The Aerodynamics of Wind Turbines

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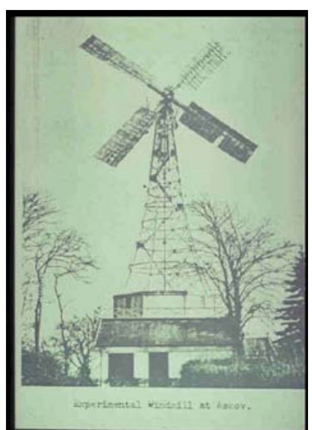
Wind Energy

Various wind turbines :



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The Danish Concept:

3-bladed upwind machine with gearbox and asynchroneous generator

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The worlds largest wind turbine
Enercon 126: P=6MW; D=126m

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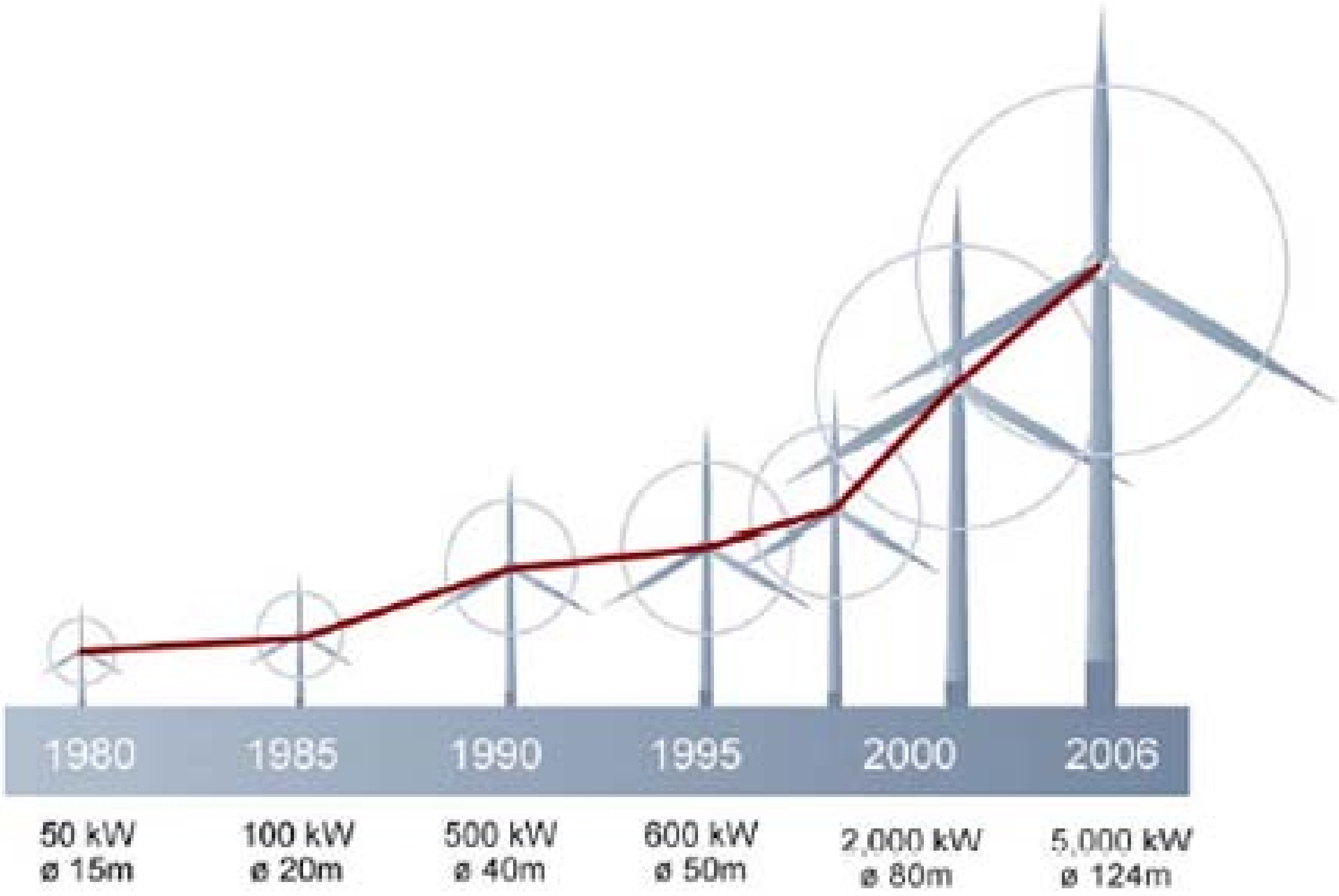
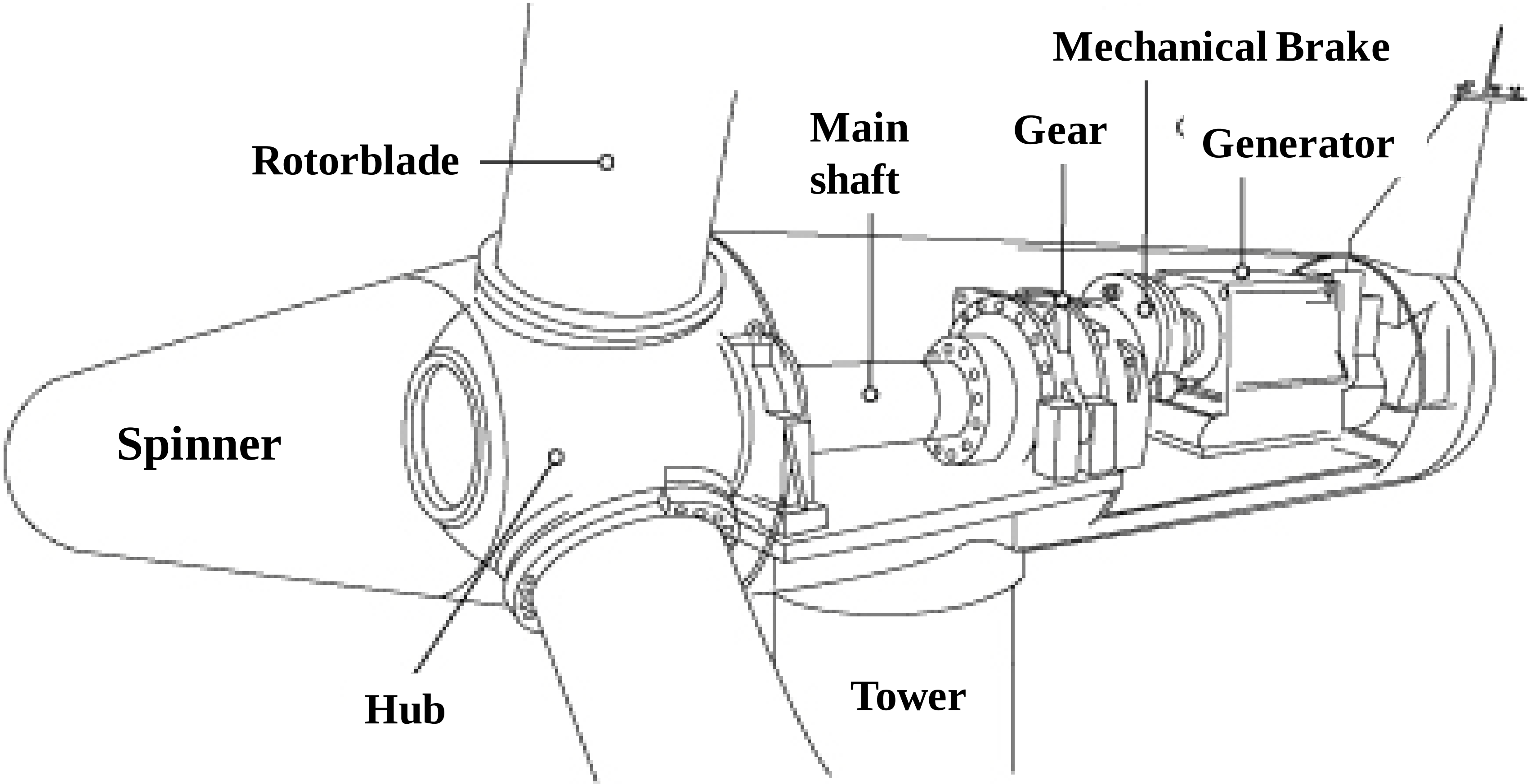
Development in wind turbine technology :



Wind turbine

Nacelle :



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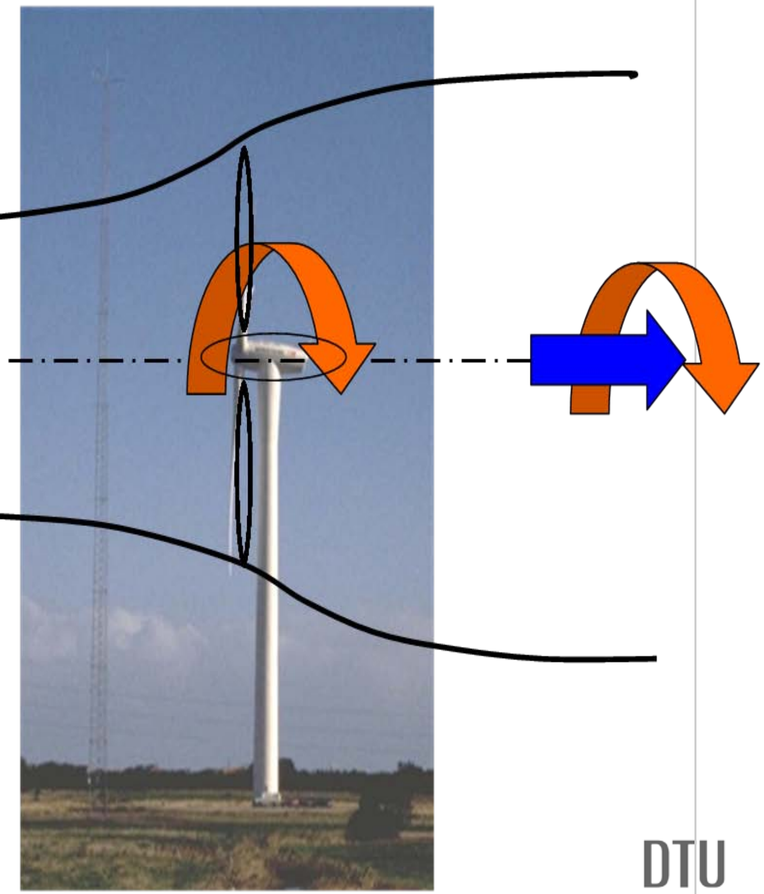
How does a wind turbine work?

- **The wind hits the rotor plane**
- **The combinition of wind speed and blade rotation results in a pressure distribution on the rotor blades**
- **The pressure distribution causes a turning moment (torque)**
- **The turning moment rotates the shaft**
- **The shaft is coupled to a generator that produces electrical power**



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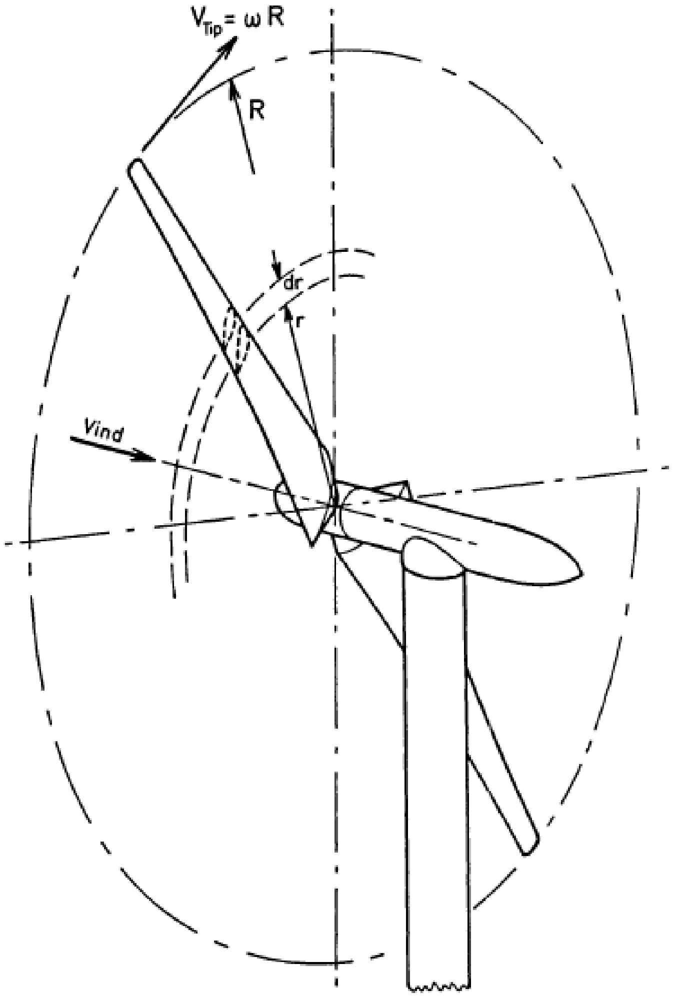
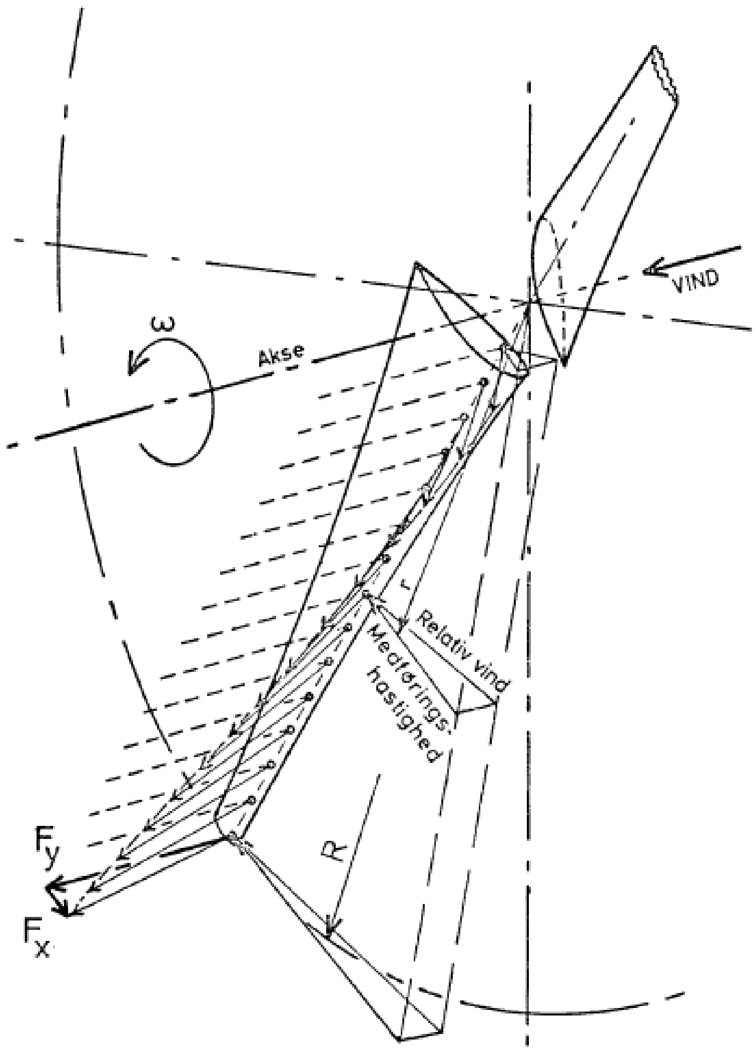
Aerodynamic forces:





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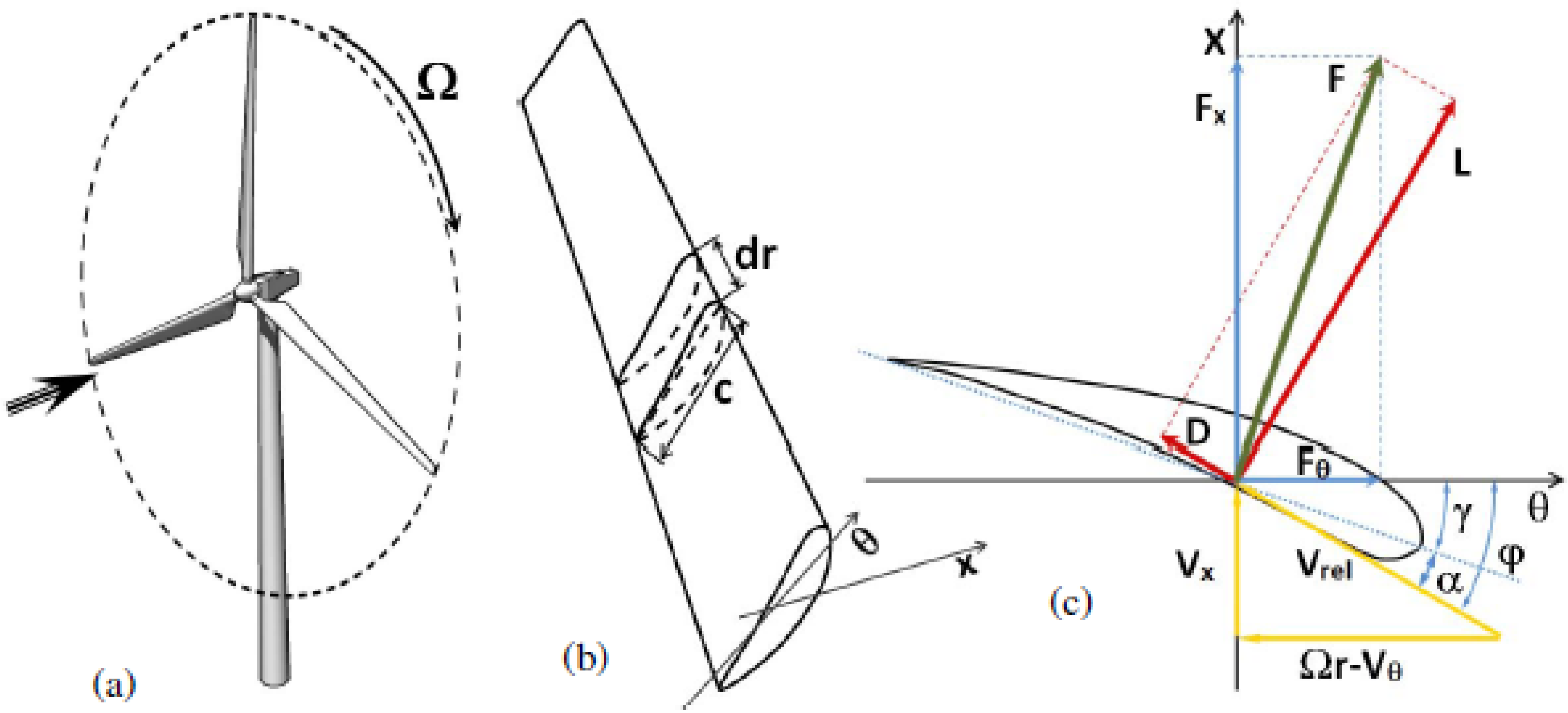
Aerodynamic forces and geometry :



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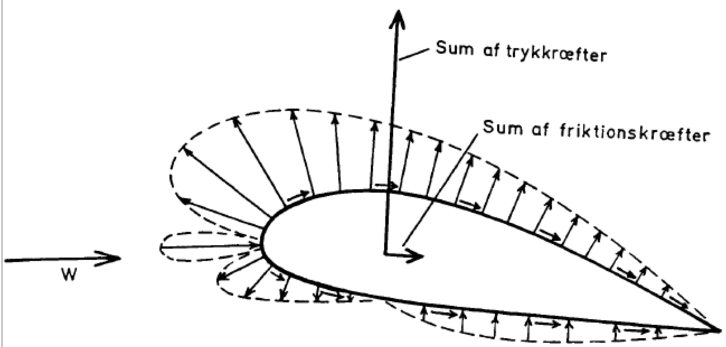
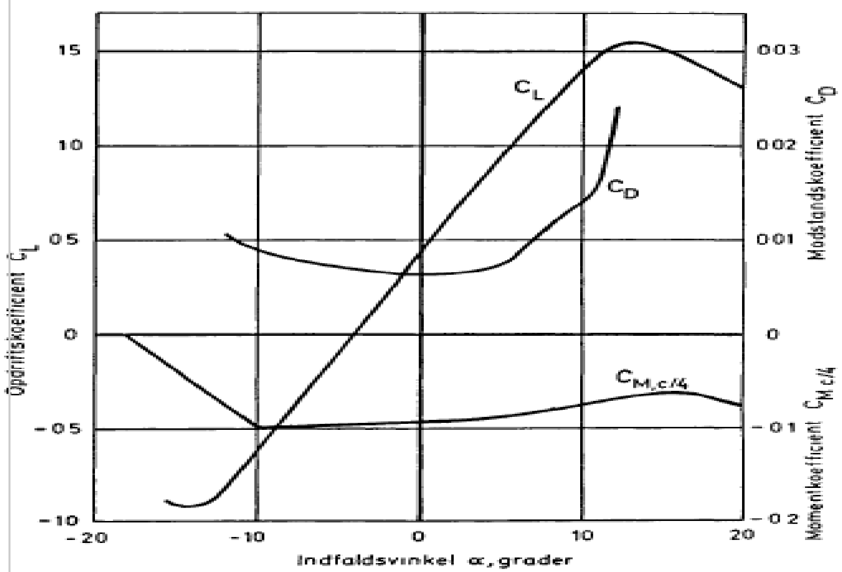
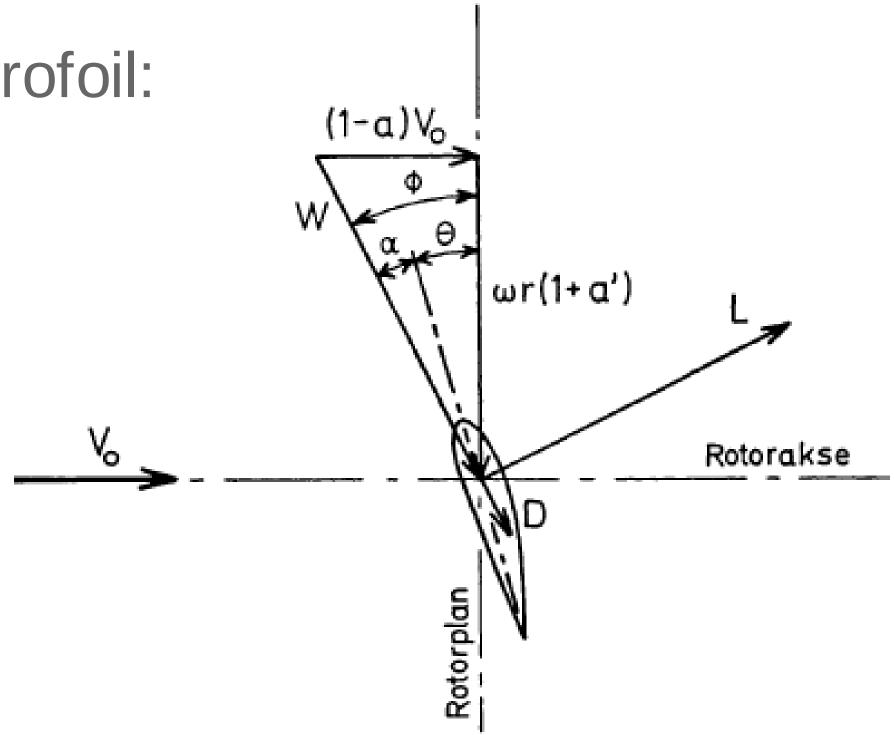
Pressure and forces on aerofoil:







$$C_y = C_L \cos\Phi + C_D \sin\Phi$$

$$C_x = C_L \sin\Phi - C_D \cos\Phi$$

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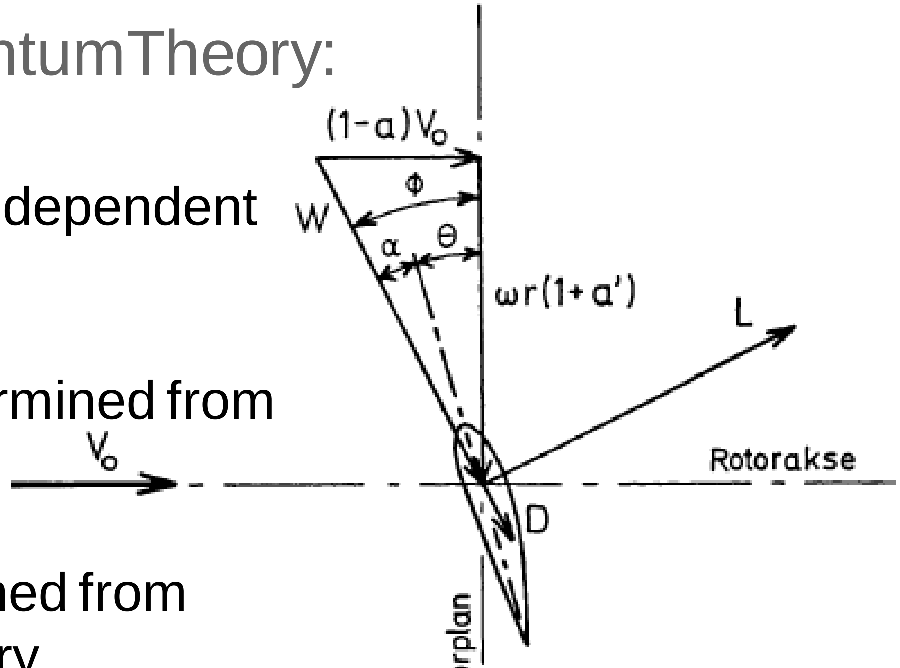
Blade-Element MomentumTheory:

- The rotor is divided into independent stream tubes
- Aerodynamic forces determined from aerofoil data
- Induced velocity determined from generel momentum theory
- Correction for finite number of blades using Prandtl tip-correction
- Various ad-hoc corrections for off-design operating conditions



$$\frac{a}{1-a} = \frac{cBC_y}{8\pi r \sin^2\phi}$$

$$\frac{a'}{1+a'} = \frac{cBC_x}{8\pi r \sin\phi\cos\phi}$$

The Optimum Rotor

- What is the optimum number of blades ?
- What is the optimum operating condition (TSR)?
- What is the maximum efficiency?

Wind Turbine Rotor Aerodynamics

What is the optimum number of blades?

3 blades ?

many blades ?

2 blades ?

1 blade ?

Why 3 blades?

- **Aerodynamics:** Close to optimum
- **Structural-dynamics:** No gyroscopic forces in yaw
- **Estetics:** Harmonic rotation
- **Historics:** Tradition of the ’Danish Concept’

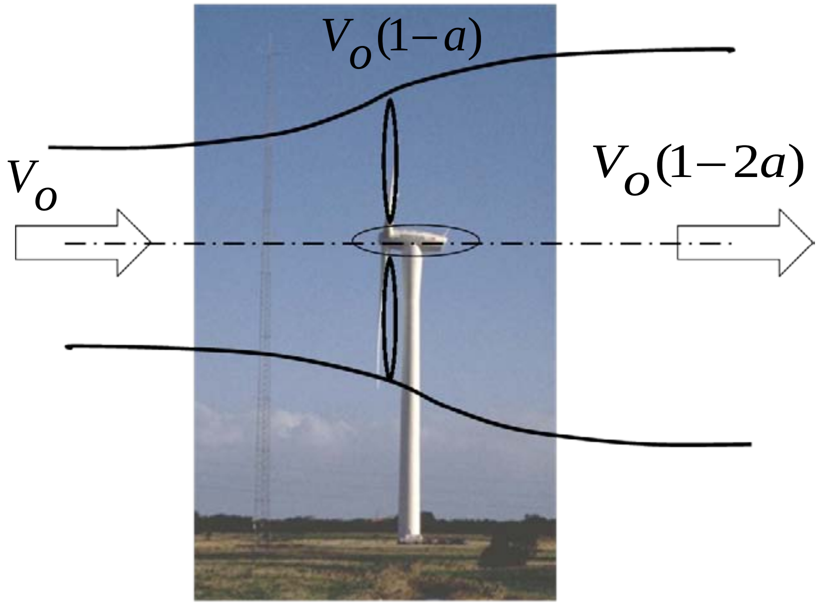
Optimum rotor with infinite number of blades

1-D Axial momentum theory:

Axial interference factor: $a = 1 - v/V_o$

Thrust Coefficient: $C_T = \dfrac{T}{\frac{1}{2}\rho A_o V_o^2}$

Power coefficient: $C_P = \dfrac{P}{\frac{1}{2}\rho A_o V_o^3}$



$$\boxed{\begin{aligned} &T = \dot{m}\Delta V \Rightarrow \quad T = 2\rho A_o v(V_o - v) \\ &P = vT \Rightarrow \quad P = 2\rho A_o v^2 (V_o - v) \end{aligned}}$$

$$\boxed{\begin{aligned} C_T &= 4a(1-a) \\ C_P &= 4a(1-a)^2 \end{aligned}}$$

Betz limit:

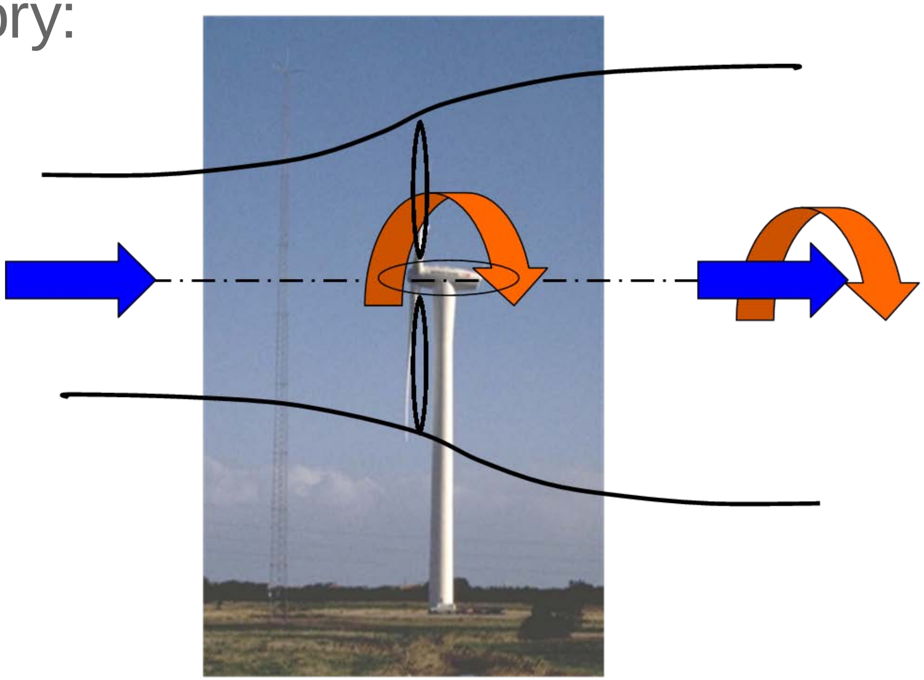
$$\boxed{a = \frac{1}{3}: \quad C_{P\max} = \frac{16}{27} = 0.593}$$

Optimum rotor with infinite number of blades

Generel momentum theory:

$$a = 1 - v_o / V$$

$$a' = \frac{u_{\theta o}}{\Omega_o r}$$



Euler's turbine equation:

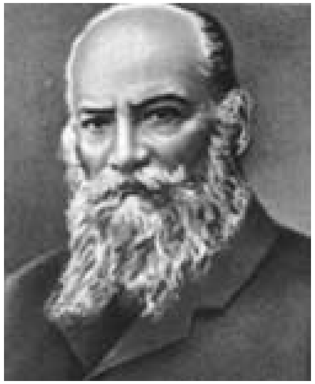
$$C_P = 8\lambda^2 \int_0^1 a'(1-a)x^3 dx$$

TSR	$C_{P\max}$
0.5	0.288
1.0	0.416
1.5	0.480
2.0	0.512
2.5	0.532
5.0	0.570
7.5	0.582
10.0	0.593

Condition for optimum operation:

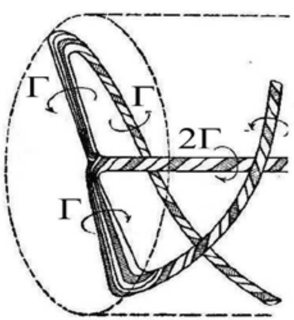
$$a' = (1-3a)/(4a-1)$$

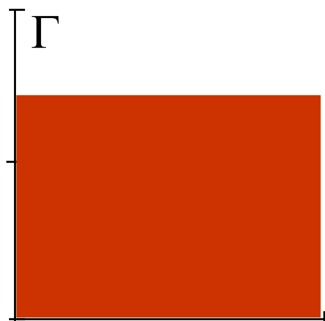
Two definitions of the ideal rotor



Joukowsky (1912)

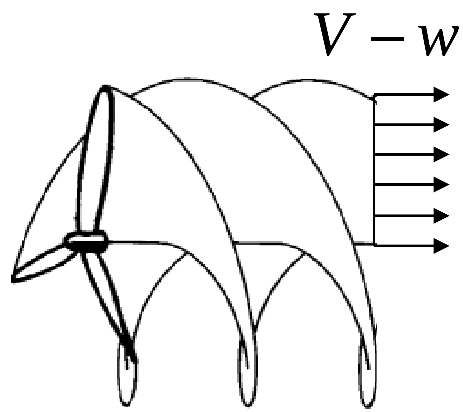
$$\Gamma = const$$

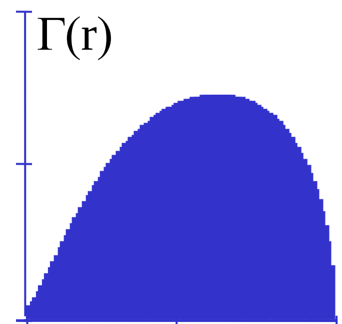




Betz (1919)

$$V - w = const$$

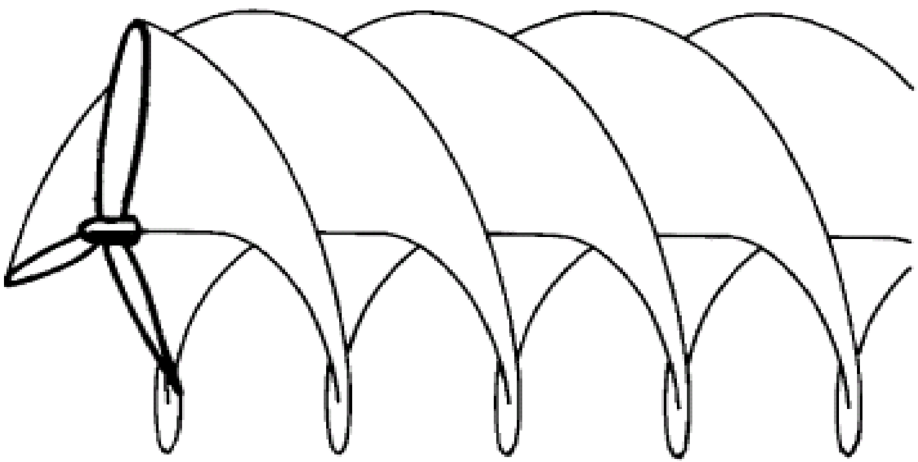




In both cases only conceptual ideas were outlined for rotors with finite number of blades, whereas later theoretical works mainly were devoted to rotors with <u>infinite</u> blades!

Betz' condition for maximum efficiency of a rotor with a finite number of blades

Maximum efficiency is obtained when the pitch of the trailing vortices is constant and each trailing vortex sheet translates backward as an undeformed regular helicoidal surface



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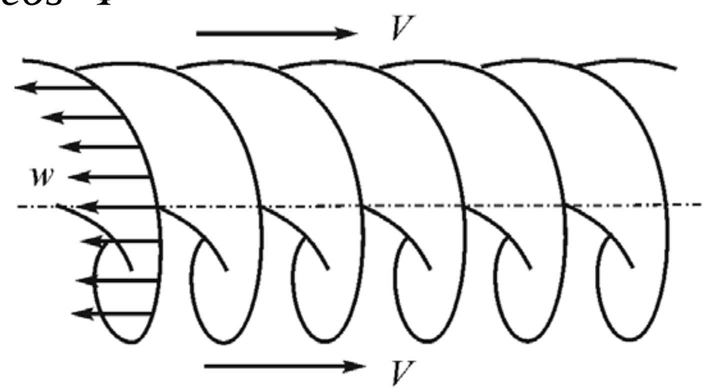
Induced velocities:

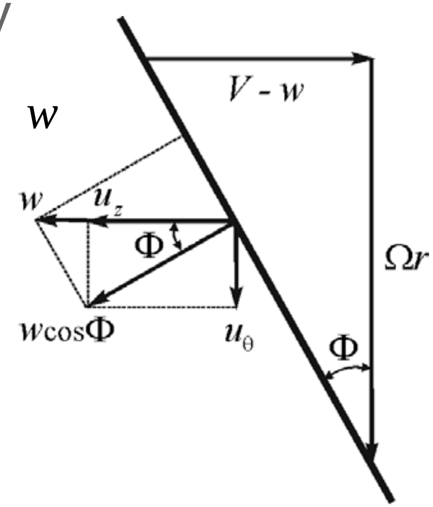
Movement of vortex sheet with constant pitch and constant velocity

Axial induced velocity:

$$u_z = w\cos^2\Phi$$



a



b

Tangential induced velocity:

$$u_\theta = w\cos\Phi\sin\Phi$$

Pitch:

$$h = 2\pi r\tan\Phi / B = 2\pi(V-w)/B\Omega$$

$$\tan\Phi = \frac{dz}{rd\theta} = (V-w)/\Omega r$$

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Optimum lift distribution:

Goldstein function: $G(r) = \Gamma(r)/hw = B\Gamma\Omega / 2\pi w(V - ½w)$

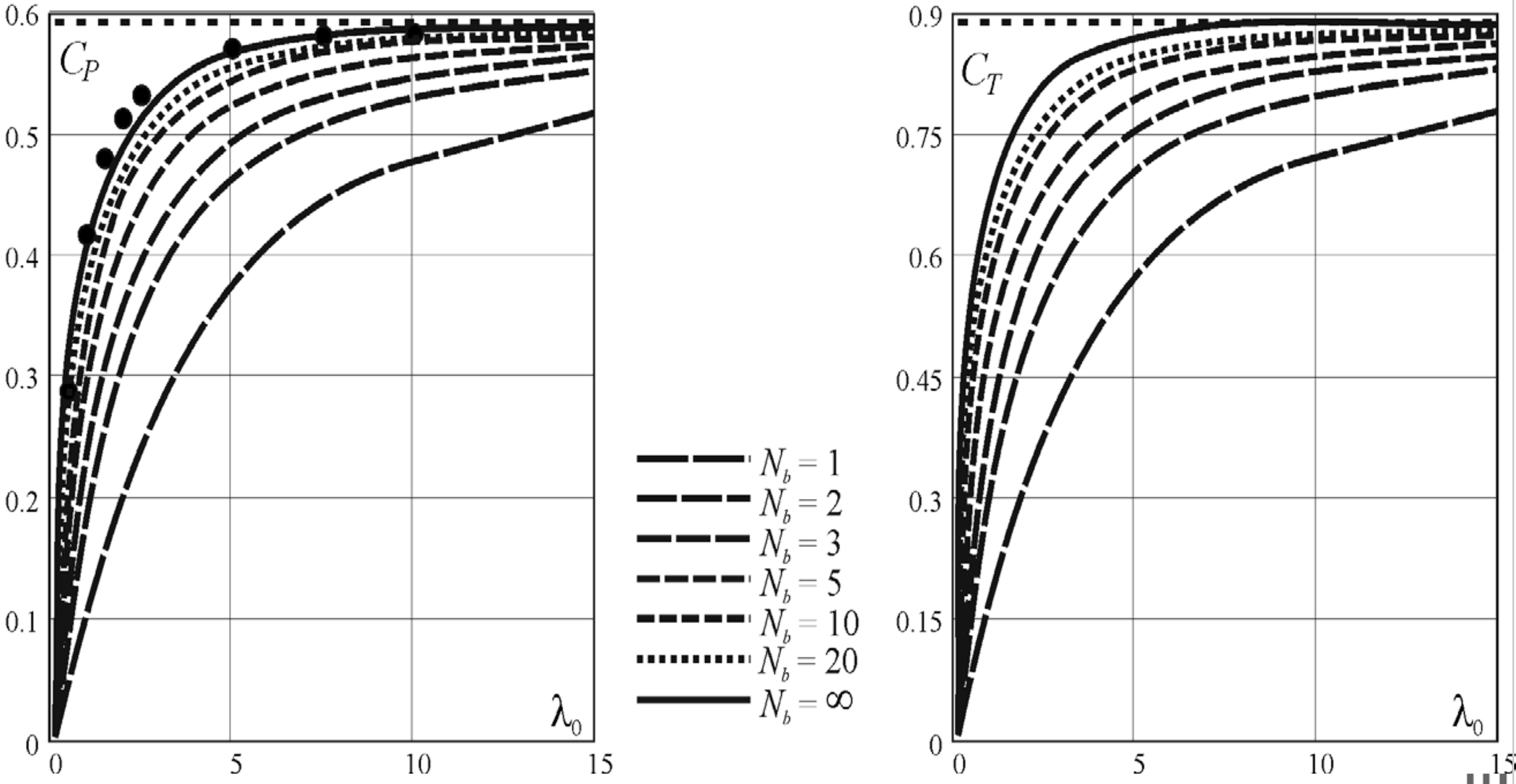
Kutta-Joukowski theorem: $\Gamma = ½ c C_L U_o$ □ $cC_L = \dfrac{2\Gamma}{U_o}$

Combining these equations, we get

$$\boxed{\sigma C_L = \frac{2\overline{w}(1-½\overline{w})G(r/R)}{\lambda(U_o/V)}}$$

Solidity: $\sigma = \dfrac{Bc}{2\pi R_o}$ Tip Speed ratio: $\lambda = \dfrac{\Omega R}{V}$

The optimum rotor:



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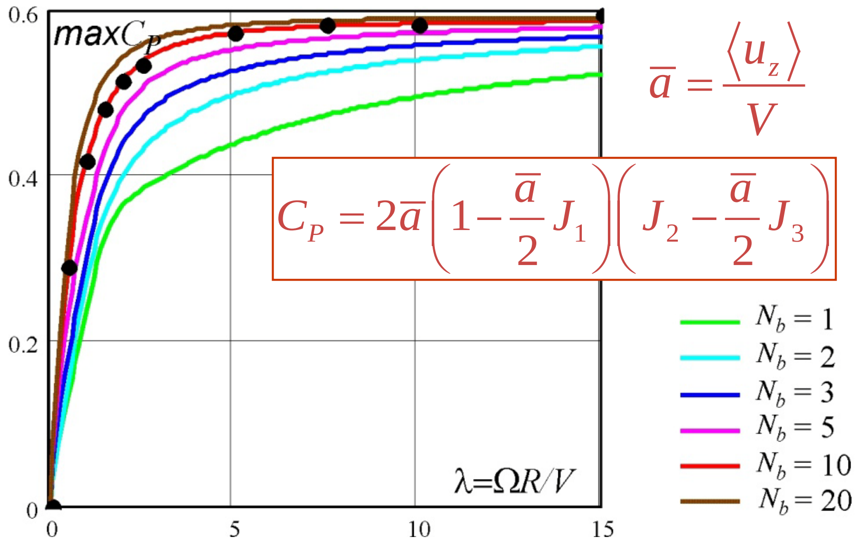
Comparison of maximum power coefficients

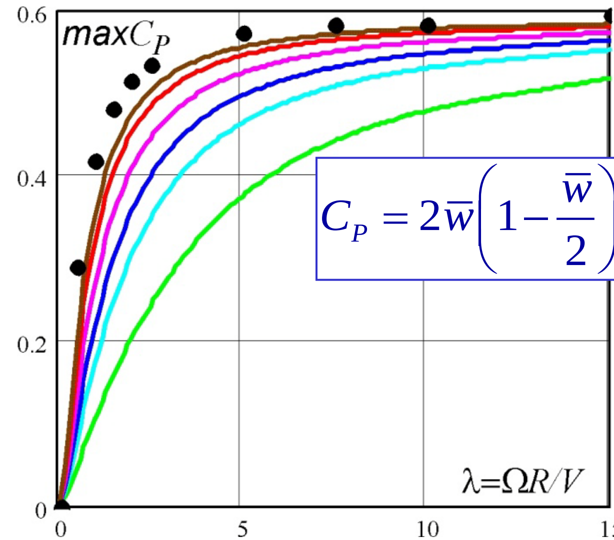
Solution of Joukowsky rotor (Okulov & Sørensen, 2010)



$$\bar{a} = \frac{\langle u_z \rangle}{V}$$

$$C_P = 2\bar{a}\left(1 - \frac{\bar{a}}{2} J_1\right)\left(J_2 - \frac{\bar{a}}{2} J_3\right)$$

Solution of Betz rotor (Okulov&Sørensen, 2008)



$$\bar{w} = \frac{w}{V}$$

$$C_P = 2\bar{w}\left(1 - \frac{\bar{w}}{2}\right)\left(I_1 - \frac{\bar{w}}{2} I_3\right)$$

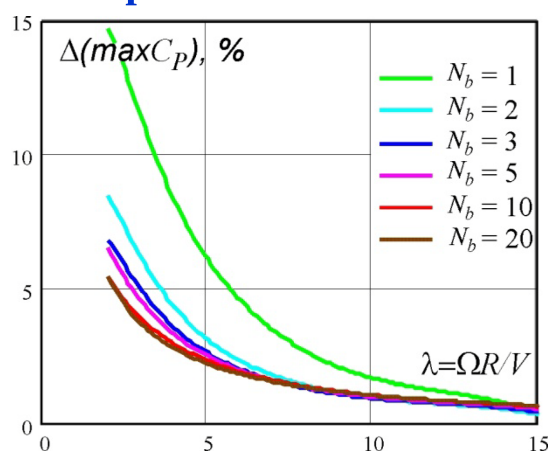
$$\bar{a} = \frac{J_1 J_2 + J_3 - \sqrt{J_1^2 J_2^2 - J_1 J_2 J_3 + J_3^2}}{3 J_1 J_3}$$

$$J_1 = 1 + 2\frac{l^2}{R^2}$$

$$J_2 = 1 + \frac{1}{2}\frac{\varepsilon}{R} + \frac{1}{6}\frac{\varepsilon^2}{R^2}$$

$$J_3 = 2l\int_0^1 \frac{\tilde{u}_z(x)}{a} x\,dx$$

Difference between power coefficients



$$\bar{w} = \frac{2\left(I_1 + I_3 - \sqrt{I_1^2 - I_1 I_3 + I_3^2}\right)}{3 I_3}$$

Mass coefficient $$I_1 = 2\int_0^1 G(x)\,x\,dx$$

"Axial loss factor" $$I_3 = 2\int_0^1 G(x)\frac{x^3\,dx}{x^2 + l^2}$$

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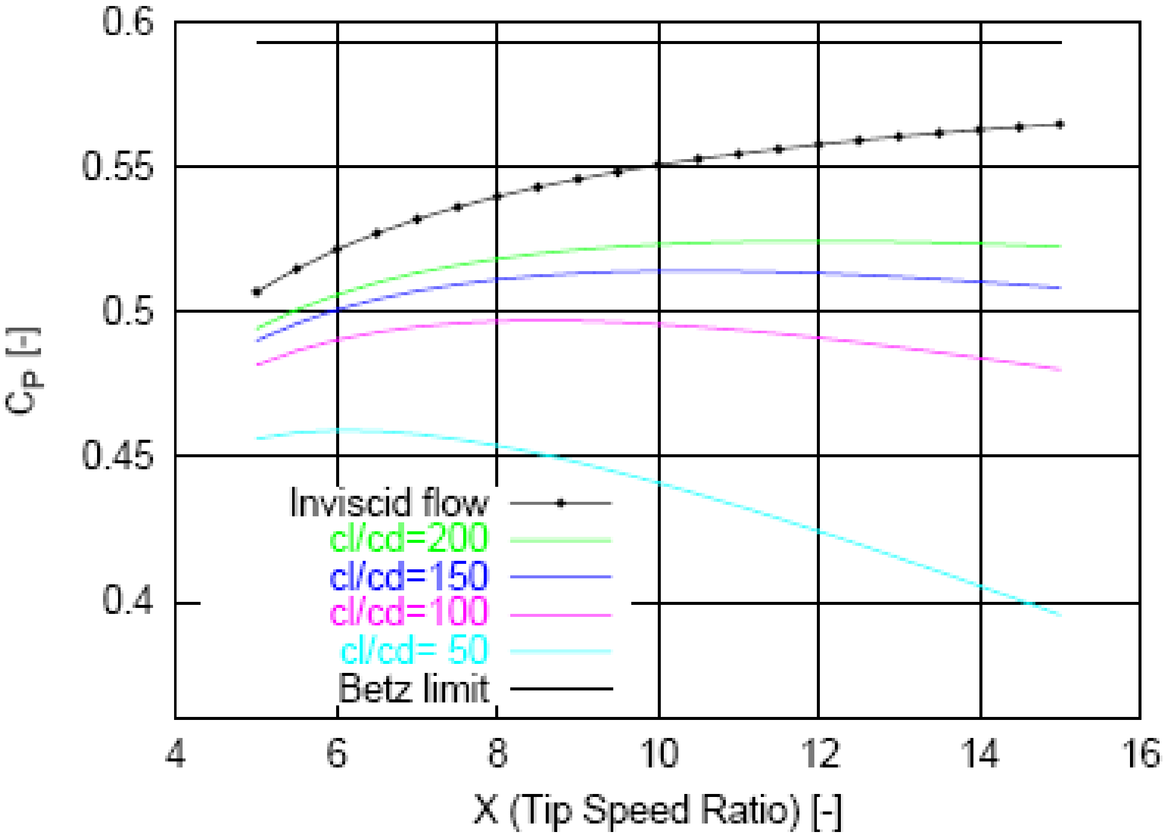
Optimum 3-bladed rotor with losses:



From C. Bak: J. Physics: Conference Series, vol. 75, 2007

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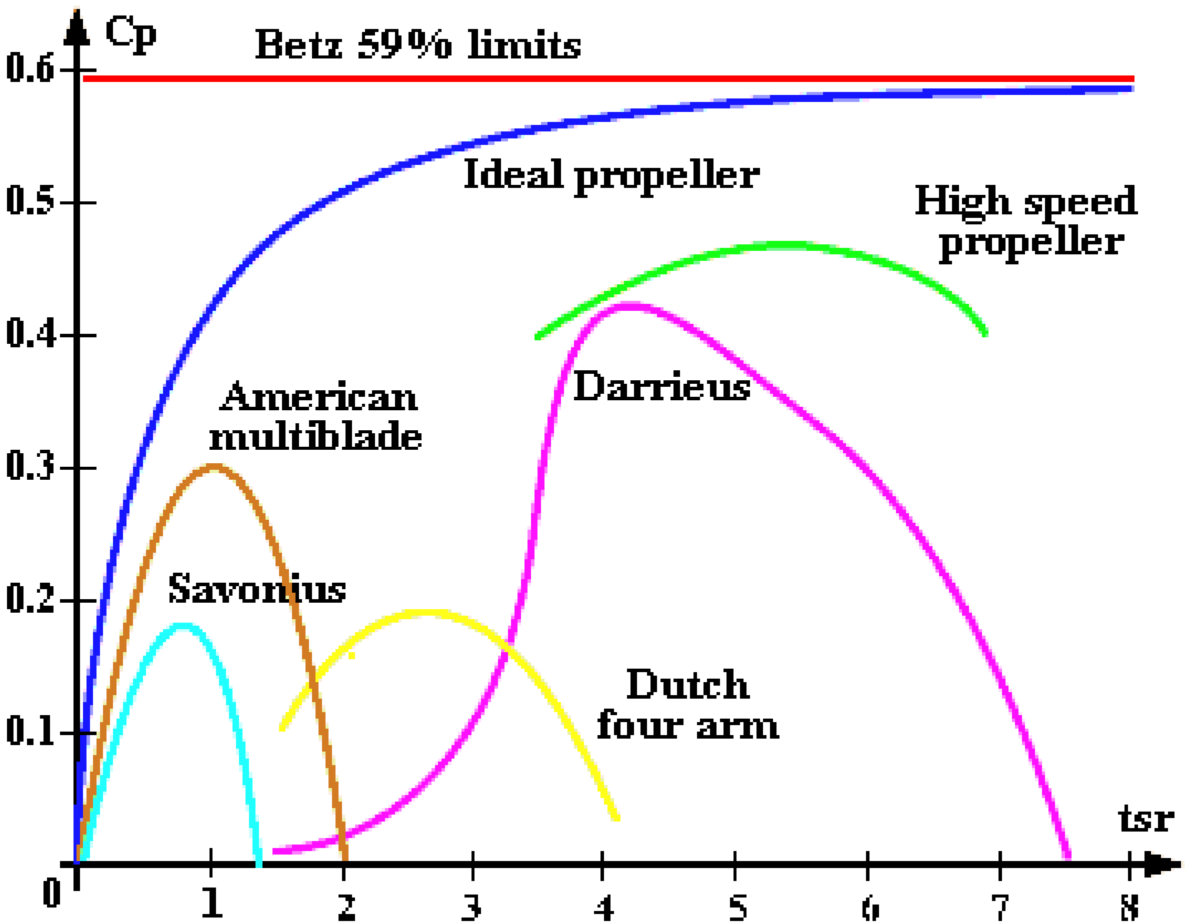
Wind turbine performance :



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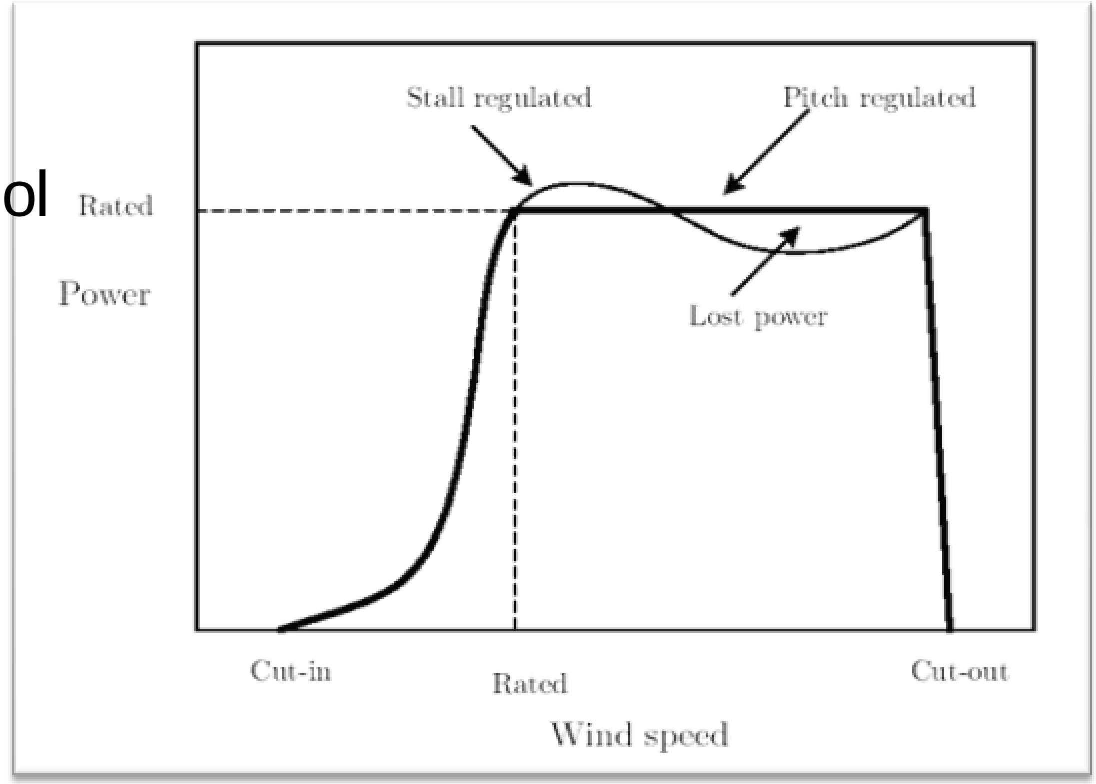
Control and regulation of wind turbines

Stall control

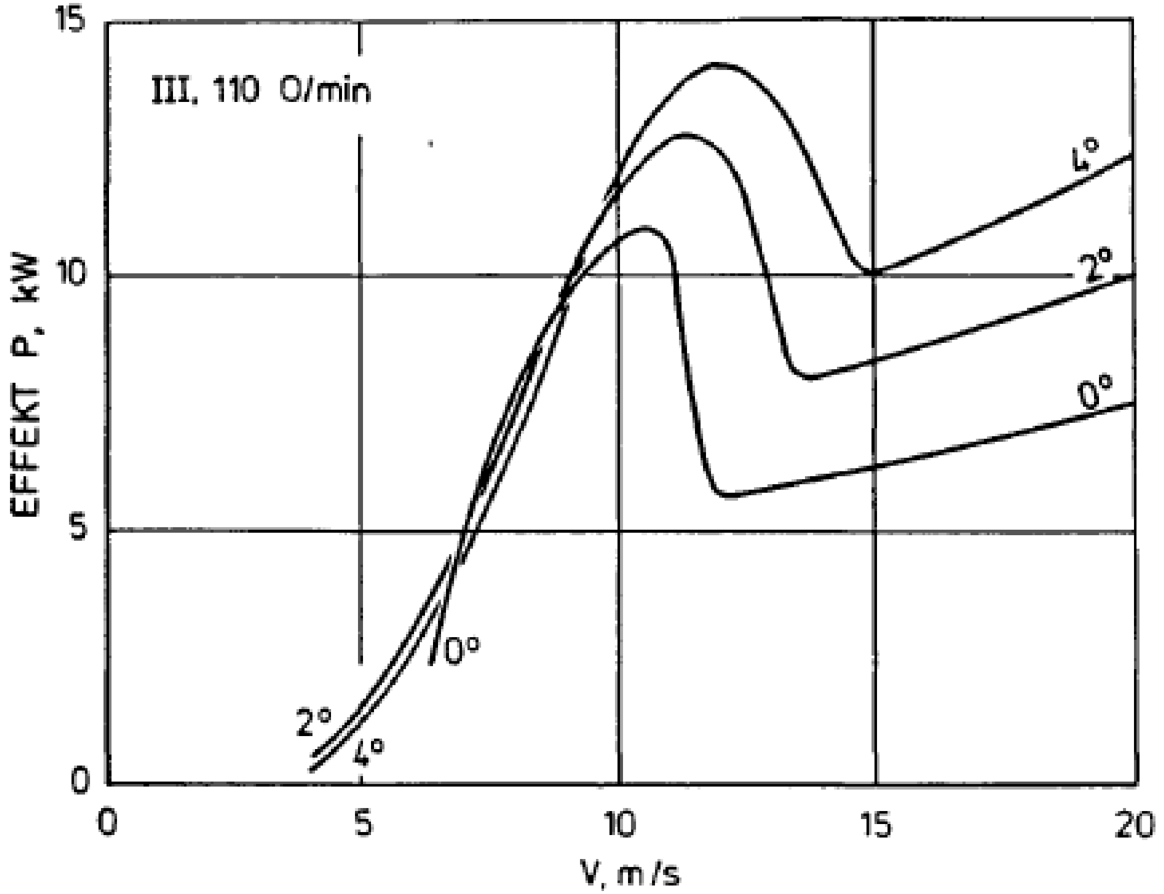
Active Stall control

Pitch control

Variable speed



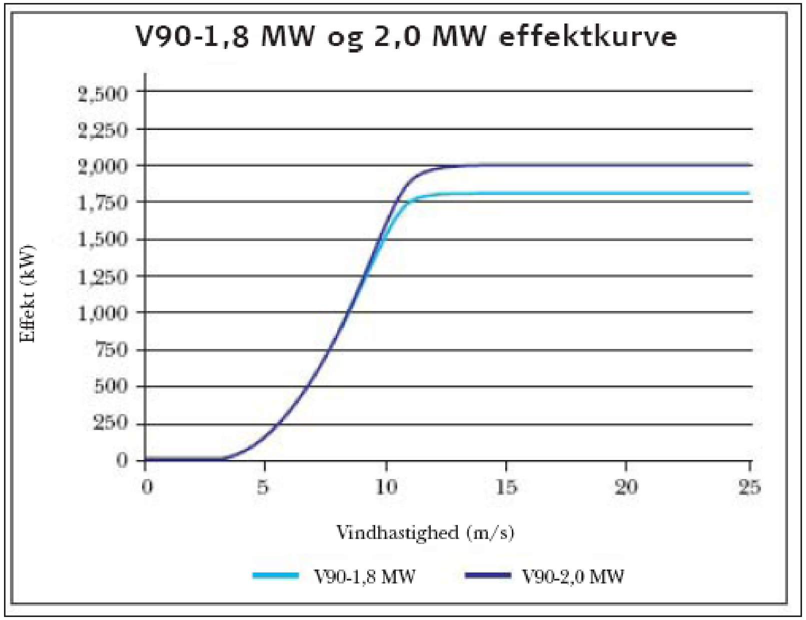
Stall-regulated wind turbine: Computed power curve

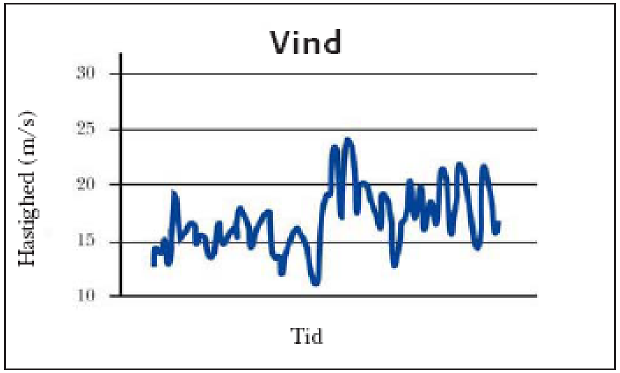


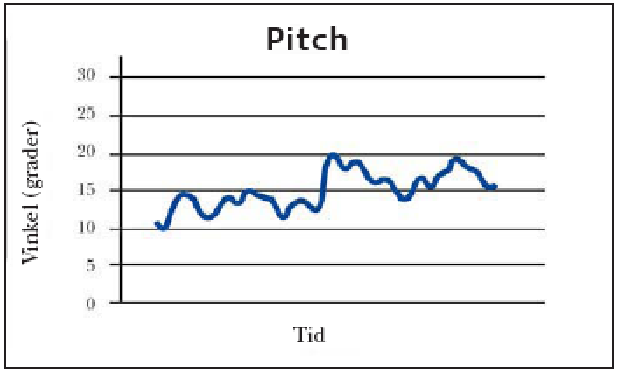
Wind Turbine

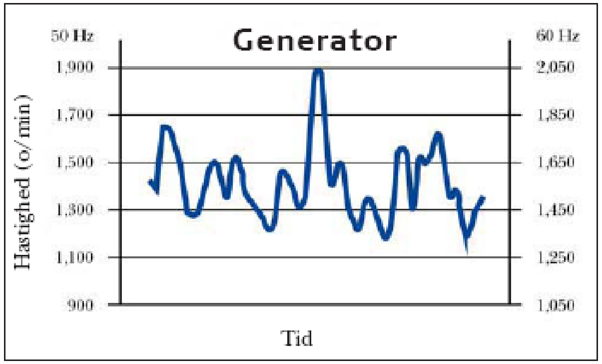
Modern Wind Turbine :

- **Pitch-regulated**
- **P=2 MW; D=90 m**
- **Nom. Tip speed.: 70 m/s**
- **Rotor: 38t, Nacelle: 68t; Tower: 150t**
- **Control: OptiSpeed; OptiTip**









Wind Turbine Aerodynamics

Need for models capable of coping with:

- **Dynamic simulations of large deformed rotors**
- **Complex geometries: Rotor tower interaction**
- **Adjustable trailing edge flaps**
- **Various aerodynamic accessories, such as vortex generators, blowing, Gourney flaps and roughness tape**
- **3-dimensional stall including laminar-turbulent transition**
- **Unsteady, three-dimensional and turbulent inflow**
- **Interaction between rotors and terrain**
- **Complex terrain and wind power meteorology**
- **Offshore wind energy: Combined wind and wave loadings**

Blade Element Momentum Model

Basic ingredients of the BEM model:

- **Based on 1-D momentum theory assuming annular independency**
- **Loading computed using tabulated static airfoil data**
- **Dynamic stall handled through ’dynamic stall’ models**
- **3-dimensional stall introduced through modifications**
- **Tip Flows based on (Prandtl) tip correction**
- **Yaw treated through simple modifications**
- **Heavily loaded rotors treated through Glauert’s approximation**
- **Wakes and park effects modelled using axisymmetric momentum theory**

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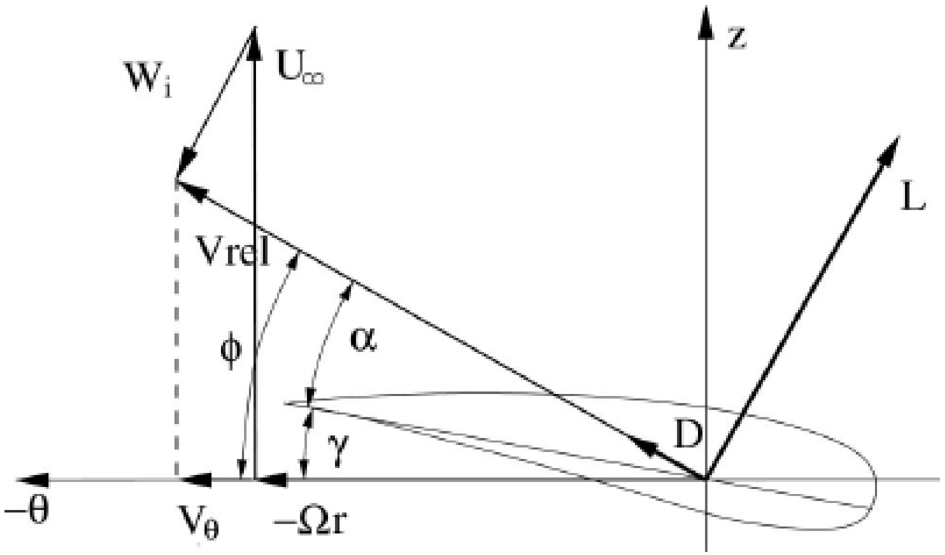
Blade Element Momentum Model

Advantages of using the BEM model:

- Extremely fast on a PC
- Can in principle cope with all flow situations
- Easy to couple with an aeroelastic conde, such as Flex
- Easy to couple with turbulent inflow model
- Many years of experience in using the model
- Performs very well at design conditions
- Capable of delivering results at off-design conditions

The BEM model is today the industrial standard used by all producers of wind turbines and wind turbine blades

Blade Element Momentum Model



Blade element

$$\frac{dT}{dr} = BF_n = \frac{1}{2}\rho c B V_{rel}^2 \cdot C_n ,$$

$$\frac{dM}{dr} = BrF_t = \frac{1}{2}\rho c B r V_{rel}^2 \cdot C_t$$

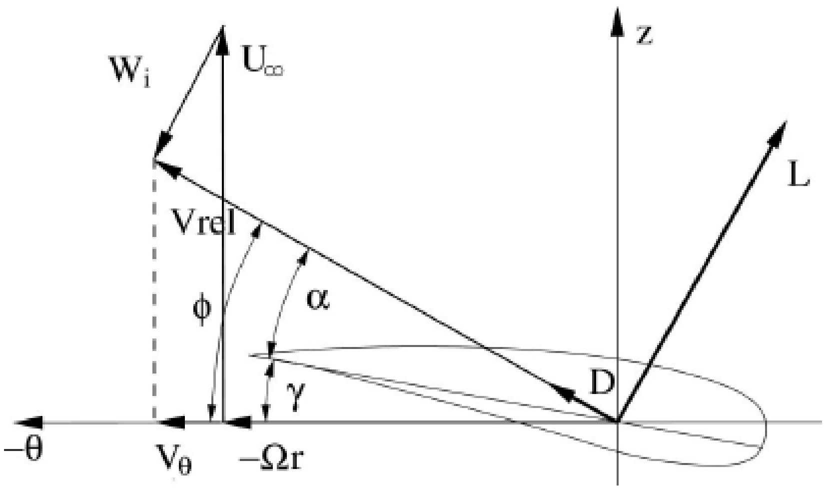
$$C_n = C_l \cos\phi + C_d \sin\phi$$

$$C_t = C_l \sin\phi - C_d \cos\phi$$

Airfoil data: $L = \frac{1}{2}\rho V_{rel}^2 c C_L$

$$D = \frac{1}{2}\rho V_{rel}^2 c C_D$$

Blade Element Momentum Model



Induced velocity:

$$W_i = (-aU_0, a'\Omega r)$$

Velocity triangle:

$$\sin\phi = \frac{U_\infty(1-a)}{V_{rel}}, \qquad \cos\phi = \frac{\Omega r(1+a')}{V_{rel}}$$

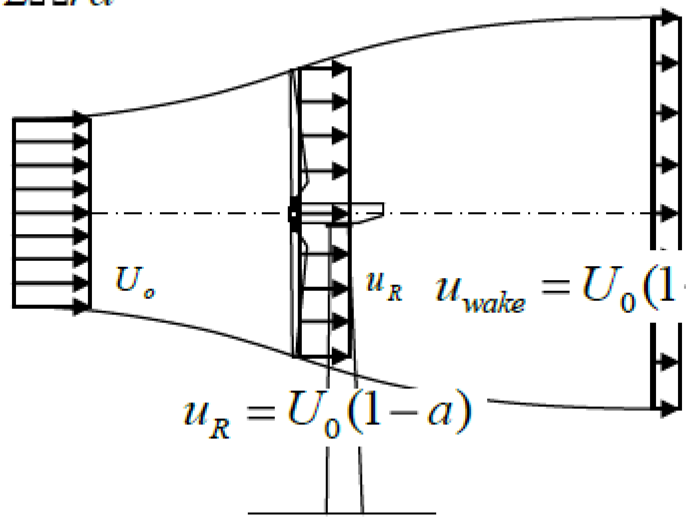
$$V_{rel}^2 = \frac{U_\infty^2(1-a)^2}{\sin^2\phi} = \frac{U_\infty(1-a)\Omega r(1+a')}{\sin\phi\cos\phi}$$

$$\frac{dT}{dr} = \frac{\rho B c U_0^2(1-a)^2}{2\sin^2\phi}\cdot C_n,$$

$$\frac{dM}{dr} = \frac{\rho B c U_0(1-a)\Omega r^2(1+a')}{2\sin\phi\cos\phi}\cdot C_t$$

Blade Element Momentum Model

Induced velocities: $a = \dfrac{U_o - u_R}{U_o}$ $\qquad u_\theta = 2\Omega r a'$



Axial momentum balance:

$$\frac{dT}{dr} = \rho(U_0 - u_{wake})2\pi r u_R = 4\pi\rho U_0^2 a(1-a)$$

Moment of momentum balance:

$$\frac{dM}{dr} = \rho r u_\theta 2\pi r u_R = 4\pi\rho r^3 \Omega U_0 a'(1-a)$$

Combining blade element and and momentum expressions:

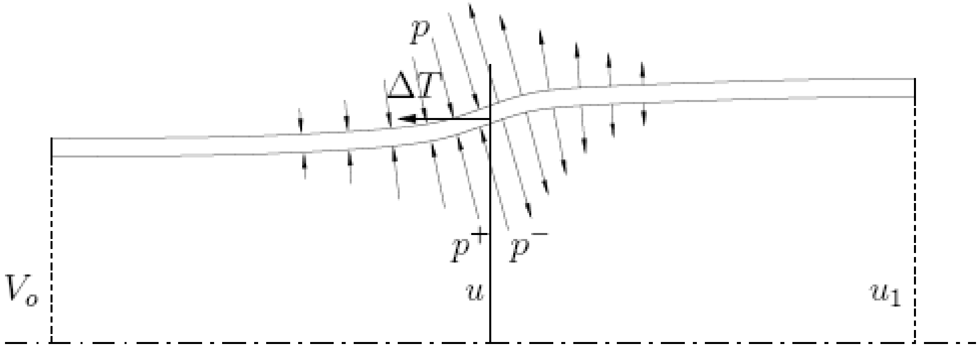
$$a = \frac{1}{4\sin^2\phi/(\sigma C_n)+1}$$

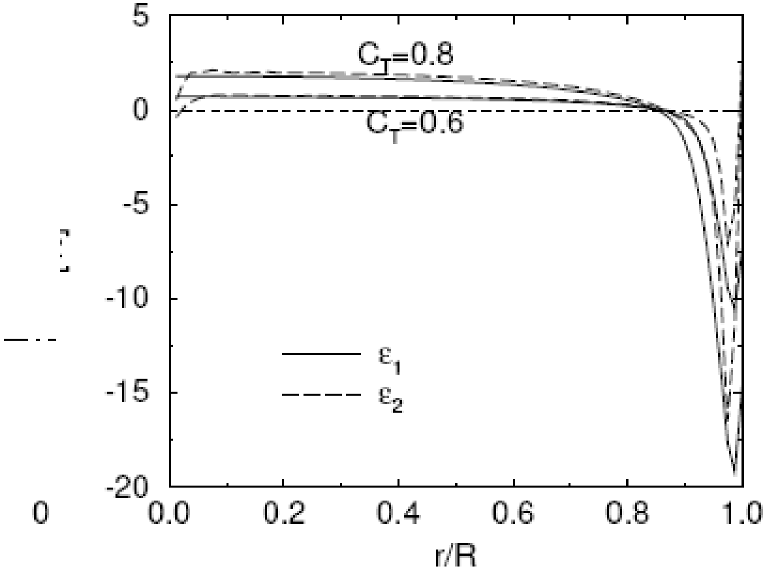
$$a' = \frac{1}{4\sin\phi\cos\phi/(\sigma C_t)-1}$$

Solidity: $\sigma = Bc/2\pi r$

Blade Element Momentum model

How good is the assumption of annular independency?





$$\Delta T = \Delta \dot{m}\left(V_o - u_1\right) + \Delta X$$

$$\Delta X = \int_{cs} p d\mathbf{S} \cdot \mathbf{e}_z$$

$$u = \tfrac{1}{2}\left(V_o - u_1\right)\left(1 - \frac{\Delta X}{\Delta T}\right)$$

$$\varepsilon_1 = \frac{u}{\tfrac{1}{2}\left(V_o - u_1\right)} - 1, \quad \varepsilon_2 = \frac{\Delta X}{\Delta T}$$

Blade Element Momentum Model

The Tip correction

The tip correction, F, corrects the axisymmetric approach to account for a finite number of blades:

$$\frac{dT}{dr} = 4\pi r \rho U_\infty^2 aF(1-a)$$

$$\frac{dM}{dr} = 4\pi \rho r^3 \Omega U_\infty a' F(1-a)$$

Prandtl tip correction formula:

$$F = \frac{2}{\pi}\cos^{-1}\left[\exp\left(-\frac{B(R-r)}{2r\sin\phi}\right)\right]$$

Modified expressions:

$$a = \frac{1}{4F\sin^2\phi/(\sigma C_n)+1}$$

$$a' = \frac{1}{4F\sin\phi\cos\phi/(\sigma C_t)-1}$$

Blade Element Momentum Model

Other corrections

Heavily loaded rotors: $$C_T = \frac{dT}{\frac{1}{2}\rho U_\infty^2 2\pi r dr} = 4aF(1-a)$$ **for a < 1/3**

$$C_T = 4aF\left(1 - \frac{a}{4}(5-3a)\right)$$ **for a > 1/3**

Yaw correction: $$w_i = w_{i0}\left(1 + \frac{r}{R}\tan(\frac{\chi}{2})\cos(\theta_{blade} - \theta_0)\right)$$

Dynamic wake: $$Rf(r/R)\frac{du_i}{dt} + 4u_i(U_0 - u_i) = \frac{\Delta T}{2\pi r \Delta r}$$

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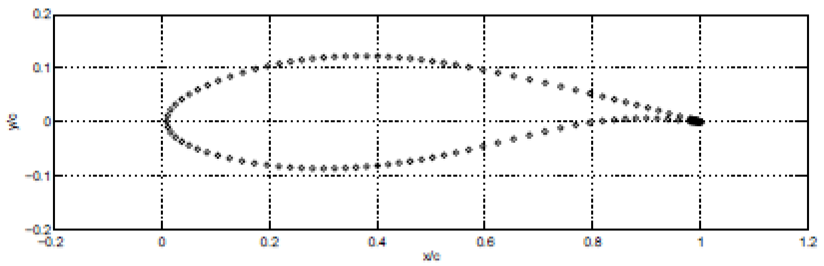
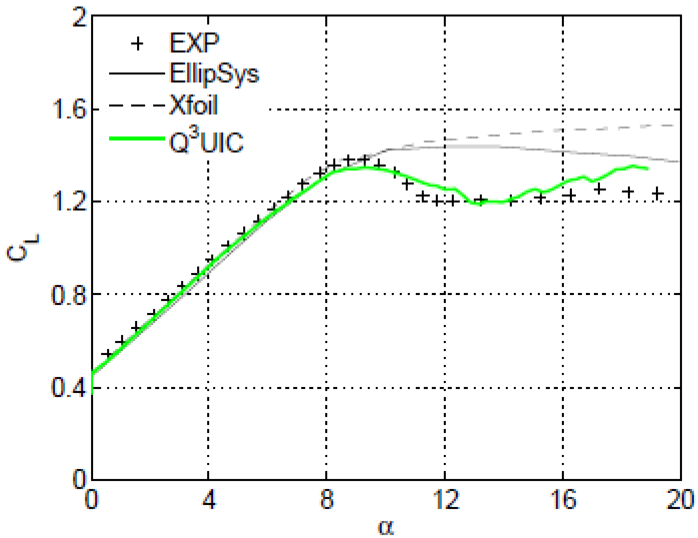
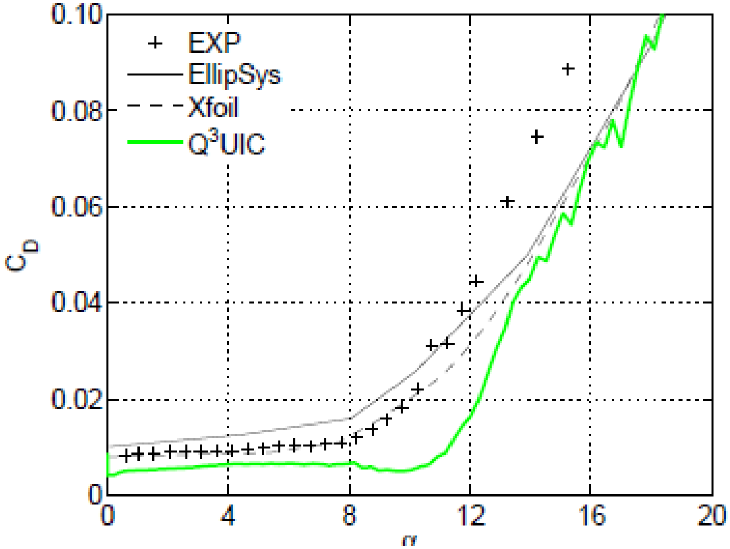
Blade Element Momentum Model

2D Airfoil data

DU-93-W-210

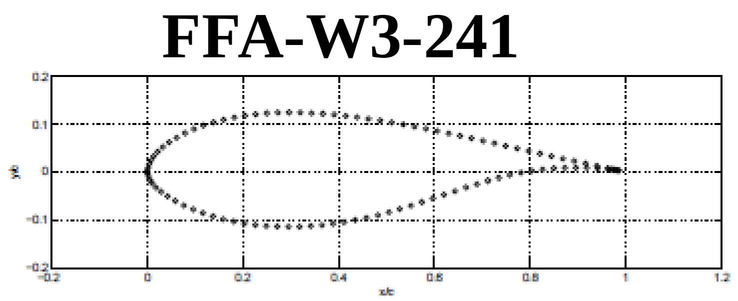


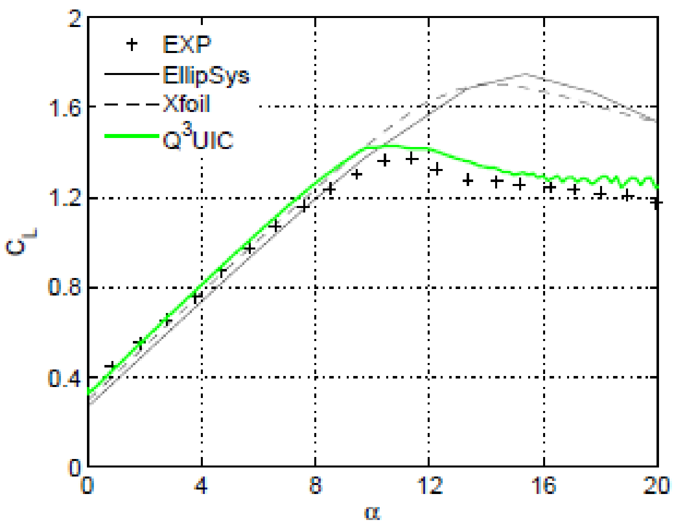


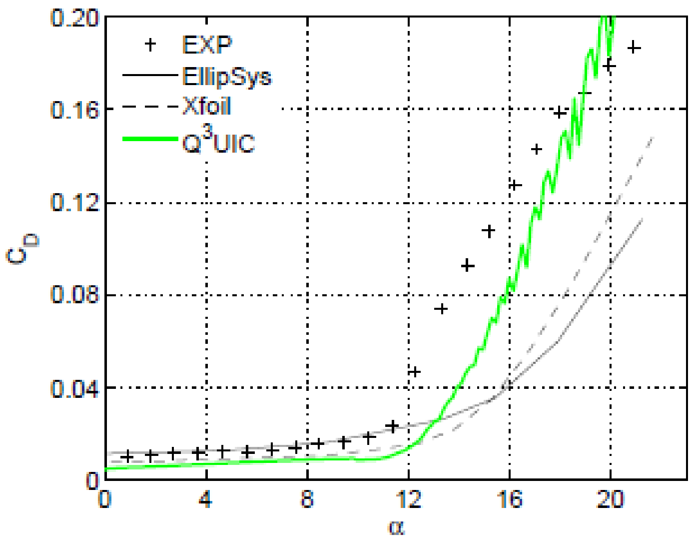


Blade Element Momentum Model

2D Airfoil data







Blade Element Momentum Model

Suggested 3D corrections for airfoil data

Correction for rotational effects (AoA < 20 degrees):

$$C_{l,3D} = C_{l,2D} + a(c/r)^b \left[C_{l,inv} - C_{l,2D} \right]$$

Correction for finite rotor size (AoA > 45 degrees):

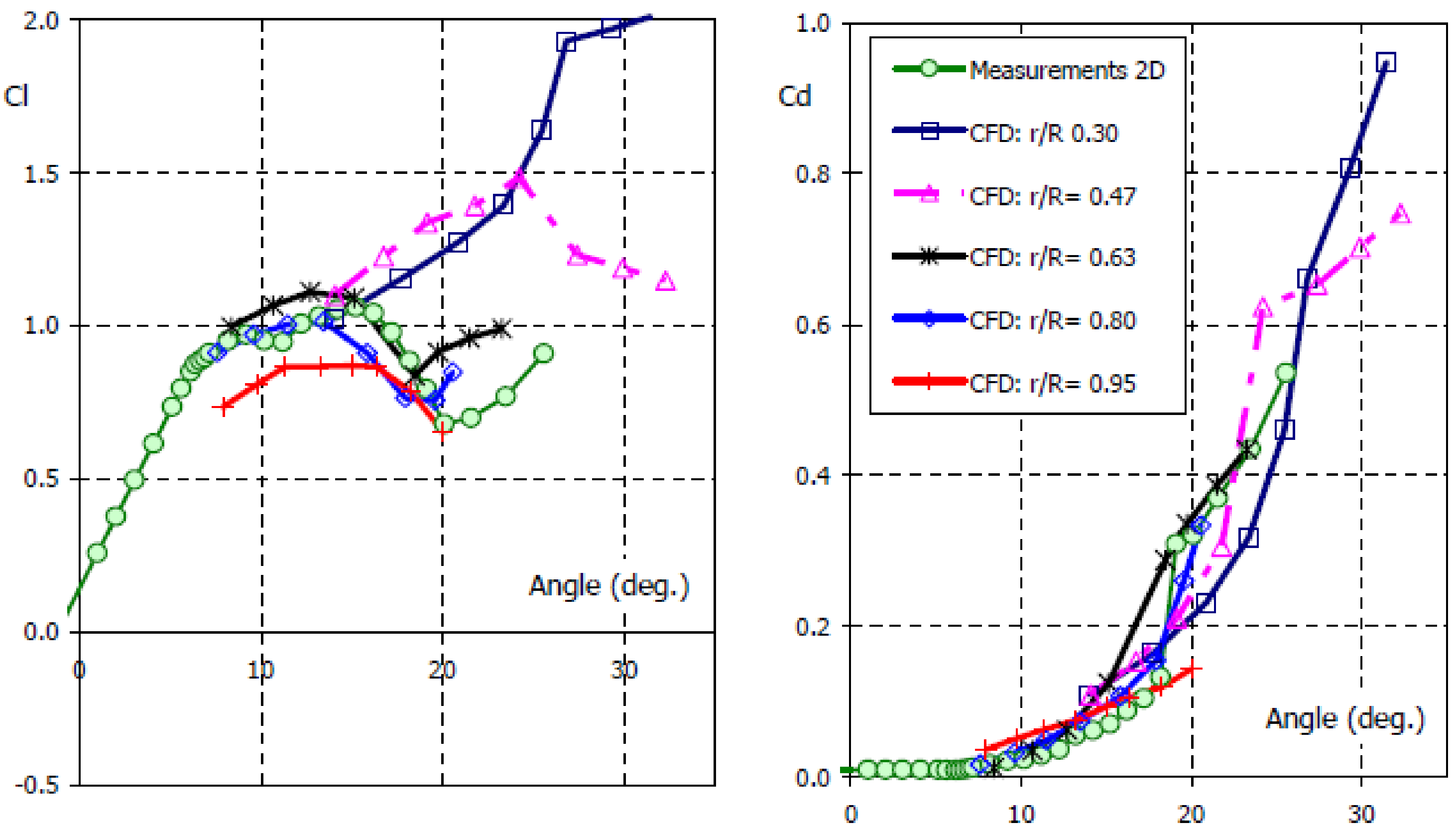
$$C_n = C_l \cos\phi + C_d \sin\phi = C_d(\alpha = 90^0)$$

$$C_t = C_l \sin\phi - C_d \cos\phi > 0$$

Linear interpolation for 20 degrees < AoA < 45 degrees

Blade Element Momentum Model

Computed 3D airfoil data



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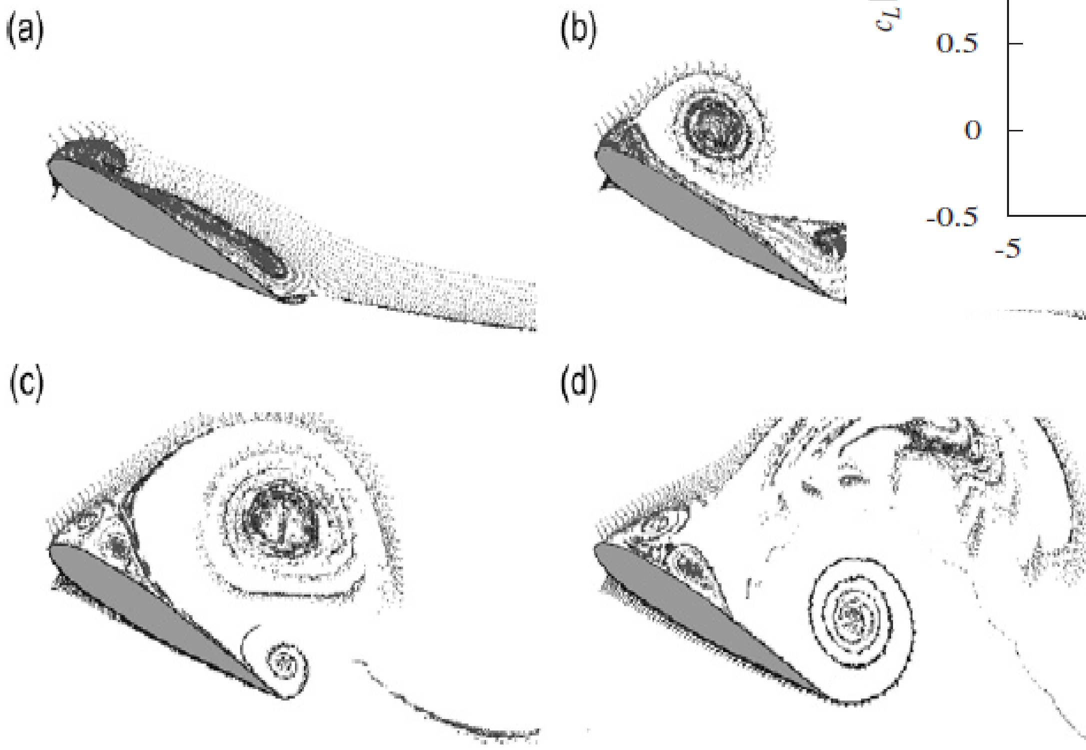
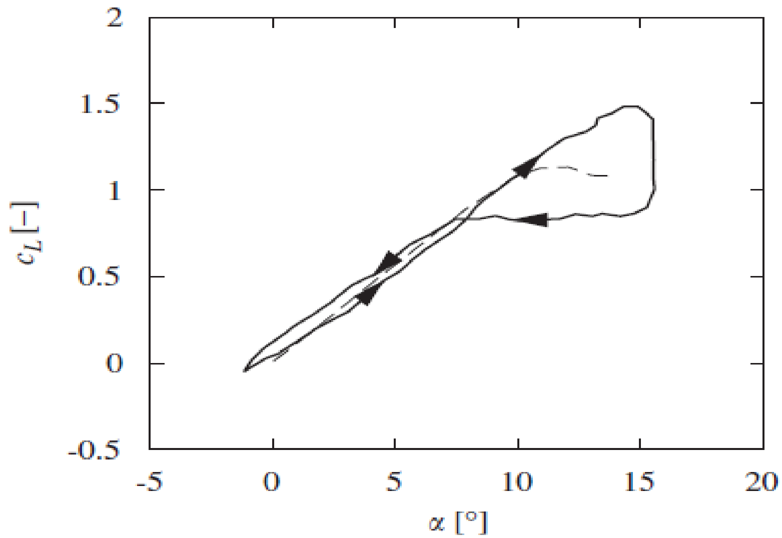
Blade Element Momentum Model

Dynamic stall



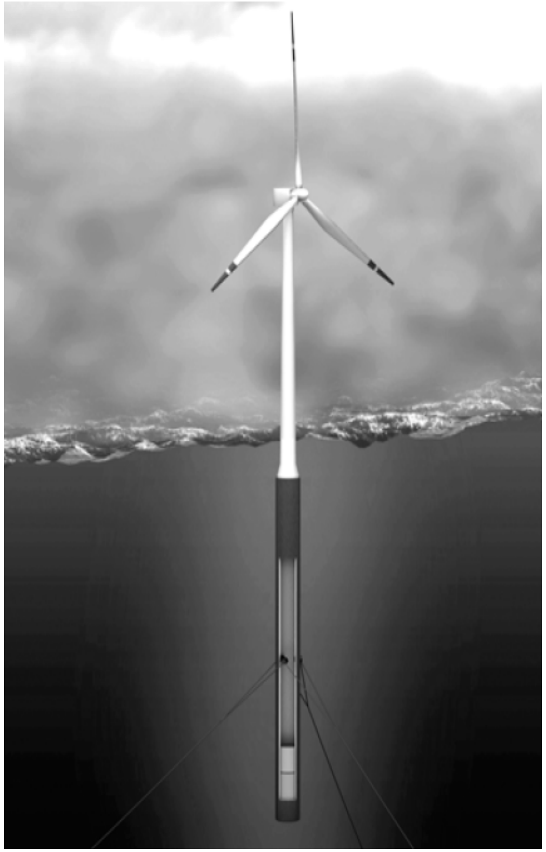


Blade Element Momentum Model

Dynamic stall

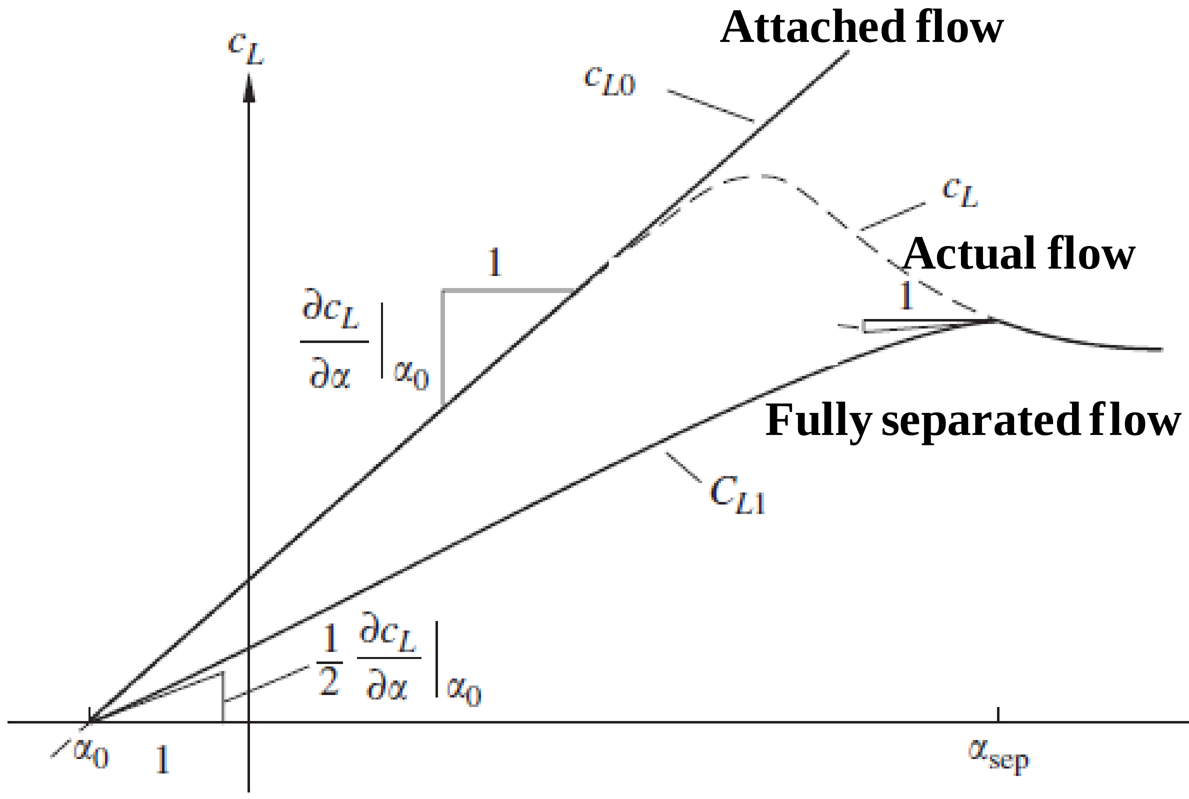
Dynamic stall is caused by:

- **Wind shear**
- **Atmospheric turbulence**
- **Tower shadow**
- **Rotors operating in yaw or tilt**
- **Dynamically deflected blades**
- **Turbine placed on floating structure**



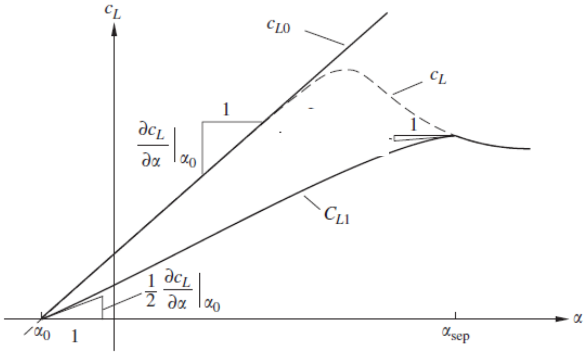
Blade Element Momentum Model

Dynamic stall model (Øye)



Blade Element Momentum Model

Dynamic stall model (Øye)



$f = 0$**: Fully separated**

$f = 1$**: Fully attached**

Linear interpolation:

$$C_L(f) = 2\pi(\alpha - \alpha_0) \cdot f + C_{L,sep} \cdot (1 - f) \Rightarrow$$

$$f_{static} = (C_{L,static} - C_{L,sep}) / (2\pi(\alpha - \alpha_0) - C_{L,sep})$$

Dynamic approach:

$$\frac{df}{dt} = \frac{f_{static} - f}{\tau}$$

Final algorithm:

$$f_i = f_{static} + (f_{i-1} - f_{static}) \cdot \exp\left(-\frac{\Delta t}{\tau}\right)$$