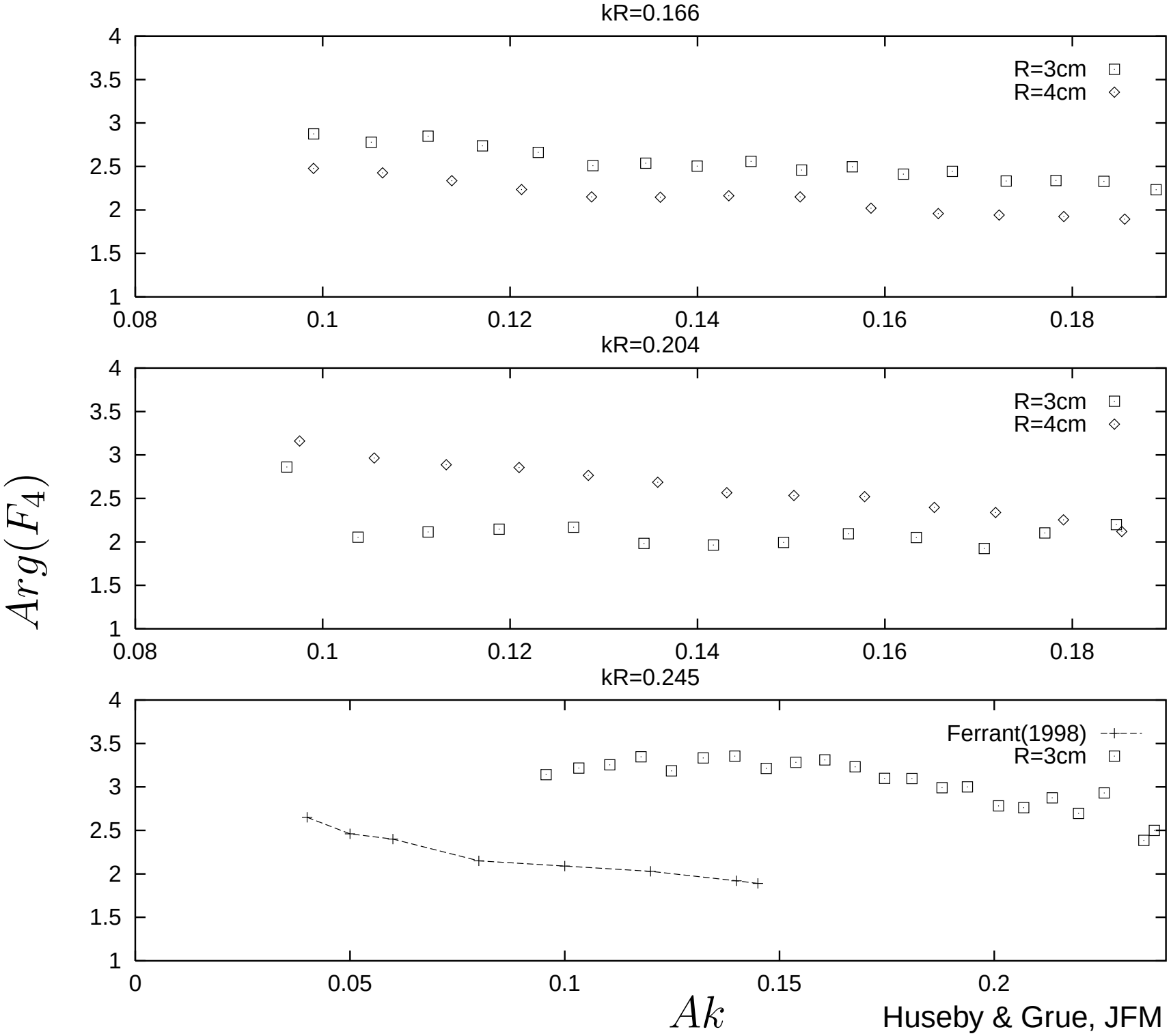
# Fourth harmonic force



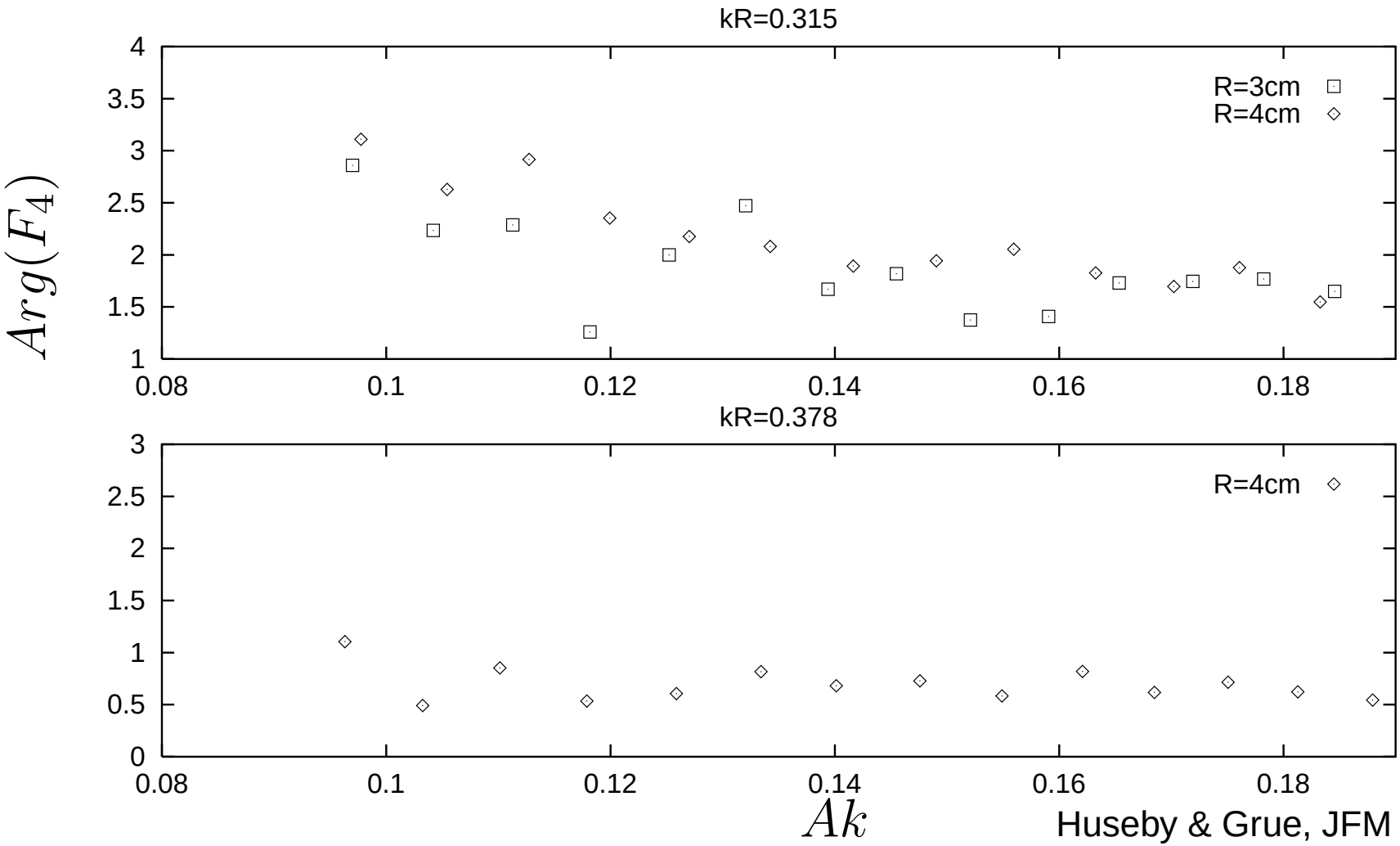

Huseby & Grue, JFM 414 (2000)




Huseby & Grue, JFM 414 (2000)

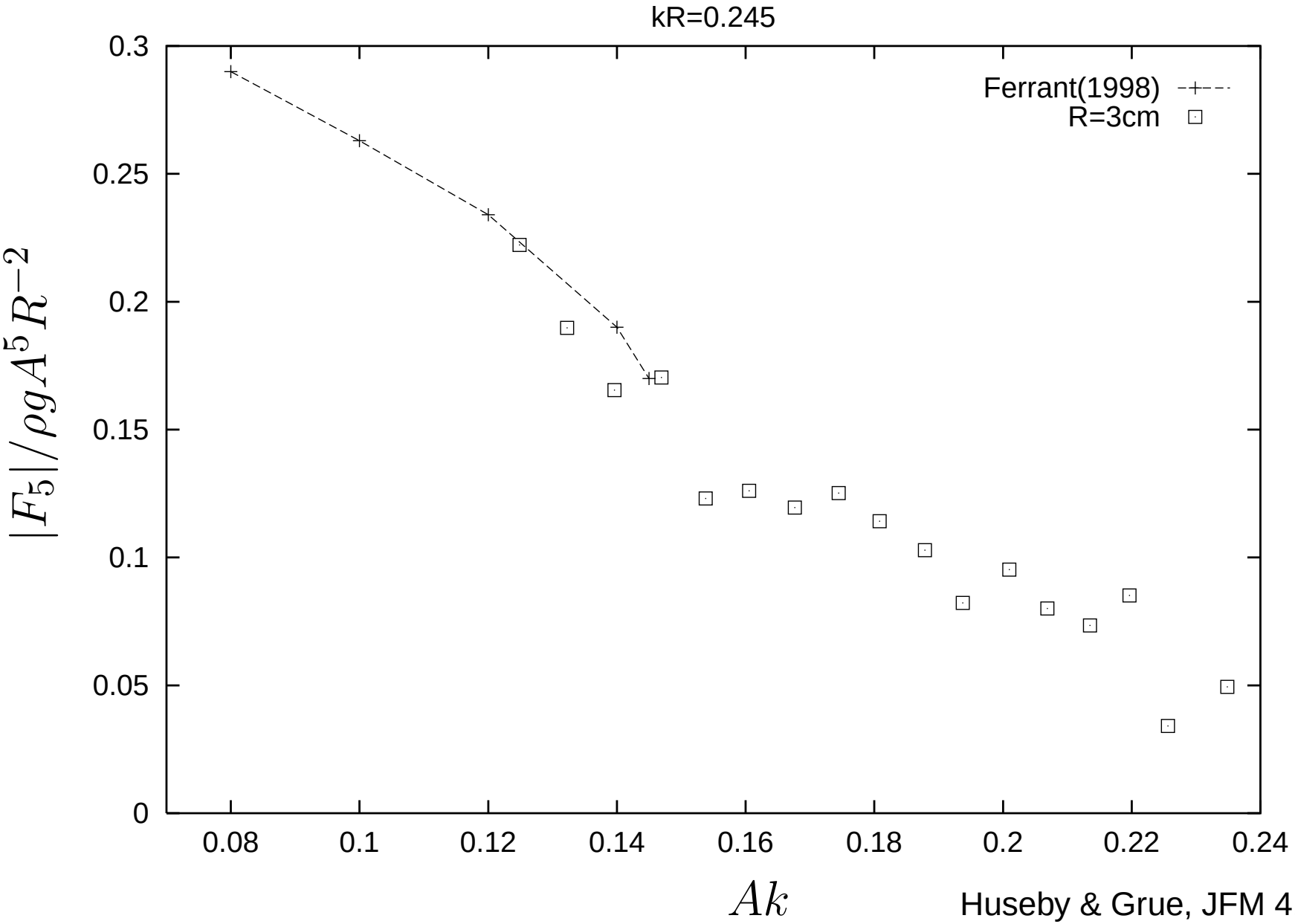



Huseby & Grue, JFM 414 (2000)

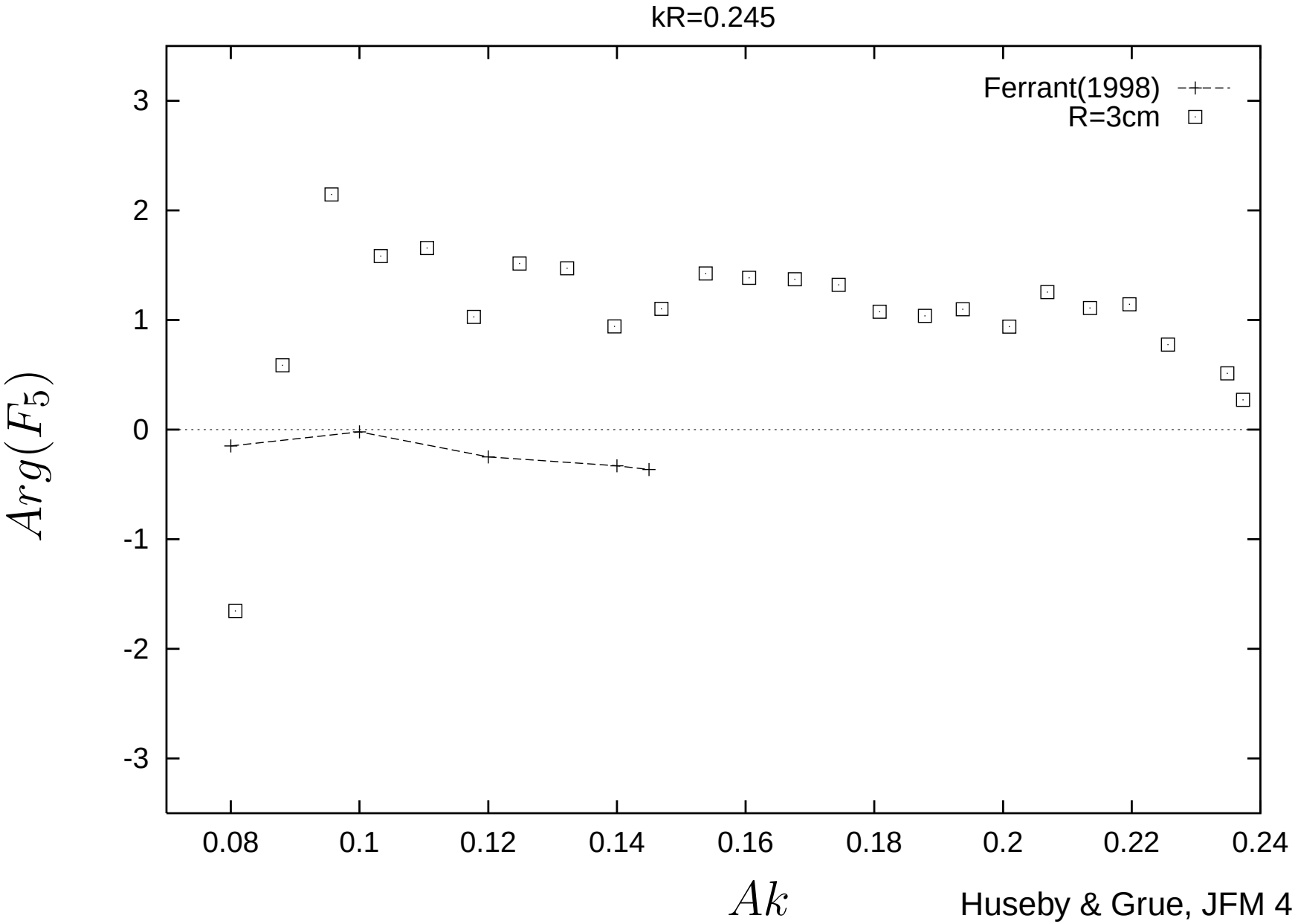



Huseby & Grue, JFM 414 (2000)

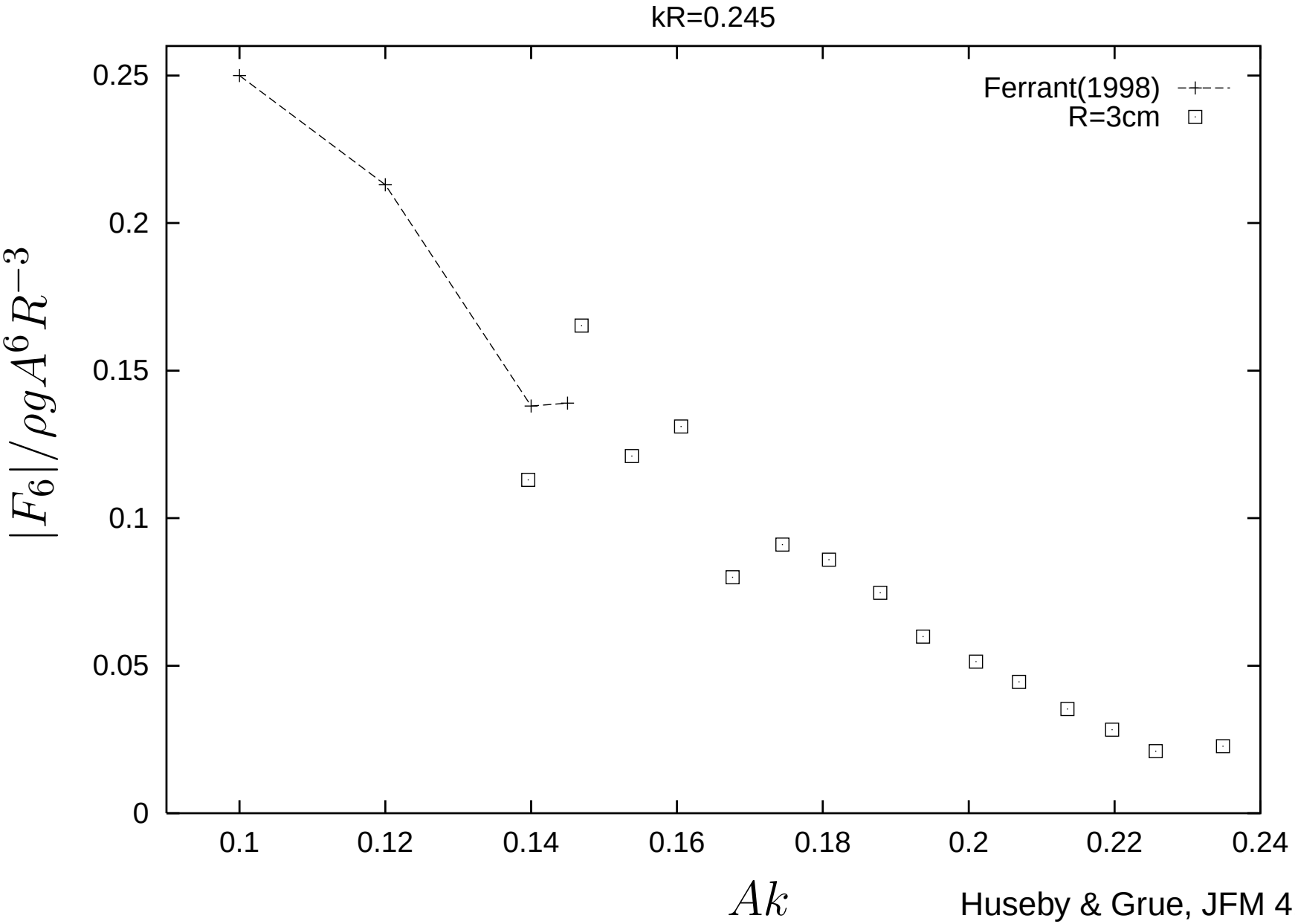
# Fifth harmonic force




Huseby & Grue, JFM 414 (2000)

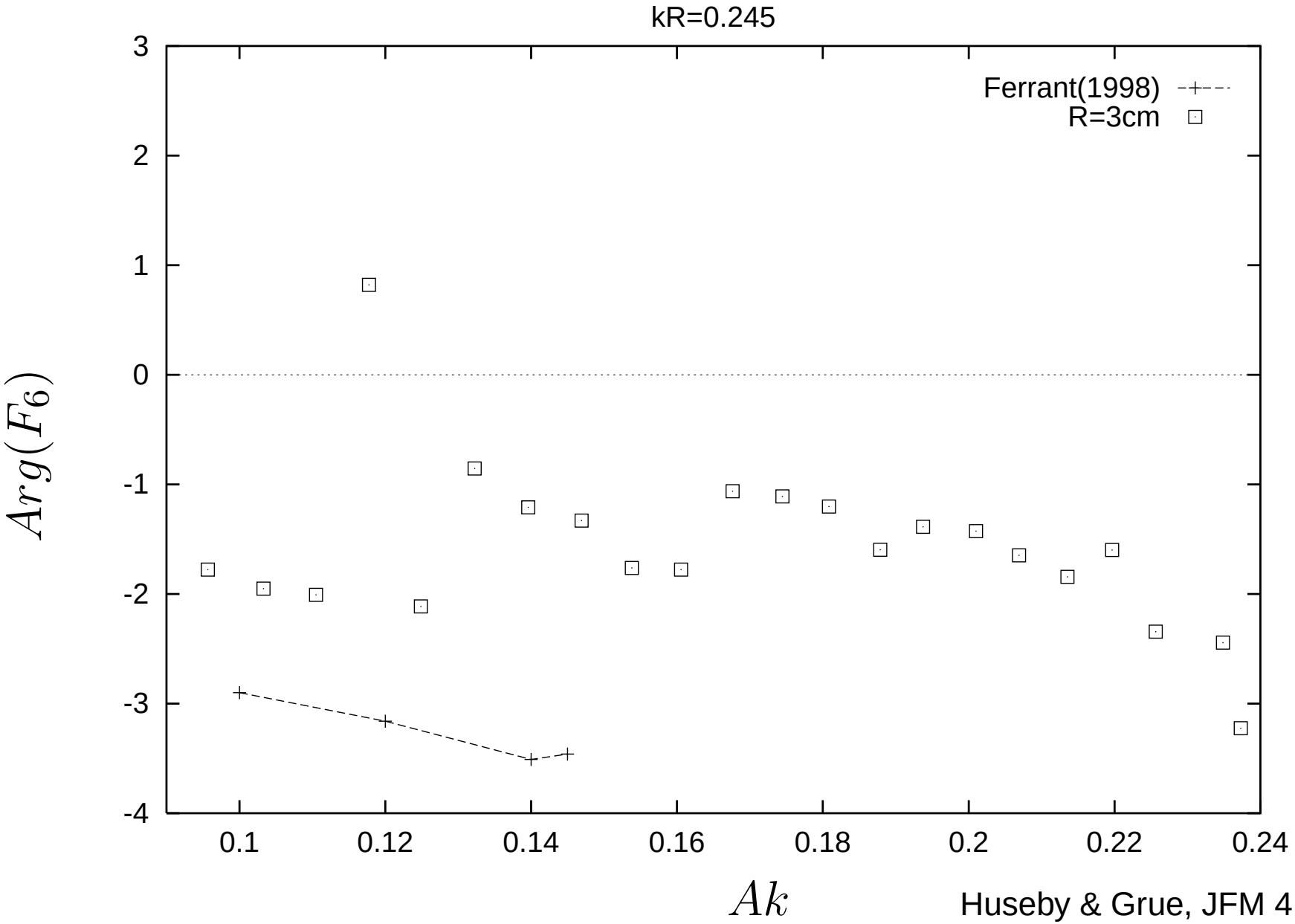



Huseby & Grue, JFM 414 (2000)



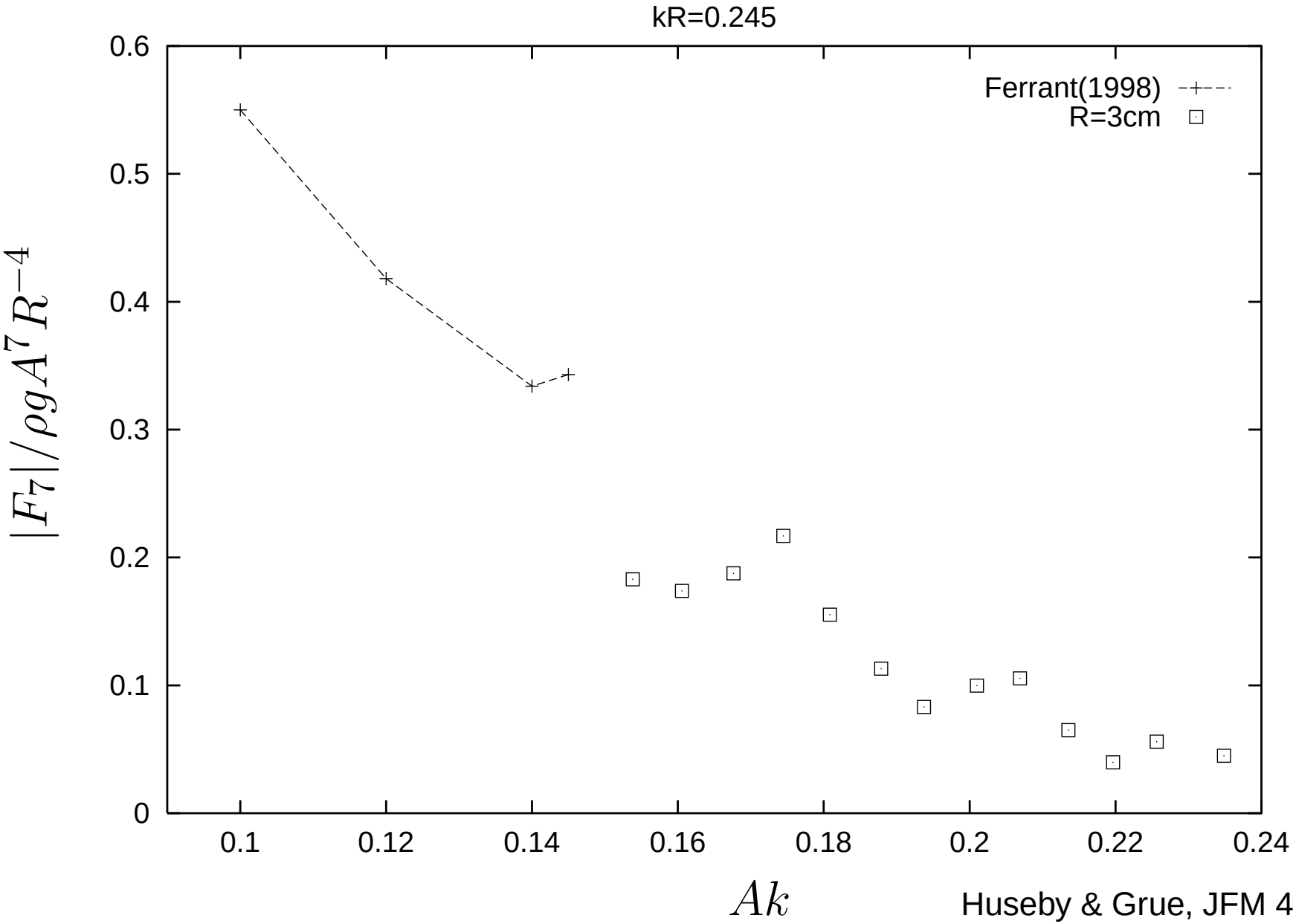

Huseby & Grue, JFM 414 (2000)

# 6th harmonic force

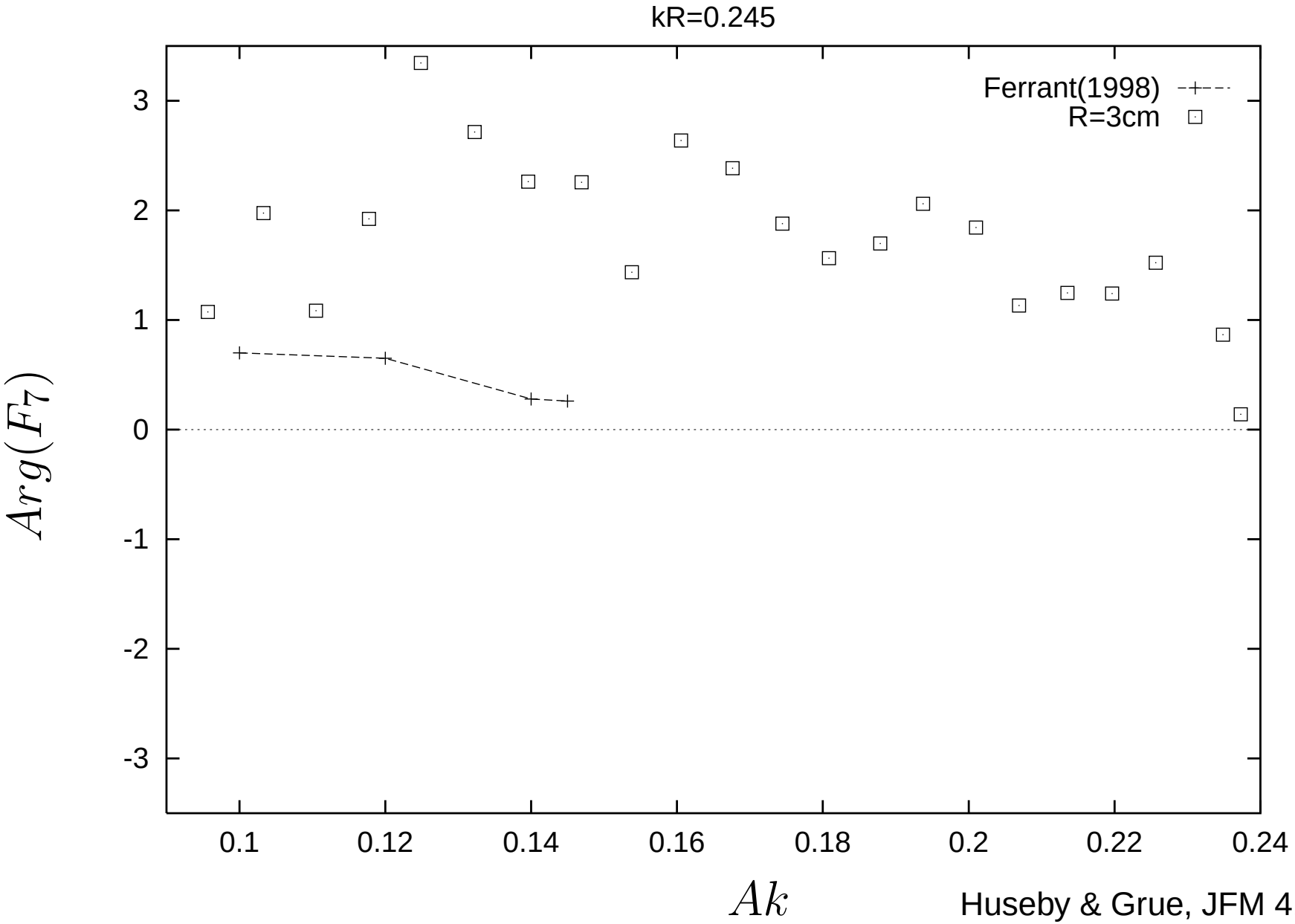



Huseby & Grue, JFM 414 (2000)

# 7th harmonic force




Huseby & Grue, JFM 414 (2000)




Huseby & Grue, JFM 414 (2000)

Comparison, forces for $Ak = 0.2$ and $kR = 0.166$:

$$\frac{|F_1|}{\rho g A R^2} \simeq 6.4$$
$$\frac{|F_2|}{\rho g A^2 R} \simeq 0.3$$
$$\frac{|F_3|}{\rho g A^3} \simeq 0.1$$
$$\frac{|F_4|}{\rho g A^4 R^{-1}} \simeq 0.1$$

(1)

This describes the force picture in regular waves, in deep water.

Perturbation methods obtain the harmonic forces, in perturbation sense.

Fully nonlinear computations are performed up to $Ak \sim 0.15$ (Ferrant, 1998). Challenges with local breaking at the column's water line. This must be circumvented in order to calculate the motion beyond this wave slope, where calculations with $Ak \sim 0.2$ are an important challenge.

References:

Faltinsen, O. M., Newman, J. N. and Vinje, T. 1995. Nonlinear wave loads on a slender vertical cylinder. J. Fluid Mech. 289, 179-199.

Ferrant, P. (1998) Fully nonlinear interactions of long-crested wave packets with a three dimensional body. 22nd ONR Symp. in Nav. Hydrod., Washington D.C. Tue/Wed Sess. Prov. Proc. pp. 59-72.

Huseby, M. and Grue, J. (2000) An experimental investigation of higher harmonic forces on a vertical cylinder. J. Fluid Mech., 414, 75-103.

McCamy, R. and Fuchs, R. (1954) Wave forces on piles: a diffraction theory. Tech. Memo No. 69, U.S. Army Corps of Engrs.

Malenica, S. and Molin, B. 1995. Third-harmonic wave diffraction by a vertical cylinder. J. Fluid Mech., 302, 203-229.