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## 1 Flow separation. Drag. Scale effects

A fixed vertical circular cylinder is exposed to oscillatory horizontal flow with velocity $U=$ $U_{m} \sin (2 \pi / T)$ where $t$ is time, $T$ period and $U_{m}$ velocity at maximum. This mimics the situation where a vertical slender cylinder is exposed to long incoming waves, with the difference that there is no decay in the velocity profile along the vertical.

The cylinder with diameter $D$ and height $h$ is exposed to a force along the oscillation direction, $F_{x_{1}}$, and force acting in the crosswise direction, $F_{x_{2}}$. Assuming that the forces per unit height are given by a Morison-type equation we evaluate coefficients of mass ( $C_{m}$ ), drag $\left(C_{d}\right)$ and lift $\left(C_{l}\right)$ from

$$
\begin{align*}
& F_{x_{1}}=\frac{1}{4} \pi \rho D^{2} C_{m} \dot{U}+\frac{1}{2} \rho D C_{d}|U| U  \tag{1}\\
& F_{x_{2}}=\frac{1}{2} \rho D C_{l} U^{2} \tag{2}
\end{align*}
$$

(Note that the cylinder diameter $D$ rather than the cylinder radius $R$ is used in the notation.) The force coefficients depend on two parameters, the Keulegan-Carpenter number (KC) and Reynolds number ( $R e$ ). The latter is better replaced by the so-called $\beta$-number which equals Re/KC.


Figure 1: Circular cylinder performs a translatoric harmonic oscillation perpendicular to its axis. Velocity $U=U_{m} \sin (2 \pi / T)\left(t\right.$ time, $T$ period and $U_{m}$ velocity at maximum). $D$ the cylinder diameter.

The $K C$-number is defined by

$$
\begin{equation*}
K C=\frac{U_{m} T}{D}=\frac{2 \pi a}{D} \tag{3}
\end{equation*}
$$

where $a$ is the excursion amplitude of the oscillation. $K C$ is independent of scale, for similar $a / D$.

The $\beta$-number is defined by

$$
\begin{equation*}
\beta=\frac{D^{2}}{\nu T} \tag{4}
\end{equation*}
$$

The $\beta$-number (and Reynolds number) is scale dependent. Assuming that the oscillation results from a wave motion, the wave period $T$ is related to the wavelength $\lambda$ by the dispersion relation for gravity waves. This is discussed earlier in the course and reads $\omega^{2}=g k \tanh k h$ where $\omega=2 \pi / T$ is frequency, $k=2 \pi / \lambda$ wavenumber and $h$ water depth. When $\tanh k h=1$ (deep water) the dispersion relation gives

$$
T=\sqrt{2 \pi \lambda / g}
$$

which means that the $\beta$-number becomes

$$
\begin{equation*}
\beta=\frac{D^{2}}{\nu T}=\frac{D^{\frac{3}{2}}}{\nu} \sqrt{\frac{g}{2 \pi \lambda / D}} \tag{5}
\end{equation*}
$$

This scaling is kept in finite water depth, although expressions become modified (see exercise).

Since the wavelength divided by the cylinder diameter $(\lambda / D)$ and water depth divided by wavelength $(h / \lambda)$ will be the same in full scale and in laboratory tests, the value of the $\beta$ number behaves according to the cylinder diameter in power $\frac{3}{2}$, assuming that the kinematic viscosity $\nu$ is the same in full scale and laboratory scale.

### 1.1 Example. $\beta$-number for Hywind. Full scale and lab.-scale.

The diameter of the Hywind spar platform is 6 m at the water line and 8.3 m of the submerged body. A typical period of the incoming waves is $T=10 \mathrm{~s}$ corresponding to a wavelength of 160 m (deep water). With $D=6 \mathrm{~m}$ and $\nu=10^{-6} \mathrm{~m}^{2} \mathrm{~s}^{-1}$ the $\beta$-number becomes

$$
\beta=\frac{D^{2}}{\nu T}=3.6 \times 10^{6}
$$

Models of Hywind have been tested in laboratory at reduced scale. With a laboratory scale of $1: 15$ we obtain $\beta=61400$. With a laboratory scale of $1: 47$ we obtain $\beta=11240$. For comparison, Hywind was tested in scale 1:50.

### 1.2 Example. $K C$ for offshore wind turbine

A wave field of significant wave height of 10 m and period 10 s , interacting with a vertical cylinder of diameter 6 m , implies a $K C$-number of about 5 and indicates an upper $K C$ value for design. The corresponding wave slope is around 0.2 . In very rare events, like the Draupner wave, the slope may double (0.4) meaning that $K C$ is up to 10 . However, very large waves appear in single events rather than in long trains.

### 1.3 Exercise

Derive an expression corresponding to (5) assuming finite water depth. Hint: Use the dispersion relation for finite water depth.

### 1.4 Exercise

Perform the calculations of the $\beta$-number in section 1.1. If in model scale tests of Hywind $\beta=1000$, what is the scale ratio? Assume same kinematic viscosity in model and full scale.

## $1.5 C_{m}, C_{d}$ and $C_{l}$. Experimental and numerical evaluation

Mass, drag and lateral force (lift) coefficients depend on $K C$ and $\beta$. There are two important regimes, the one where flow separation occurs, and the other where flow separation does not occur. In the former regime, the flow coefficients are basically determined by the pressure field. The viscosity of the fluid plays an important role in the flow separation process. There is little or no direct effect of the viscosity in the cylinder's boundary layer on the forces, particularly in large scale.

In the regime where flow separation does not occur there is practically speaking no effect of viscosity, in large scale. In laboratory scale, the effect of viscosity is measurable, contributing to the mass and drag coefficients, and is given by the classical Stokes-Wang solution (Stokes, 1851; Wang, 1968). The solution valid for $K C \ll 1$, and $\beta \gg 1$, can be reduced to the following form

$$
\begin{align*}
C_{d} & \left.=\frac{3 \pi^{3}}{2 K C}\left[(\pi \beta)^{-\frac{1}{2}}+(\pi \beta)^{-1}-\frac{1}{4}(\pi \beta)^{-\frac{3}{2}}\right)\right]  \tag{6}\\
C_{m} & =2+4(\pi \beta)^{-\frac{1}{2}}+(\pi \beta)^{-\frac{3}{2}} \tag{7}
\end{align*}
$$

We note that when the cylinder is oscillating in fluid otherwise at rest, $\hat{C}_{m}=1+4(\pi \beta)^{-\frac{1}{2}}+$ $(\pi \beta)^{-\frac{3}{2}}$.
There are other features in the non-separating regime, such as formation of the Honji instability and the Honji rolls.

### 1.6 Regime with flow separation

An important result of experiments by Otter (1990) is that flow separation does not occur when $K C$ is less than about 2. His experiments were performed with $\beta=61400$. Recent LES-computations of a circular cylinder performing a translational harmonic oscillation perpendicular to its axis, with $\beta=11240$, by Rashid, Vartdal and Grue (2011), obtained also that there was no flow separation when $K C<2$ (figure 2).

Evaluation of the drag coefficient in the $K C$-regime between 2 and 4, experimentally by Otter (1990) $(\beta=61400)$ and computationally by Rashid, Vardal and Grue (2011) $(\beta=11240)$, shows a conforming picture, even though there is some span in the $\beta$-number. $C_{d}$ is 0.44 for $K C=4$ (figure 3). The force coefficients for $K C=4$ and $\beta=11240$ are: $C_{m}=1.95$, $C_{d}=0.44$ and $C_{l}=0.011$ and illustrate that the mass term totally dominates (figure 3). The lateral force (lift) is only 2.5 per cent of the drag force. This comparison indicates that flow separation is not a dominant contribution to the force picture of offshore wind turbines.

Drag coefficients $C_{d}$ by Otter and Rashid et al. are close to those in U-tube measurements by Sarpkaya (1986) ( $\beta=11240$ ) but significantly overpredicts $C_{d}$ for the smaller $K C$.

The various force contributions become all significant when the scale is reduced with the $\beta$-number around 1000. According to computations, flow separation does not occur when $K C<2$. Force coefficients evaluated for $\beta=1035$ and $K C=8.5$ are all comparable: $C_{m}=1.52, C_{d}=1.59$ and $C_{l}=1.75$.


Figure 2: a) Drag coefficient divided by the Stokes-Wang solution in eq. (6) denoted by W vs. $K C$. LES computations with $\beta=1035$ (red squares); $\beta=11240$ (red triangles); $\beta=300$ and 197 (red dots); $\beta=61400$ (large red circle); Otter's experiments (1990) $\beta=61400$ (blue crosses). From Rashid, Vardal and Grue (2011). b) $C_{d}$ vs. $K C$. Experiments by Otter (1990) for $\beta=61400(\times)$, computations by Rashid, Vartdal and Grue (2011) for $\beta=11240$ (squares), experiments by Sarpkaya (1986) for $\beta=11240(\star)$.


Figure 3: In-line force $F_{x_{1}}$ (blue) and lateral force $F_{x_{2}}$ (red) vs. time for $\beta=11240$ and $K C=4$. LES-computations by Rashid, Vartdal and Grue (2011).

### 1.7 References

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