Simulation of Wind Farms

by

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Wake and Wind Farm Aerodynamics

Basic questions and issues:

- **How important is the dynamics of the vortex system**
- **Relationship between strength of the vortices and the blade load**
- **Conditions for instability**
- **How far downstream do ’near-wake’ and ’far-wake’ refer to**
- **What is the relationship between vortex dynamics and meandering**
- **How does the added turbulence intensity relate to the loading**
- **Performance predictions of wind farms**
- **Life time estimation of turbines in wind farms**
- **Influence of stability of the atmospheric boundary layer**
- **Estimation of wind resources in wind farm to wind farm interaction**

Wake and Wind Farm Aerodynamics

Simulation models

- **Momentum theory (Frandsen)**
- **Linearized Navier-Stokes (Fuga)**
- **Parabolised Navier-Stokes (Ainslie, UMPWAKE)**
- **Reynolds Averaged Navier-Stokes (RANS)**
- **Detached Eddy Simulation (DES)**
- **Large Eddy Simulation (LES)**
- **Actuator Disc/Line - LES (AD/L-LES)**

Wind Turbine Wake Aerodynamics

Horns Rev offshore wind farm:

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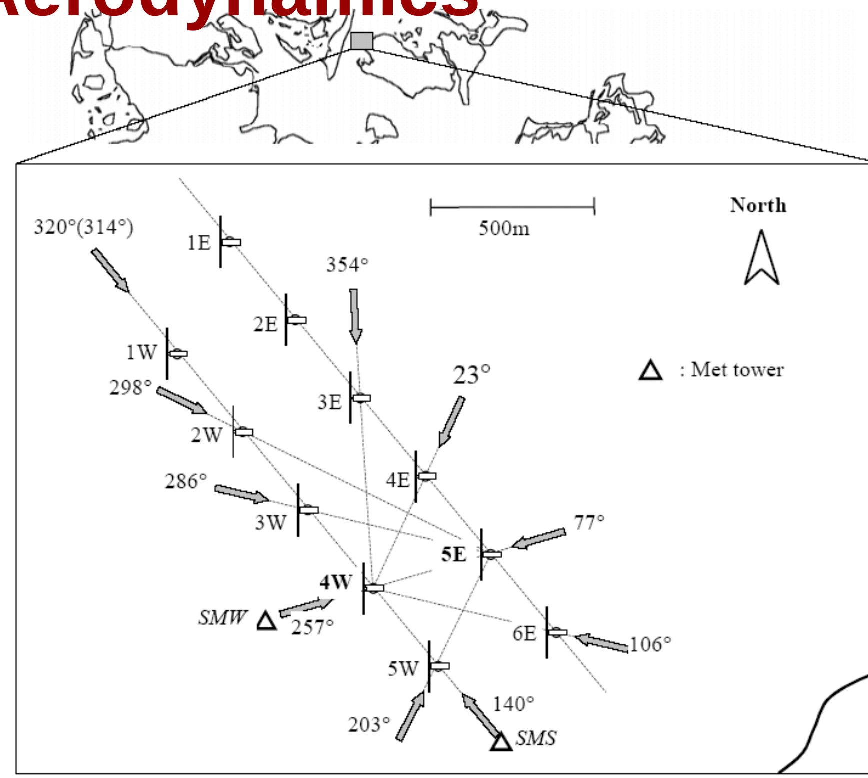
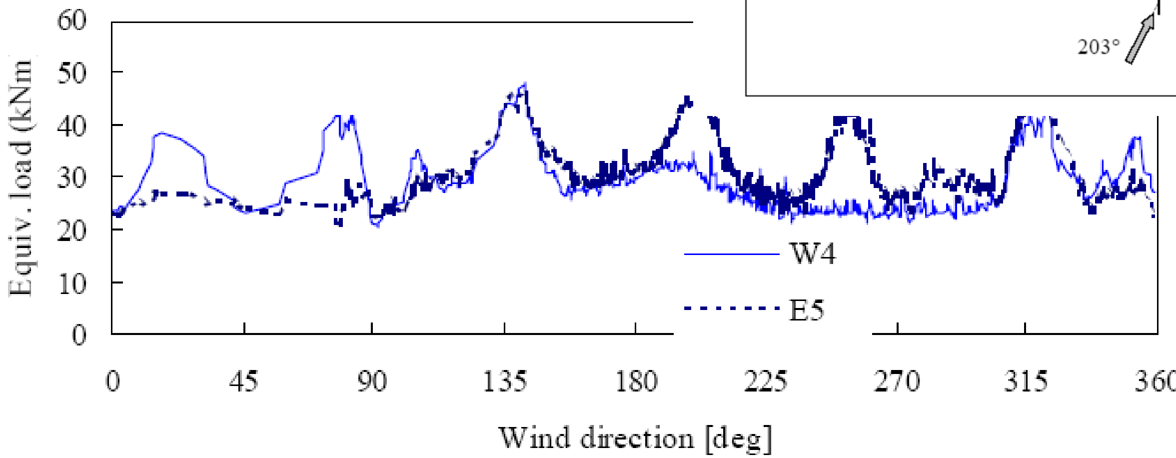
Wind Turbine Wake Aerodynamics

Vindeby off-shore wind farm:





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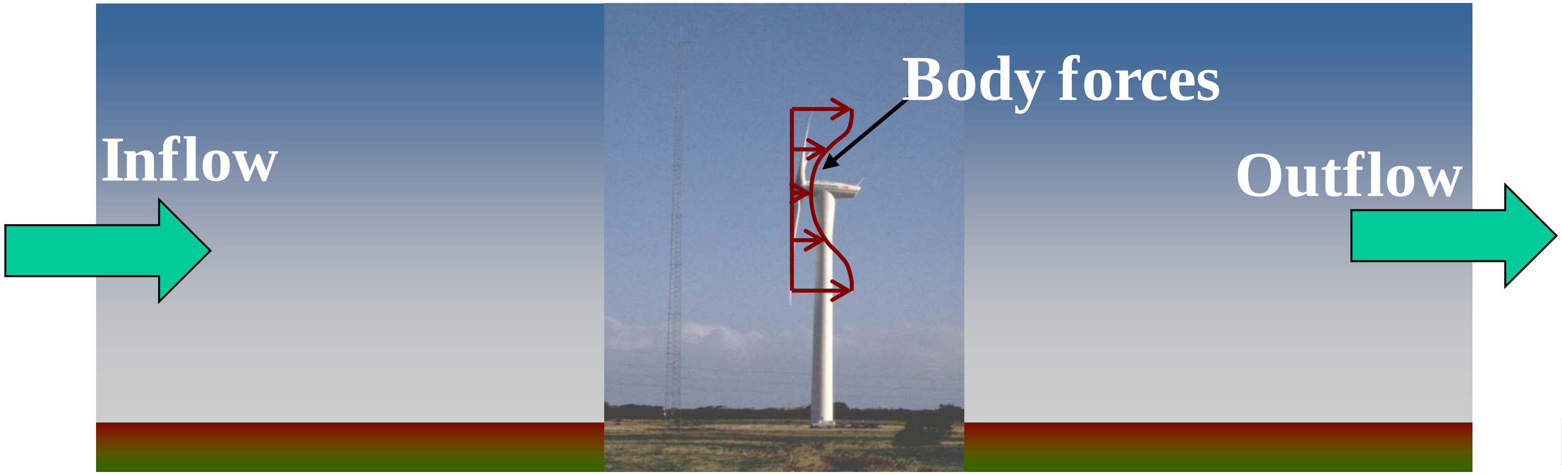
The actuator line technique

Basic idea:

- **Replace rotor blades by body forces**
- **Determine body forces from aerofoil data**
- **Simulate flow domain using DNS or LES**
- **Computing code: *EllipSys3D***



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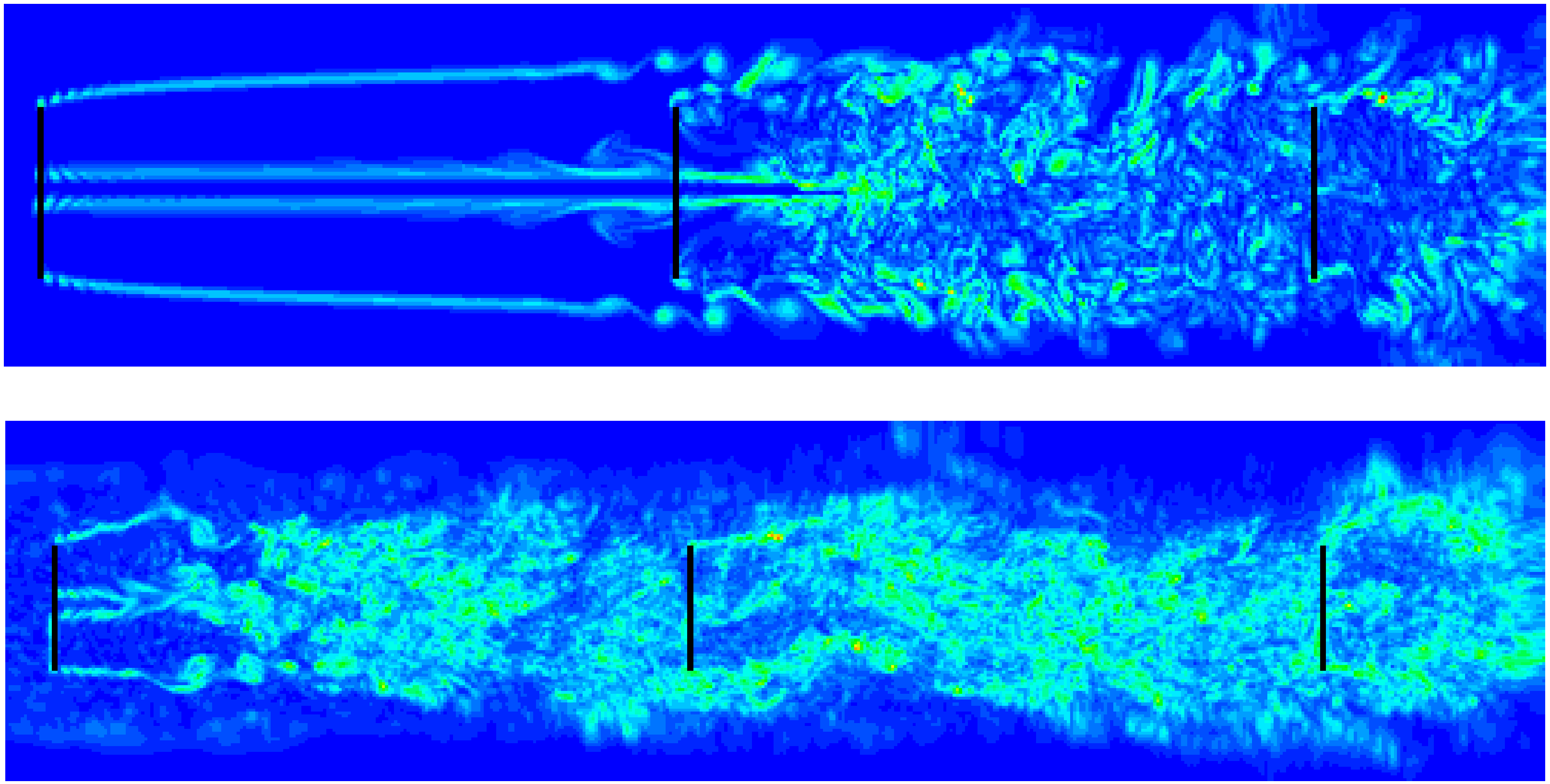
Vortex structures in the wake of a row of rotors



Development of wake behind three rotors in a row at W_0 = 10 m/s; Turbine spacing 6 rotor radii. A) Constant inflow; B) Turbulent inflow.

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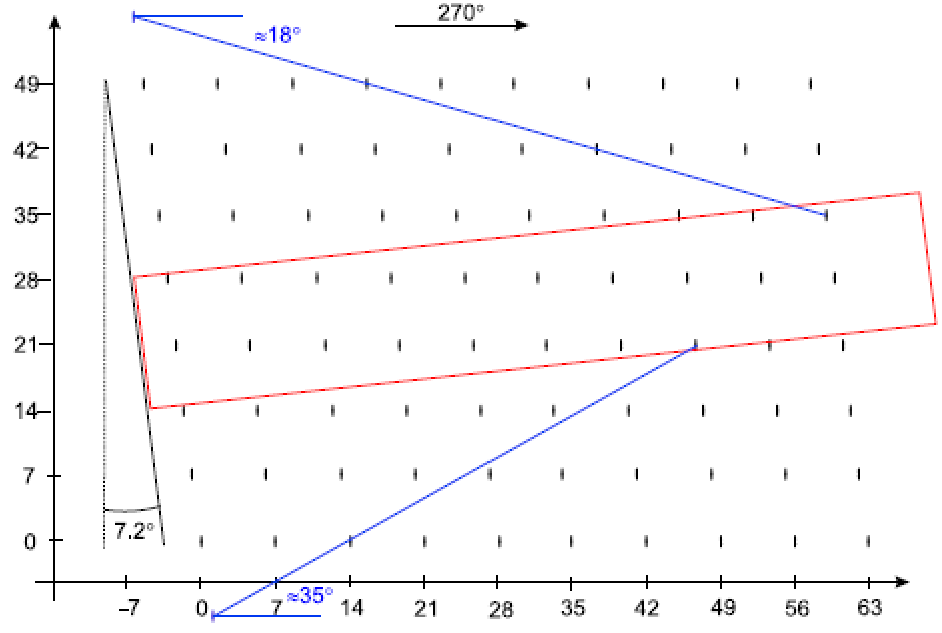


FIGURE 1. Layout of Horns Rev Wind Farm

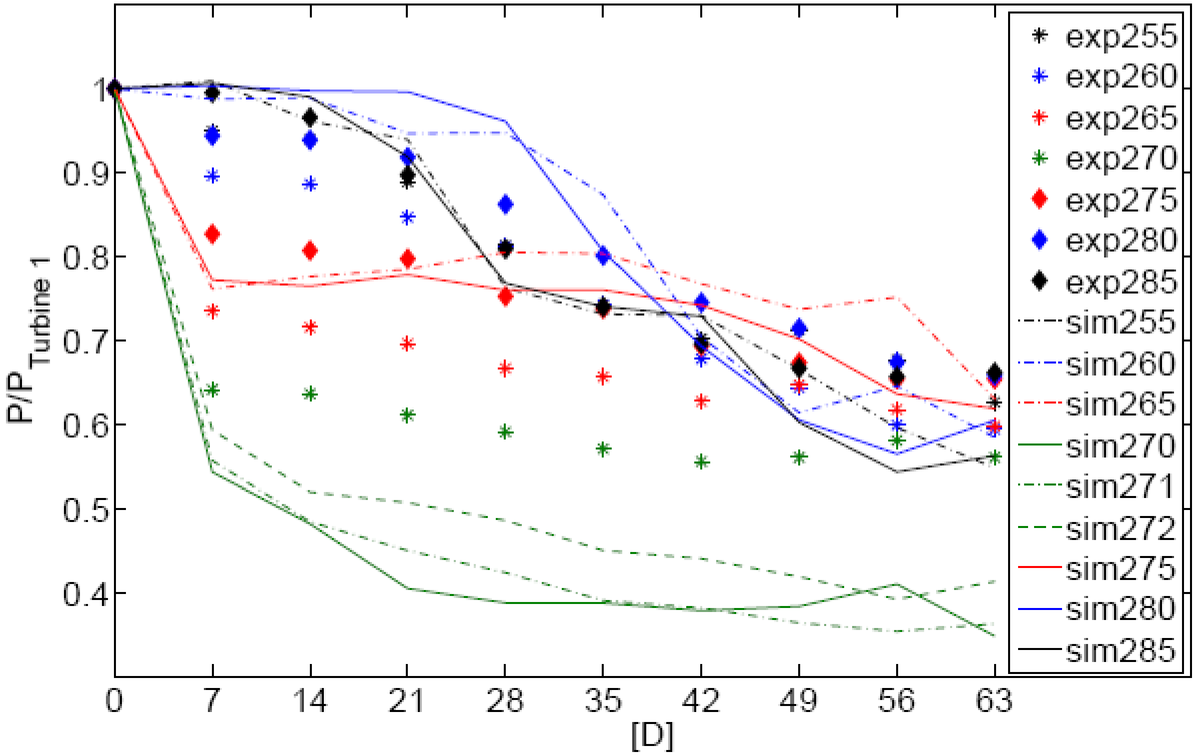
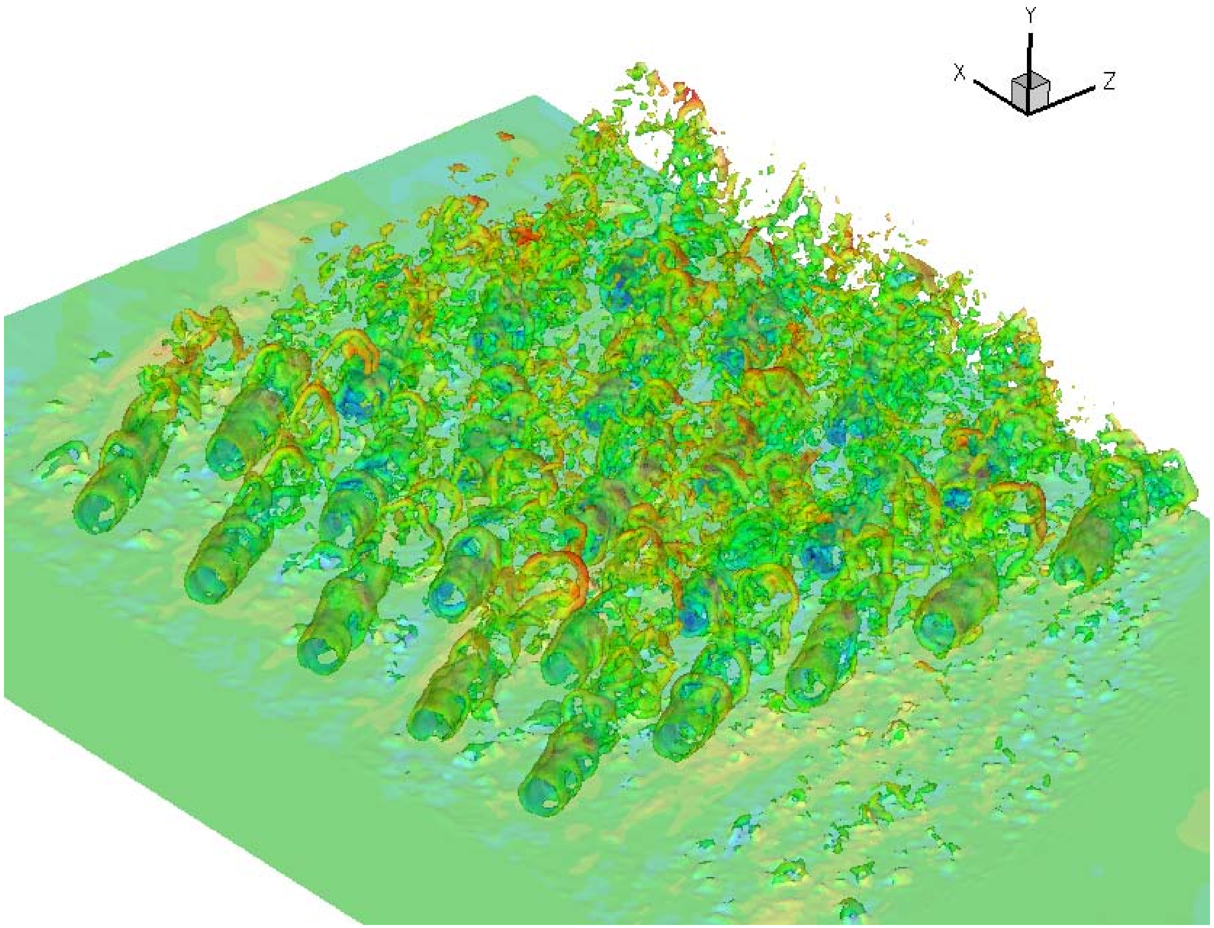


FIGURE 13. Simulation results compared with measurements. Results from both simulations and measurements are shown for inflow angles between 255 and 285 degrees, i.e., ± 15 degrees from the westerly direction.

Modelling of Turbulent and Atmospheric Turbulence

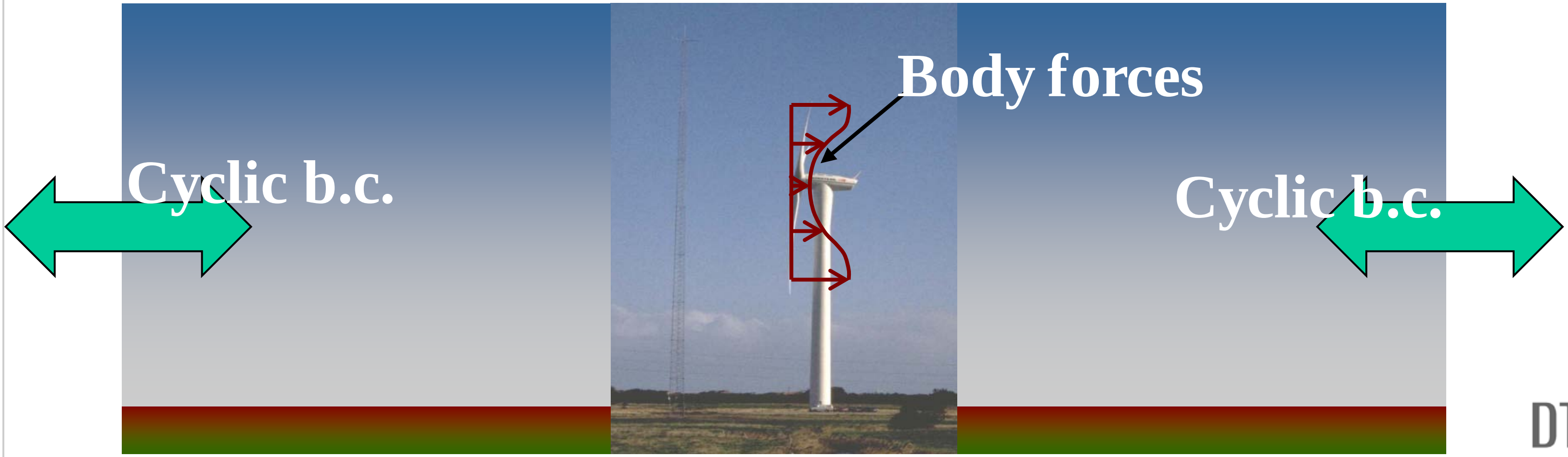


Vorticity shed from 5x5 turbines in a farm computed by actuator disk method

Simulation of turbulence inside wind farm

Basic idea:

- **Replace rotor blades by body forces**
- **Determine body forces from aerofoil data**
- **Simulate an 'infinite' row of turbines using cyclic boundary conditions**



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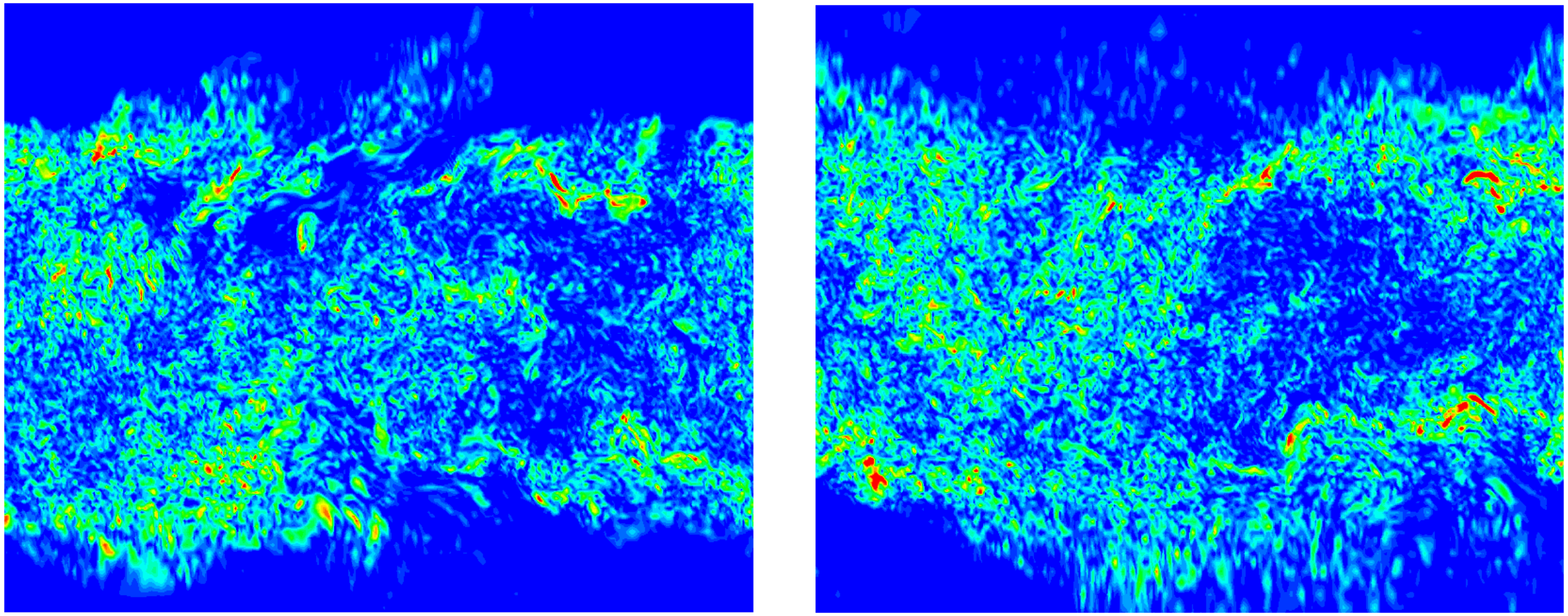
Simulation of turbulence inside wind farm

Cross sectional turbulent flow fields:



Iso-vorticity contours in the final stage

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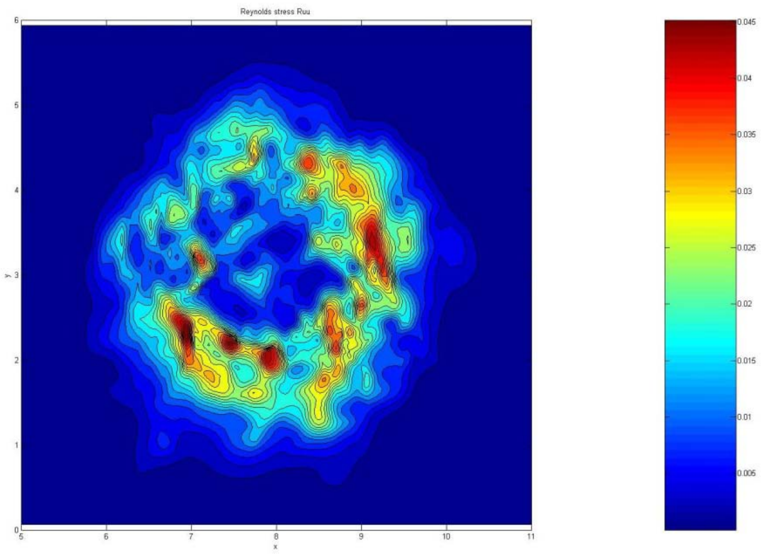
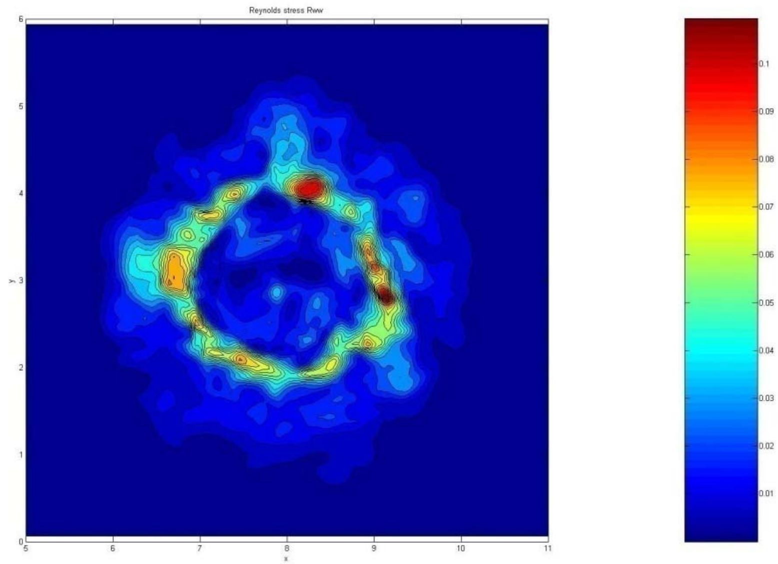
Simulation of turbulence inside wind farm

Reynolds stresses:



$\overline{u'u'}$



$\overline{w'w'}$

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Simulation of Wind Farms

ABL flow equations:

$$\frac{\partial \widetilde{u}_i}{\partial x_i} = 0,$$

$$\frac{\partial \widetilde{u}_i}{\partial t} + \widetilde{u}_j \left(\frac{\partial \widetilde{u}_i}{\partial x_j} - \frac{\partial \widetilde{u}_j}{\partial x_i} \right) = -\frac{1}{\rho} \frac{\partial \widetilde{p}^*}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \nu \frac{\partial^2 \widetilde{u}_i}{\partial x_j^2}$$
$$+ \delta_{i3} g \frac{\widetilde{\theta} - \langle \widetilde{\theta} \rangle}{\theta_0} + f_c \varepsilon_{ij3} \widetilde{u}_j - \frac{f_i}{\rho} + \mathcal{F}_i,$$

$$\frac{\partial \widetilde{\theta}}{\partial t} + \widetilde{u}_j \frac{\partial \widetilde{\theta}}{\partial x_j} = -\frac{\partial q_j}{\partial x_j} + \alpha \frac{\partial^2 \widetilde{\theta}}{\partial x_j^2},$$

SGS fluxes:

$$\tau_{ij} = \widetilde{u_i u_j} - \widetilde{u}_i \widetilde{u}_j$$
$$q_j = \widetilde{u_j \theta} - \widetilde{u}_j \widetilde{\theta}.$$

Simulation of Wind Farms

Dynamic Sub-Grid Scale model:

SGS stress: $\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\nu_{sgs}\widetilde{S}_{ij}$

SGS heat flux: $q_j = -\dfrac{\nu_{sgs}}{Pr_{sgs}}\dfrac{\partial\widetilde{\theta}}{\partial x_j}$

Pr_{sgs} is the SGS Prandtl number

Resolved strain rate: $\widetilde{S}_{ij} = (\partial\widetilde{u}_i/\partial x_j + \partial\widetilde{u}_j/\partial x_i)/2$

Smagorinsky mixing length model:

$\nu_{sgs} = C_s^2\widetilde{\Delta}^2|\widetilde{S}|$, where $|\widetilde{S}| = (2\widetilde{S}_{ij}\widetilde{S}_{ij})^{1/2}$

Problem: C_s and Pr_{sgs} **?**

Simulation of Wind Farms

Dynamic SGS model:

(Germano (1991); Lilly (1992))

Least squares minimization of error:

$$C_s^2(\tilde{\Delta}) = \frac{\langle L_{ij} M_{ij} \rangle_{\mathcal{L}}}{\langle M_{ij} M_{ij} \rangle_{\mathcal{L}}},$$

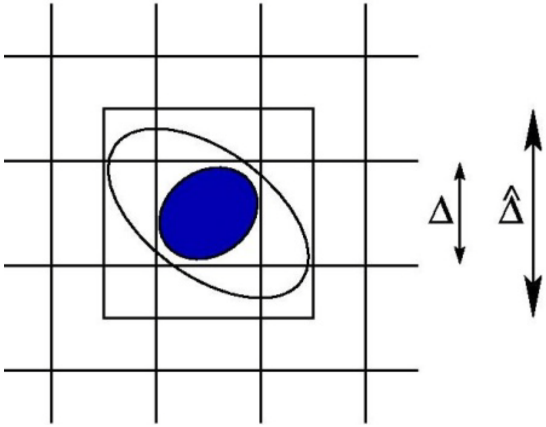
$$C_s^2 Pr_{sgs}^{-1}(\tilde{\Delta}) = \frac{\langle K_i X_i \rangle_{\mathcal{L}}}{\langle X_i X_i \rangle_{\mathcal{L}}}$$

Scale dependence parameters:

$$\beta = C_s^2(\alpha\tilde{\Delta}) / C_s^2(\tilde{\Delta})$$

$$\beta_\theta = C_s^2 Pr_{sgs}^{-1}(\alpha\tilde{\Delta}) / C_s^2 Pr_{sgs}^{-1}(\tilde{\Delta})$$

Typically: $\beta = 1$ and $\beta_\theta = 1$



Test filter:

$\overline{\Delta} = \alpha\tilde{\Delta}$ (with typically $\alpha = 2$)

Where:

$$L_{ij} = \overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$

$$M_{ij} = 2\tilde{\Delta}^2 \left(\overline{|\tilde{S}|\tilde{S}_{ij}} - \alpha^2 \beta |\bar{\tilde{S}}| \bar{\tilde{S}}_{ij} \right)$$

$$K_i = \overline{\tilde{u}\tilde{\theta}} - \bar{\tilde{u}}_i \bar{\tilde{\theta}}$$

$$X_i = \tilde{\Delta}^2 \left(\overline{|\tilde{S}| \partial\tilde{\theta}/\partial x_i} - \alpha^2 \beta_\theta |\bar{\tilde{S}}| \partial\bar{\tilde{\theta}}/\partial x_i \right)$$

Simulation of Wind Farms

Dynamic scale-dependent SGS model:

(Meneveau et al. (1996); Porte-Agel et al. (2011))

Least squares minimization of error:

$$C_s^2(\widetilde{\Delta}) = \frac{\langle L'_{ij} M'_{ij} \rangle_{\mathcal{L}}}{\langle M'_{ij} M'_{ij} \rangle_{\mathcal{L}}},$$

$$C_s^2 Pr_{sgs}^{-1}(\widetilde{\Delta}) = \frac{\langle K'_i X'_i \rangle_{\mathcal{L}}}{\langle X'_i X'_i \rangle_{\mathcal{L}}}$$

Scale dependence parameters:

$$\beta = \frac{C_s^2(\alpha\widetilde{\Delta})}{C_s^2(\widetilde{\Delta})} = \frac{C_s^2(\alpha^2\widetilde{\Delta})}{C_s^2(\alpha\widetilde{\Delta})},$$

$$\beta_\theta = \frac{C_s^2 Pr_{sgs}^{-1}(\alpha\widetilde{\Delta})}{C_s^2 Pr_{sgs}^{-1}(\widetilde{\Delta})} = \frac{C_s^2 Pr_{sgs}^{-1}(\alpha^2\widetilde{\Delta})}{C_s^2 Pr_{sgs}^{-1}(\alpha\widetilde{\Delta})}$$

Where:

$$L'_{ij} = \widehat{\widetilde{u}_i \widetilde{u}_j} - \widehat{\widetilde{u}}_i \widehat{\widetilde{u}}_j$$

$$M'_{ij} = 2\widetilde{\Delta}^2 \left(\widehat{|\widetilde{S}|\widetilde{S}_{ij}} - \alpha^2 \beta^2 |\widehat{\widetilde{S}}| \widehat{\widetilde{S}}_{ij} \right)$$

$$K'_i = \widehat{\widetilde{u}\widetilde{\theta}} - \widehat{\widetilde{u}}_i \overline{\widetilde{\theta}}$$

$$X'_i = \widetilde{\Delta}^2 \left(\widehat{|\widetilde{S}| \partial\widetilde{\theta}} / \partial x_i - \alpha^2 \beta_\theta^2 |\widehat{\widetilde{S}}| \partial\widehat{\widetilde{\theta}} / \partial x_i \right)$$

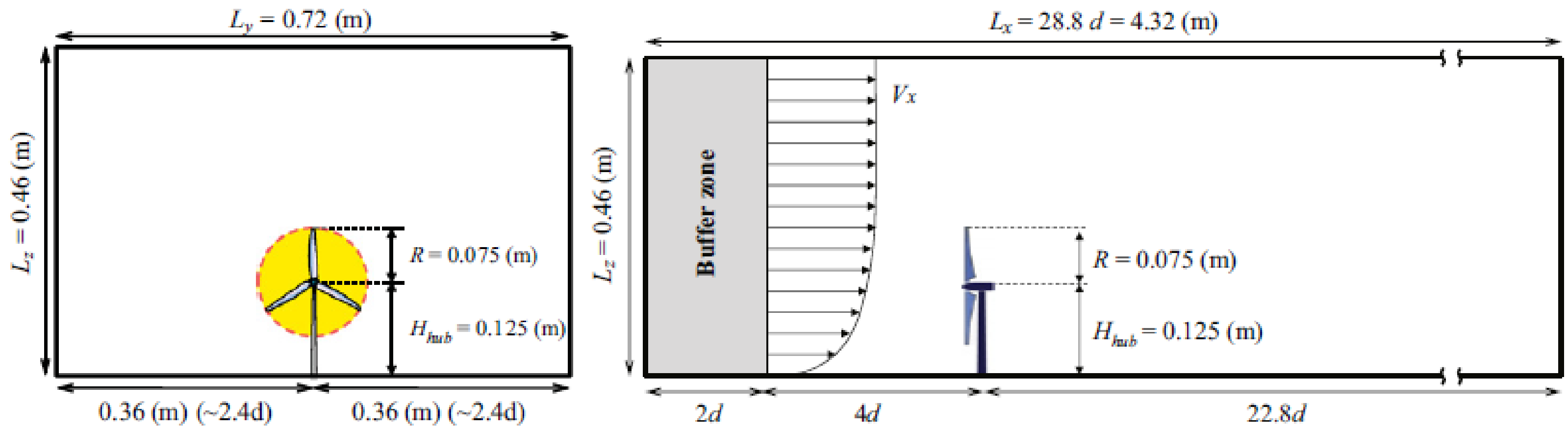
Second test filter:

$$\widehat{\Delta} = \alpha^2 \widetilde{\Delta}$$

Simulation of Wind Farms

Some results of ABL-LES computations:
(Porte-Agel et al. (2011))

Flow domain



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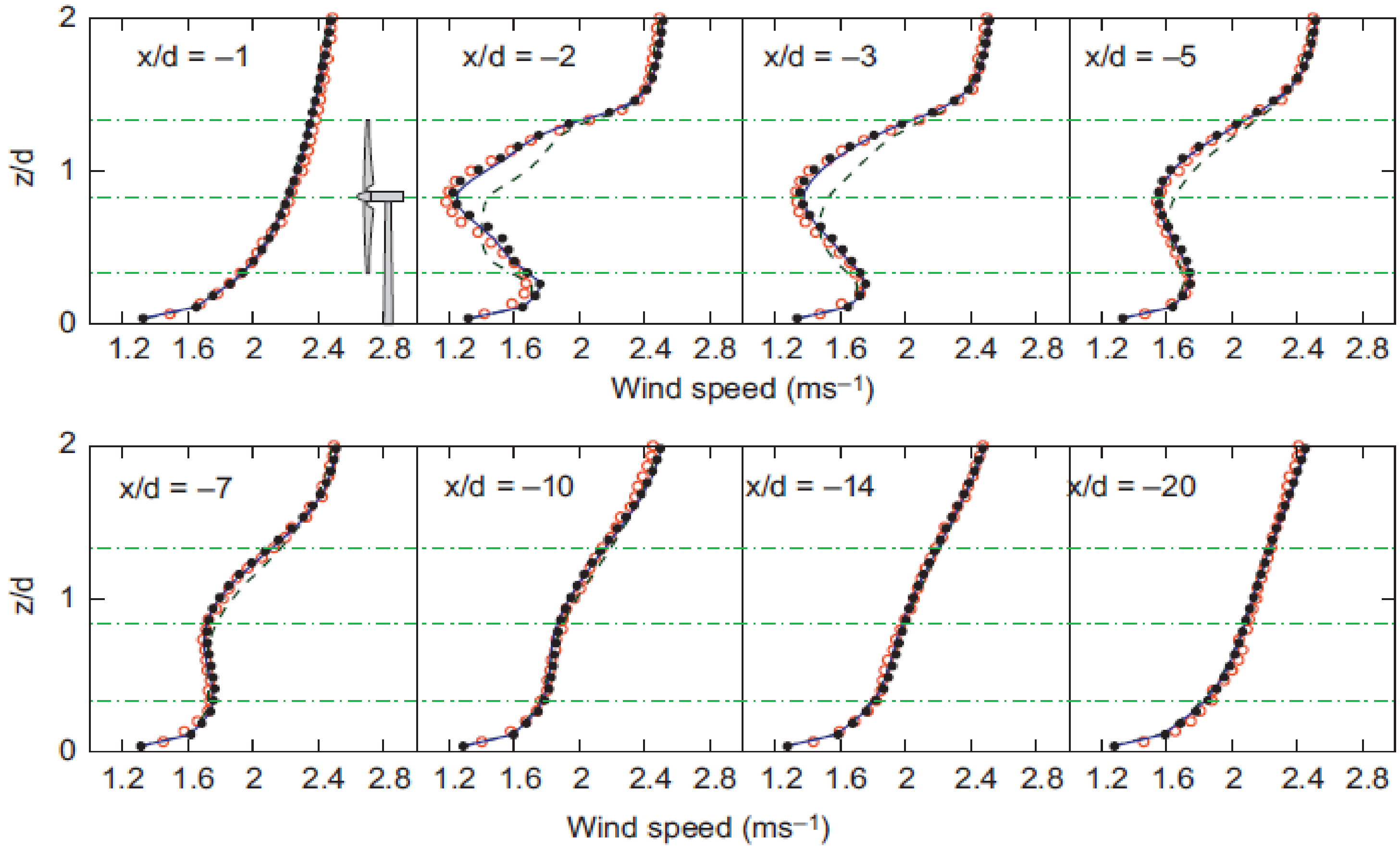
Comparison of LES computations with measurements
(Porte-Agel et al. (2011))



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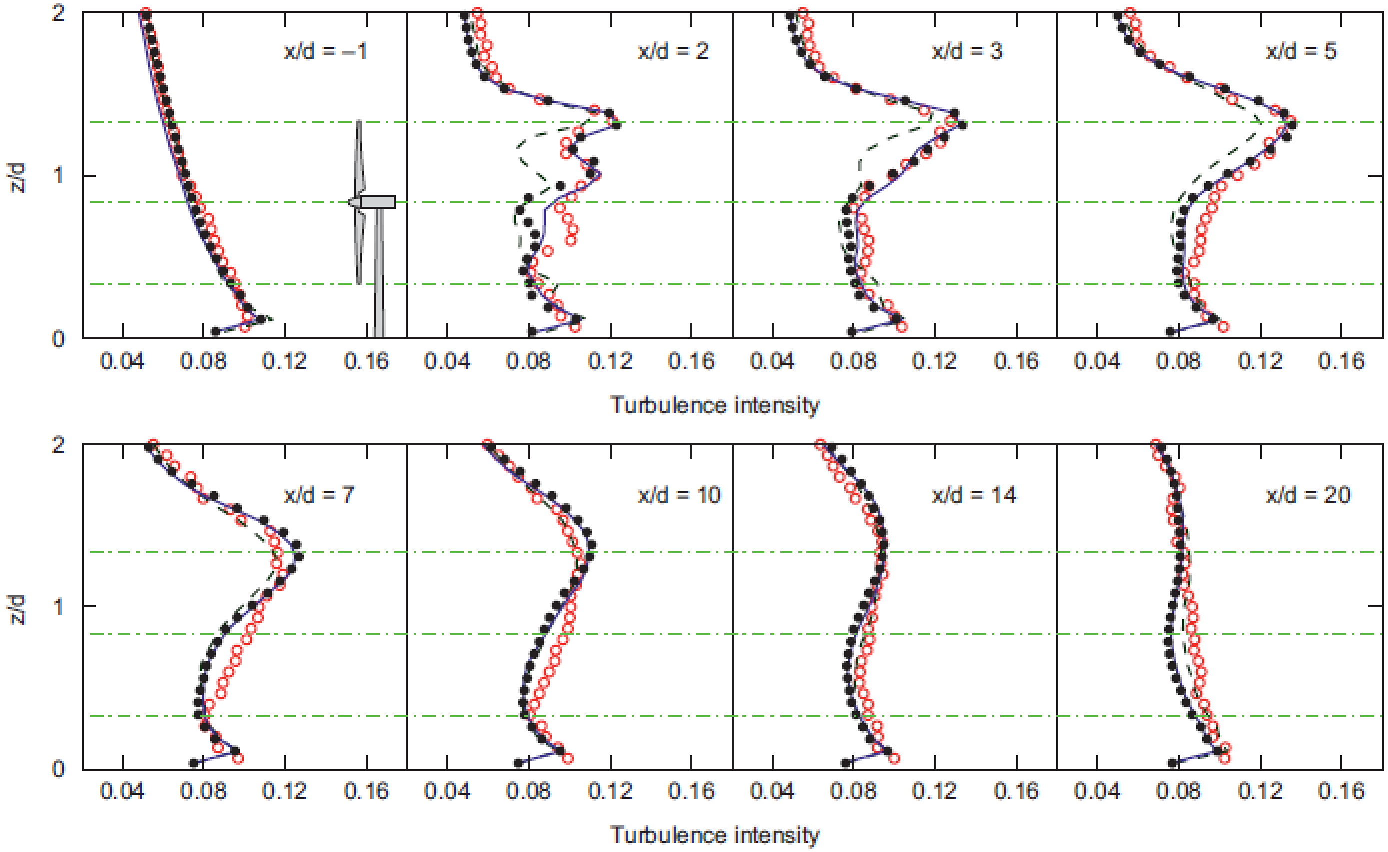
Comparison of LES computations with measurements
(Porte-Agel et al. (2011))



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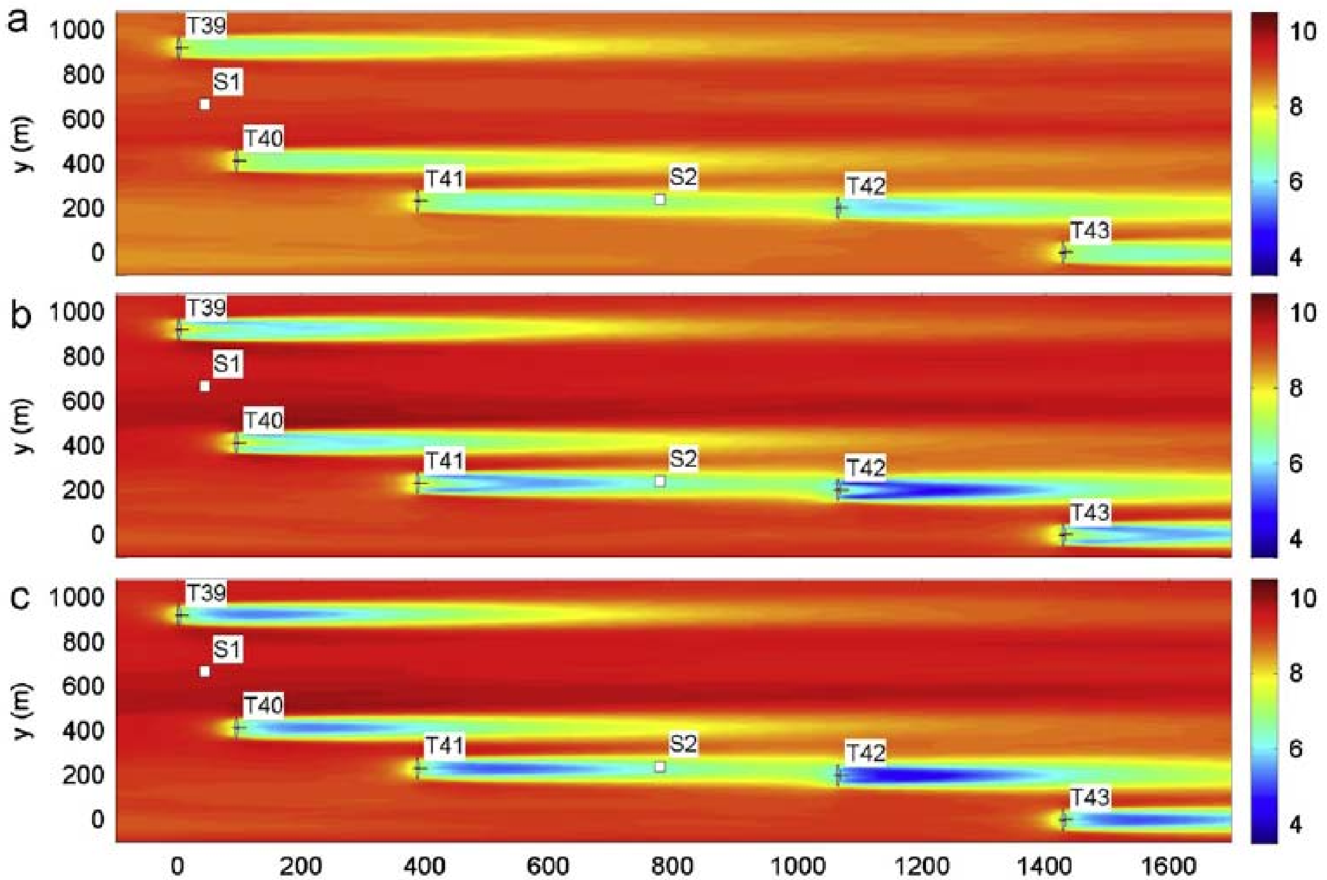
Simulation of Wind Farms

Comparison between actuator disc and actuator line models (Porte-Agel et al. (2011)):



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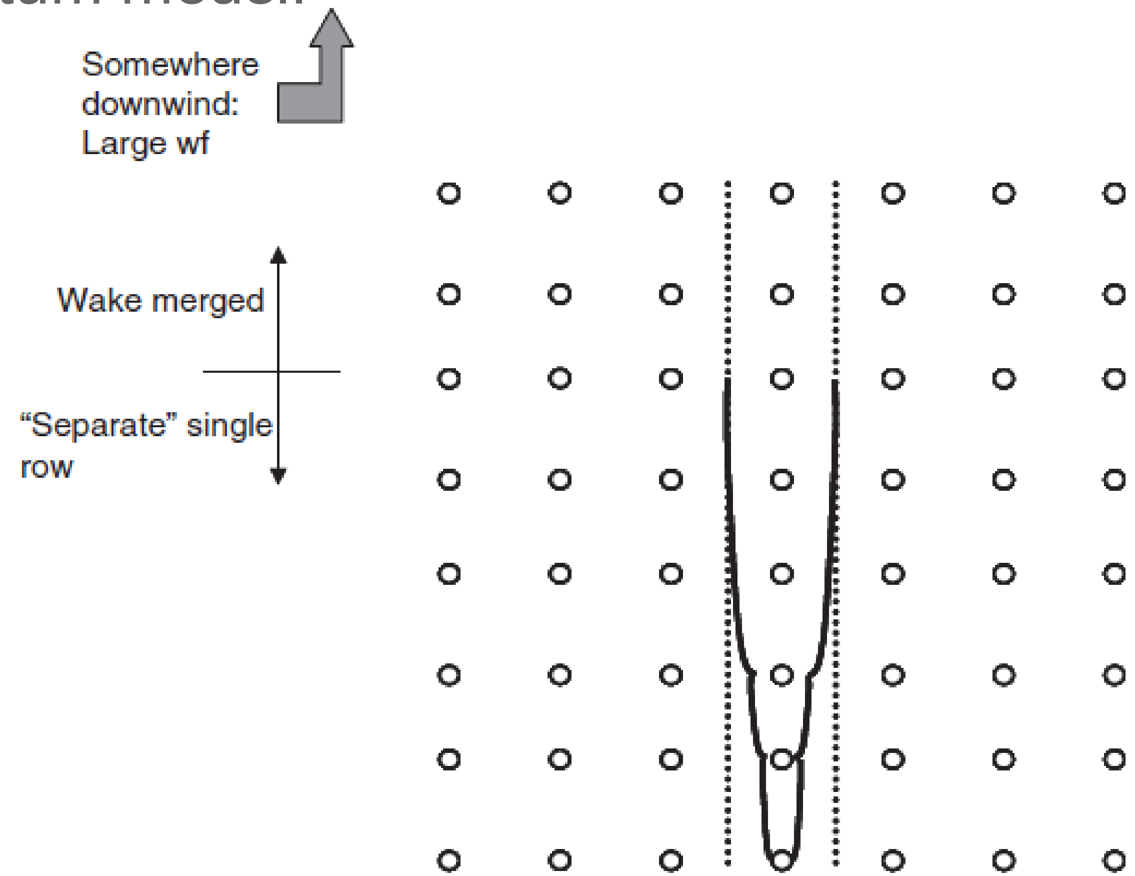
Momentum model:



From Frandsen et al.: Wind Energy vol. 9, 2006

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Momentum model:

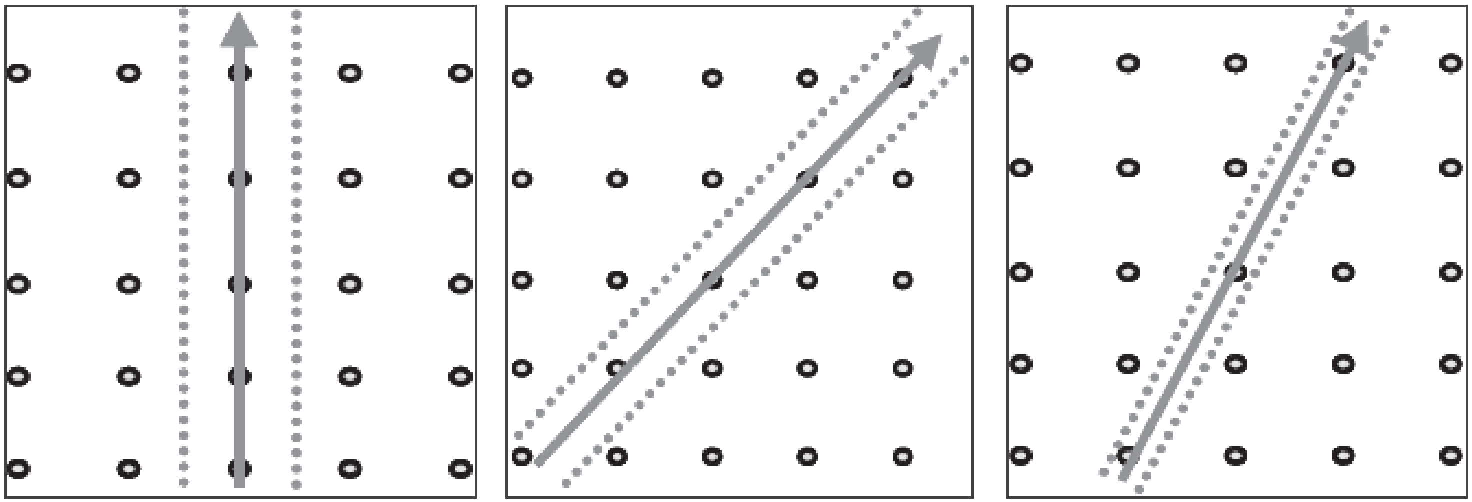

Figure 2. Examples of wind turbine patterns for different wind directions

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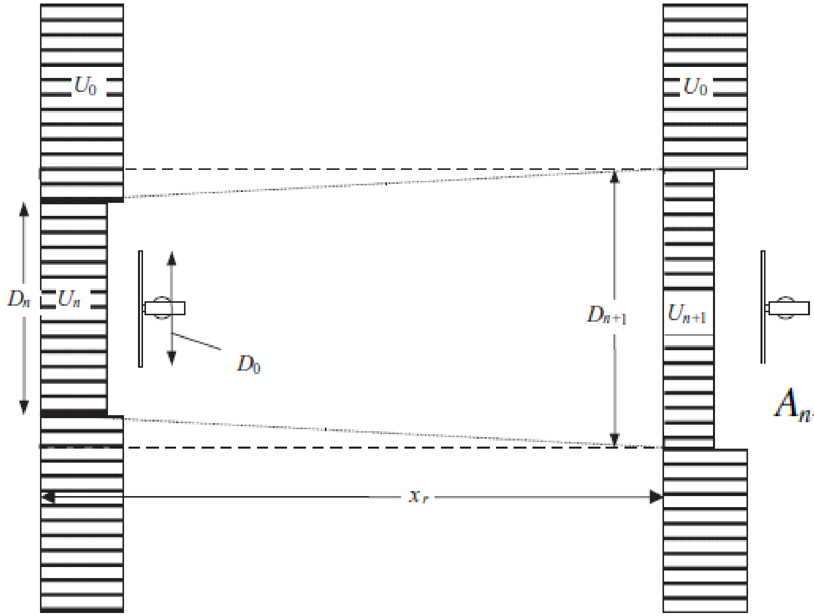
Multiple wakes, single row:



$$T = \int_A \rho U (U_0 - U) \mathrm{d}A$$

$$\rho A_{n+1} U_{n+1} (U_0 - U_{n+1}) = \rho (A_{n+1} - A_n) U_0 (U_0 - U_0) + \rho A_n U_n (U_0 - U_n) + T \quad \Rightarrow$$

$$A_{n+1} U_{n+1} (U_0 - U_{n+1}) = A_n U_n (U_0 - U_n) + \frac{1}{2} A_R U_n^2 C_T \quad \Rightarrow$$

$$c_{n+1}(1 - c_{n+1}) = \frac{A_n}{A_{n+1}} c_n (1 - c_n) + \frac{1}{2} \frac{A_R}{A_{n+1}} c_n^2 C_T$$

$$c_n = \frac{U_n}{U_0}, \qquad c_{n+1} = \frac{U_{n+1}}{U_0}$$

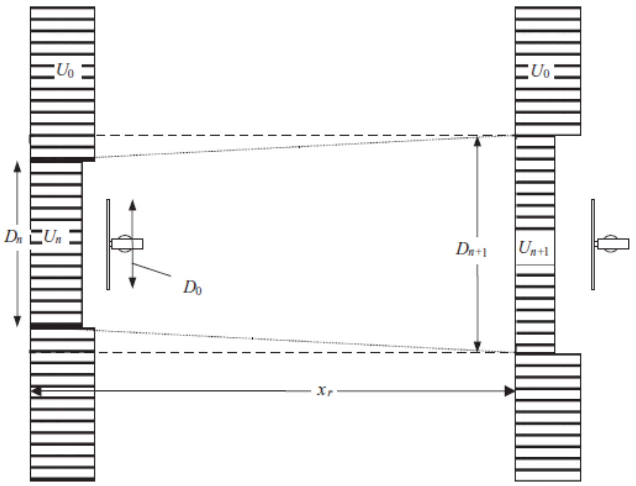
$$c_{n+1} = 1 - \left(\frac{A_n}{A_{n+1}} (1 - c_n) + \frac{1}{2} \cdot \frac{A_R}{A_{n+1}} C_T c_n \right)$$

$$A_n = A_n(s) = A_n(n s_r), \qquad s_r = x_r / D_0$$

Simulation of Wind Farms

Momentum model:

$n \to \infty$



$$c_w(1-c_w) = \frac{A_n}{A_{n+1}} c_w(1-c_w) + \frac{1}{2}\frac{A_R}{A_{n+1}} c_w^2 C_T \quad \Rightarrow$$

$$A_{n+1} - A_n = \frac{1}{2} A_R \frac{c_w}{1-c_w} C_T$$

where $\frac{1}{2} A_R \frac{c_w}{1-c_w} C_T$ **is a constant** $\Rightarrow$

The wake cross-sectional area is expanding linearly with x

Frandsen model:

$$A_{n+1} - A_n = \frac{\pi}{4} D_0^2[\beta + \alpha s_r(n+1)] - \frac{\pi}{4} D_0^2(\beta + \alpha s_r n) = A_R \alpha s_r$$

where $\alpha = \frac{1}{2}\frac{C_T}{s_r}\frac{c_w}{1-c_w}$

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Simulation of Wind Farms

Validation of model:

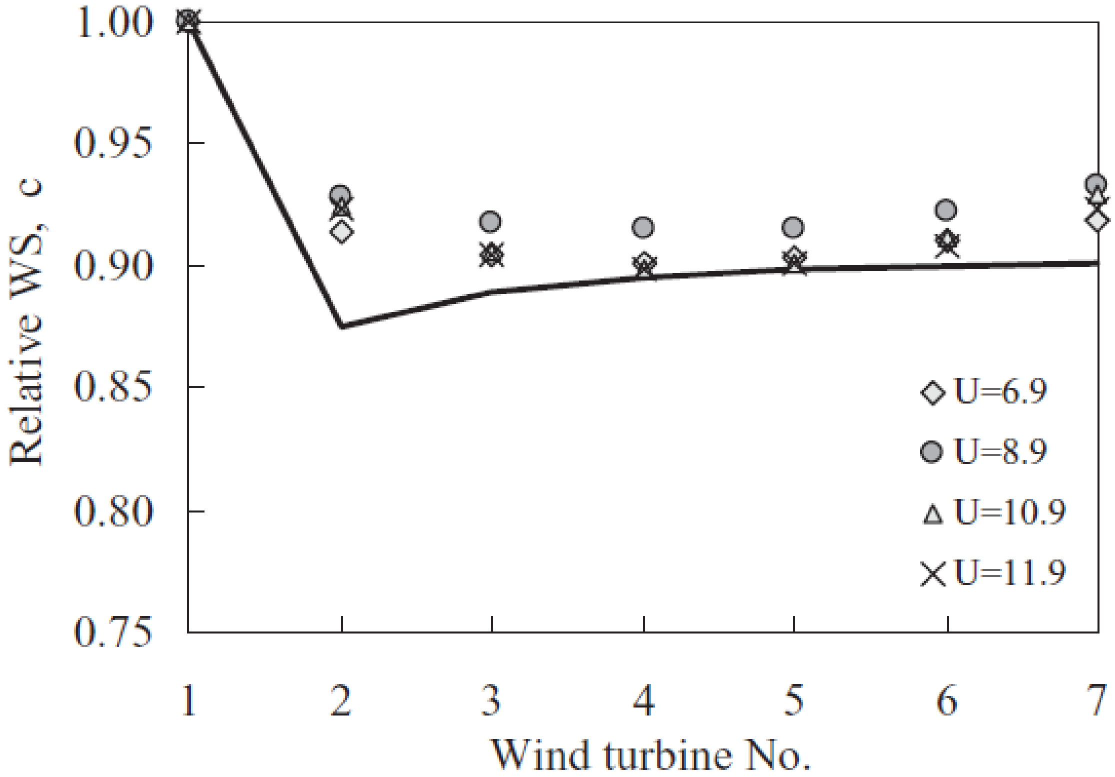
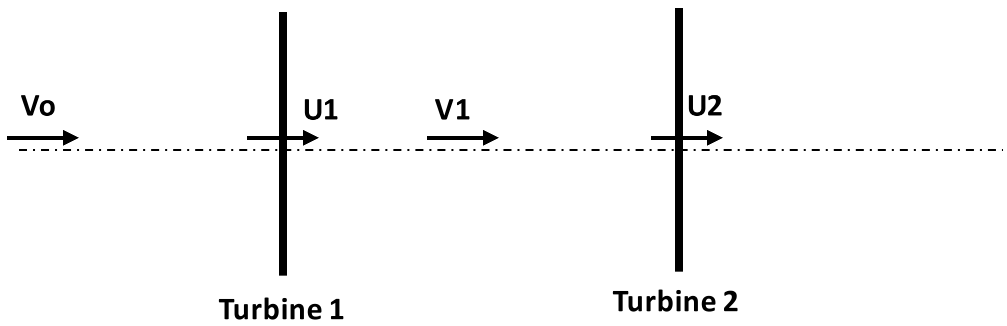


Figure 6. Measurement of wind speed ratio c_i at Nørrekær Enge II. Wind speeds are derived from power signals. Average is taken over six rows, each with seven units. $s_r \approx 7$. The wind farm consists of 42,300 kW units

Simulation of Wind Farms

Assignment:

Determine the maximum performance of two turbines in tandem:



Assume that U1 =½(Vo + V1), and introduce a1 = 1 – U1/Vo and a2 = 1 – U2/V1.

1. **Derive an expression for the power coefficient Cp = Cp(a1,a2)**
2. **What is the optimum Cp and operating conditions for the rotor system**
3. **Can the model be extended to an arbitrary number of wind turbines**

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