Turbulence and Spectra for wind field simulation

Jakob Mann

Risø DTU, Roskilde, Denmark

August, 2011 – PhD Course: LES in Hydrodynamics and Offshore Wind Energy
Outline of Part I: Turbulence structure

1. Turbulence for wind turbine load modeling
   - Basic properties and assumptions
   - Validity of assumptions
Outline of Part I: Turbulence structure

1. Turbulence for wind turbine load modeling
   - Basic properties and assumptions
   - Validity of assumptions

2. Three dimensional turbulence structure
   - Description by a spectral tensor
   - Rapid Distortion Theory and the Mann model
   - Testing the model
Outline of Part I: Turbulence structure

1. Turbulence for wind turbine load modeling
   - Basic properties and assumptions
   - Validity of assumptions

2. Three dimensional turbulence structure
   - Description by a spectral tensor
   - Rapid Distortion Theory and the Mann model
   - Testing the model

3. Parameter variations
   - Høvsøre site
   - Diabatic observations
   - Extensions to complex terrain
The purpose is to describe spatial and temporal fluctuations with relevance for wind turbine load calculations.
Stationarity and homogeneity

A stochastic process $X(t)$ is completely described in term of all joint probabilities

$$p(x_1, t_1; x_2, t_2; \ldots; x_n, t_n) \quad \text{for all } n$$

or equivalently (under some conditions) all moments

$$\langle X(t_1)X(t_2)\ldots X(t_n) \rangle$$

It is stationary if

$$p(x_1, t_1; x_2, t_2; \ldots; x_n, t_n) = p(x_1, t_1 + t; x_2, t_2 + t; \ldots; x_n, t_n + t)$$

or

$$\langle X(t_1)X(t_2)\ldots X(t_n) \rangle = \langle X(t_1 + t)X(t_2 + t)\ldots X(t_n + t) \rangle \quad \forall t$$
Atmospheric time series and stationarity

Stationary?

Non-stationary?
Homogeneity

A stochastic field $X(x)$ is *homogeneous* if

$$
\langle X(x_1)X(x_2)\ldots X(x_n) \rangle = \langle X(x_1 + r)X(x_2 + r)\ldots X(x_n + r) \rangle
$$

i.e. “stationary in space.”
Homogeneity: Example

Wind direction at three heights (°)

35 m
290
280
270
260

29.3 m
290
280
270
260

19.7 m
290
280
270
260
A Gaussian variable

\[ p(v) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{1}{2} \frac{v^2}{\sigma^2} \right) \]

The zero mean gaussian variable \( v \) is simulated by the Box-Müller method.
An $n$-dimensional Gaussian variable

\[ p(\mathbf{v}) = \frac{1}{\sqrt{(2\pi)^n \det(\mathbf{R})}} \exp \left( -\frac{1}{2} \mathbf{v} \cdot \mathbf{Qv} \right) \]

\[ \mathbf{v} = \{ v_1, v_2, ..., v_n \}, \quad \langle \mathbf{v} \rangle = 0 \]

\[ \mathbf{R} = \sigma^2 \begin{pmatrix}
1 & \rho_1 & \rho_2 & \cdots \\
\rho_1 & 1 & \rho_1 & \cdots \\
\rho_2 & \rho_1 & 1 & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix} \]

\[ \mathbf{Q} = \mathbf{R}^{-1} \]

is simulated by Fourier techniques, essentially treating the eigenvectors of $\mathbf{R}$ independently.
Coordinate systems

\[ U: \text{Mean wind speed.} \]
\[ u: \textbf{Longitudinal} \text{ fluctuations.} \]
\[ v: \textbf{Transversal} \text{ fluctuations.} \]
\[ w: \textbf{Vertical} \text{ fluctuations.} \]
\[ x_i: \text{Space coordinates.} \]
The logarithmic velocity profile

\( \rho \)  Air density (kg/m\(^3\))

\( \tau \)  Frictional force on a unit area of the surface (kg m\(^{-1}\) s\(^{-2}\))

\( z \)  Distance from the surface.

The only combination giving the dimension of velocity gradient is

\[
\frac{dU}{dz} = \text{const} \sqrt{\frac{\tau}{\rho z^2}} = \frac{u_*}{\kappa z},
\]

(1)

where the friction velocity \( u_* \) is defined by

\[
\tau = \rho u_*^2
\]

and \( \kappa \approx 0.4 \) is the dimensionless von Kármán constant. Other turbulence quantities relate to \( u_*^2 = -\langle uw \rangle \):

\[
\sigma_u \approx 2.4 u_* \quad \sigma_v \approx 1.9 u_* \quad \sigma_w \approx 1.25 u_*
\]
The logarithmic velocity profile

Solving (1) we get

\[ U(z) = \frac{u^*}{\kappa} \log \left( \frac{z}{z_0} \right) \]

where the *roughness length* \( z_0 \) is an integration constant.
The influence of stability

Stable and unstable flows
A much used parameter in fluid dynamics

\[ Ri = \frac{g}{T} \frac{d\Theta/dz}{(dU/dz)^2}, \]

where \( \Theta \) is the mean potential temperature.

In surface layer meteorology the parameter \( z/L \) where

\[ L = \frac{T}{g\kappa} \frac{u_3^3}{\langle w\theta \rangle} \] (2)

is the Monin-Obukhov length, is widely used. Departures from neutral profiles can, at least close to the ground, be written as empirical functions of \( z/L \).

The finite height of the boundary layer will limit the linear growth of the eddy diffusivity making the profile look stable. Analysis of profiles from Høvsøre up to 160 m confirm this.
Are homogeneity, stationarity, gaussianity and neutral atmospheric stratification valid?

Lack of stationarity at an off-shore location. Wind speed constant $\approx 15 \text{ m/s}$ in $\Delta \theta = 75^\circ$ in 30 s.
Stationarity?

Off-shore frontal passage: $\Delta u = 19 \text{m/s}$ in $\Delta t_u = 60 \text{s}$. 
Independence of stability at strong winds?

Spectra of $w$ from the Great Belt Coherence Experiment. Mean wind speeds are between 16 and 20 m/s and directions are in a narrow interval around the South. Dashed spectra have slightly unstable stratification, gray have stable, and the thin have neutral.
Gaussianity

Pdf of instantaneous velocity differences between two heights within the rotor plane. Smooth curves are expectations from standards.
Suppose the velocity field is homogeneous. Taylor’s frozen turbulence hypothesis

\[ \tilde{u}(x, y, z, t) = \tilde{u}(x - Ut, y, z, 0) \]

Covariance tensor

\[ R_{ij}(r) = \langle u_i(x)u_j(x + r) \rangle \]

For \( r = 0 \) the diagonal elements of \( R_{ij} \) are \( \sigma_u^2, \sigma_v^2, \sigma_w^2 \). For \( |r| \to \infty \) \( R_{ij} \to 0 \).
Technical preliminaries of the Mann model

Spectral tensor

\[ \Phi_{ij}(k) = \frac{1}{(2\pi)^3} \int R_{ij}(r) \exp(-i k \cdot r) dr \]

One-dimensional spectrum

\[ F_i(k_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{ij}(x, 0, 0) e^{-i k_1 x} dx \]

Cross-spectrum

\[ \chi_{ij}(k_1, \Delta y, \Delta z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{ij}(x, \Delta y, \Delta z) e^{-i k_1 x} dx \]

Coherence

\[ \text{coh}_{ij}(k_1, \Delta y, \Delta x) = \frac{|\chi_{ij}(k_1, \Delta y, \Delta z)|^2}{F_i(k_1)F_j(k_1)} \]
Symmetries

From *symmetries* it is possible to determine if some cross-spectra are real, purely imaginary or zero. The symmetry group of a turbulent field is the set of all orthonormal transformations $T$ for which the second order statistics of $u_i(x)$ is the same as $T_{ij}u_j(Tx)$. Consequences for the correlation tensor:

$$R_{ij}(r) \equiv \langle u_i(x)u_j(x+r) \rangle$$

$$= \langle T_{ik}u_k(Tx)T_{jl}u_l(Tx+Tr) \rangle$$

$$= T_{ik}\langle u_k(Tx)u_l(Tx+Tr) \rangle T_{jl}$$

$$= T_{ik}R_{kl}(Tr)T_{jl}$$
Symmetries

Example

\[ T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow R_{23}(x, 0, z) = -R_{23}(x, 0, z) \Rightarrow R_{23}(x, 0, z) = 0 \]
Properties of the spectral tensor

\[
\Phi_{ij}(k) \equiv \frac{1}{(2\pi)^3} \int R_{ij}(r) \exp(-ik \cdot r) \, dr
\]

\[
R_{ij}(r) \equiv \int \Phi_{ij}(k) \exp(ik \cdot r) \, dk
\]

\[
R_{ij}(r) = R_{ji}(-r) \Rightarrow \Phi_{ij}(k) = \Phi_{ji}^*(k),
\]

where * denotes complex conjugation.

\[
R_{ij}(r) = T_{ik} R_{kl}(Tr) T_{jl} \Leftrightarrow \Phi_{ij}(k) = T_{ik} \Phi_{kl}(Tk) T_{jl}
\]

where * also denotes the adjoint, i.e. in the case of a real matrix the transpose.
\[ \nabla \cdot \mathbf{u}(\mathbf{x}) = \frac{\partial u_i}{\partial x_i} = 0 \]

\[ \Leftrightarrow k_i u_i(\mathbf{k}) = 0. \]

\[ \frac{\partial}{\partial r_j} R_{ij}(\mathbf{r}) = \frac{\partial}{\partial r_j} \langle u_i(\mathbf{x}) u_j(\mathbf{x} + \mathbf{r}) \rangle = \left\langle u_i(\mathbf{x}) \frac{\partial}{\partial r_j} u_j(\mathbf{x} + \mathbf{r}) \right\rangle = 0 \]

\[ \Leftrightarrow k_j \Phi_{ij}(\mathbf{k}) = 0. \] This property implies zero direct backscatter of acoustical beams under neutral stratification.
Isotropy

All orthonormal transformations $T$ leaves the statistics of the velocity field $u$ unchanged. For a moment think of a scalar field $\theta(x)$ and consider the second order statistics $R(r) = \langle \theta(x)\theta(x + r) \rangle$. Isotropy here implies that $R(r)$ can only depend on $r = |r|$. An isotropic, symmetric second order tensor as the velocity correlation function can only depend on

$$\delta_{ij} \quad k_i k_j$$

We are left with

$$\Phi_{ij}(k) = f_1(k)\delta_{ij} + f_2(k)k_i k_j$$
\[ \Phi_{ij}(k) = \frac{E(k)}{4\pi k^2} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \]

where \( E(k) \) is half the variance of the wind velocity fluctuations whose magnitude of the wave vector is in the range \((k, k + dk)\).
Kolmogorov and von Kármán

Kolmogorov (1941) (dimensional analysis) for large $k$ but still smaller than the wave length corresponding to the viscous scale. 

$$E(k) = \alpha \varepsilon^{2/3} k^{5/3}$$

The value of “the spectral Kolmogorov constant” $\alpha$ is $\approx 1.7$.

Implies

$$F_{22}(k_1) = F_{33}(k_1) = \frac{4}{3} F_{11}(k_1)$$

as in the IEC standard.

Isotropy implies $\sigma_u^2 = \sigma_v^2 = \sigma_w^2$. Also $\chi_{13} = 0$.

Von Kármán proposed

$$E(k) = \alpha \varepsilon^{2/3} L^{5/3} \frac{(Lk)^4}{(1 + (Lk)^2)^{17/6}}$$
Navier-Stokes equations and higher order moments

\[
a = \frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u
\]

From this it can be shown (Kolmogorov, 1941) that

\[
\langle \delta u ||(r)^3 \rangle = -\frac{4}{5} \varepsilon r
\]

which is in direct conflict with gaussianity.
Rapid distortion theory (RDT) was originally formulated to calculate turbulence in a wind tunnel contraction. It was later used to model the response of turbulence to shear.
The basic idea is to divide the flow into a mean and a fluctuating part. In the Navier-Stokes equations the interaction (or products) between fluctuating parts are ignored. This allows for a Fourier transform of the equations, resulting in linear differential equations with no coupling between wave-vectors.
The Mann model

Symmetry group of the Mann model

\[
\left\{ I, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, -I \right\}
\]
The linearization is unrealistic; stretched ‘eddy’ will break up (interaction between fluctuations). Equilibrium is postulated where eddies of size $\propto |k|^{-1}$ are stretched by the shear over a time proportional to their life time $\tau$. In the inertial subrange $\tau \propto k^{-2/3}$. We introduce a parameter $\Gamma$, such that the dimensionless life time, $\beta$, can be written as $\beta \equiv \frac{dU}{dz}\tau = \Gamma (kL)^{-\frac{2}{3}}$. A more general model of the dimensionless eddy life time $\beta$ outside the inertial subrange is established in Mann (1994).
Evolution of individual wave-vectors:

\[ \mathbf{k}_0 = (k_1, k_2, k_{30}) \quad \text{with} \quad k_{30} = k_3 + \beta k_1 \]
‘Initial condition’ $dZ^{iso}(k_0)$ has the statistics of the isotropic von Kármán tensor. The sheared tensor is then given by

$$dZ(k) = \begin{bmatrix} 1 & 0 & \zeta_1 \\ 0 & 1 & \zeta_2 \\ 0 & 0 & k_0^2/k^2 \end{bmatrix} dZ^{iso}(k_0)$$

where

$$\zeta_1 = C_1 - k_2 C_2/k_1 , \quad \zeta_2 = k_2 C_1/k_1 + C_2$$

with

$$C_1 = \beta k_1^2 (k_0^2 - 2k_30^2 + \beta k_1 k_30) / k^2(k_1^2 + k_2^2)$$

and

$$C_2 = \frac{k_2 k_0^2}{(k_1^2 + k_2^2)^{3/2}} \arctan \left[ \frac{\beta k_1 (k_1^2 + k_2^2)^{1/2}}{k_0^2 - k_30 k_1\beta} \right],$$
Compared to the isotropic tensor the extra parameter $\Gamma$ implies

- $\sigma_u^2 > \sigma_v^2 > \sigma_w^2$
- $\langle uw \rangle < 0$
- Length scale of $u$ much larger than $w$
Comparison with data: Great Belt Coherence Experiment
Comparison with data: Great Belt Coherence Experiment

Figure 1: Two minutes time series of the three components of velocity measured in the same point 70 m over the Great Belt. The connection between the components of the spectral tensor and the cross-spectra is

\[
\begin{align*}
S_{XX}(k) &= \frac{2}{\pi} \int \frac{S_{YY}(k) \cos(\theta) + S_{ZZ}(k) \sin(\theta)}{(k^2 + \theta^2)^{3/2}} \, d\theta, \\
S_{XY}(k) &= \frac{1}{\pi} \int \frac{S_{YY}(k) \sin(\theta) - S_{ZZ}(k) \cos(\theta)}{(k^2 + \theta^2)^{3/2}} \, d\theta,
\end{align*}
\]

When the two indices \(XX\) and \(YY\) are the same and \(YY\) becomes the one-point spectrum

\[
S_{YY}(k) = \frac{2}{\pi} \int \frac{S_{YY}(k) \cos(\theta)}{(k^2 + \theta^2)^{3/2}} \, d\theta.
\]

To distinguish between spectra as functions of wave number \(YY\) and frequency \(XX\) we use \(YY\) for the former and \(XX\) for the latter, i.e.

\[
S_{YY}(k) = \frac{2}{\pi} \int \frac{S_{YY}(k) \cos(\theta)}{(k^2 + \theta^2)^{3/2}} \, d\theta.
\]

The coherence is defined as

\[
C_{XX}(\theta) = \frac{S_{XX}(k) \cos(\theta) + S_{YY}(k) \sin(\theta)}{(S_{XX}(k)^2 + S_{YY}(k)^2)^{1/2}},
\]

Davenport (1961) found that the coherence of the components vertically separated by a distance \(YY\) could be well approximated by

\[
C_{XX}(\theta) = \frac{1}{\theta^2 + \theta^2},
\]

where \(YY\) is the mean wind speed at an average height and \(XX\) a constant of the order of 8. It can be shown both experimentally and theoretically, that the coherence does not go to zero for \(XX\) for \(YY\) and that the coherence is smaller for large distances than indicated by the Davenport model (Kristensen and Jensen 1979, Mann 1994). In spite of these shortcomings \(XX\) is widely used and has been extended to coherences of the other wind components (with other values of \(XX\)) and to horizontal separations (Panofsky and Dutton 1984, Simiu and Scanlan 1996, Dyrbye and Hansen 1997).
Spectra and one-point cross-spectra

![Graph showing spectra and one-point cross-spectra]

The spectra shown in figure 2 calculated from the entire two-hour record also imply that the turbulence is certainly not isotropic. Note also that the wavelength containing the most variance (or more or less equivalently, the integral scale) is largest for $u$ and smallest for $w$. The $u$- and the $w$-signals are anti-correlated, which can be difficult to judge from figure 1.

One 2 hour run.
Turbulence for wind turbine load modeling
Three dimensional turbulence structure
Parameter variations

Description by a spectral tensor
Rapid Distortion Theory and the Mann model
Testing the model

Figure 3: A two minute fraction of measured wind speeds at positions separated in the horizontal direction perpendicular to the wind. The instruments used for the two lowest plots are separated by $\Delta y = 15$ m, the upper and the middle by 32.5 m, and the upper and the lower by 47.5 m.

The component spectra also obey Kolmogorov's $5/3$-law for high frequencies or wavenumbers and the associated law that the ratio between the $w$- or $v$-spectrum to the $u$-spectrum should be $4/3$ (Kolmogorov 1941, Landau and Lifshitz 1987). Integrating the real part of the cross-spectrum (the thin solid line in figure 2) from $-\infty$ to $\infty$ we get by definition $-u_2^\ast$.

The often used model spectra of Simiu and Scanlan (1996) have the same functional shapes as Kaimal, Wyngaard, Izumi and Coté's (1972) but the numerical constants are different:

\[
\begin{align*}
    f_{S_U}(f) & = 100 \left(1 + 50 n\right)^{5/3}, \\
    f_{S_V}(f) & = 7 \left(1 + 9.5 n\right)^{5/3}, \\
    f_{S_W}(f) & = 1.68 \left(1 + 10 n\right)^{5/3},
\end{align*}
\]

(14) and

\[
\begin{align*}
    f_{S_W}(f) & = 1.68 \left(1 + 10 n\right)^{5/3},
\end{align*}
\]

(16)

where $n = f z / U$. These spectra obey closely the $5/3$- and $4/3$-laws mentioned above and also fit the observations at the Great Belt quite well.

Figure 3 shows simultaneously recorded wind histories separated in the horizontal direction by various distances. The coherences calculated from these time series are shown in figure 4. At $\Delta y = 47.5$ m is less than implied by the Davenport model.

Note, that these spectra are two-sided, i.e. we get the variance by integrating from $-\infty$ to $\infty$. 

8
Normalized two-point cross-spectra: coherences

![Normalized two-point cross-spectra: coherences](image-url)
Figure 3: Average $u$, $v$, $w$, and cross-spectra of all the neutral runs present in figure 2. The ragged curves are measurements while the smooth are the model spectra. The model has zero imaginary part of the cross-spectrum (quadrature spectrum).

which are identical to (3.14) — (3.17) of (Mann 1994b).

Compared to the isotropic tensor model we have an extra parameter $\Gamma$ which determines the anisotropy of the tensor. Integrating the spectral tensor over the entire wave vector space we obtain the (co-)variances as a function of $\Gamma$. It can be shown that when anisotropy in this way is introduced, $\sigma^2_u \sigma^2_v \sigma^2_w \text{ and } \sigma_{uw} \text{ which is confirmed by observations.}$ The larger $\Gamma$ the larger the difference between the variances.

Four experimental tests of the model has been carried out. Two are atmospheric, one over water (Mann et al. 1991, Mann 1994b) and one over flat terrain (Courtney 1988), giving the parameters $L_z 0 \text{ and } \Gamma 2 6$, respectively. The third is based on data from the Martin Jensen boundary layer wind tunnel (Smitt and Brinch 1992) at Danish Maritime Institute (DMI) giving $L_z 0 \text{ and } \Gamma 2 2$, implying that the turbulence is closer to being isotropic compared to the atmospheric turbulence, (Mann 1994a). The fourth test took place in DMI’s wind tunnel used for bridge section model tests. In the setup used for these tests there is almost no shear and $\Gamma 0 76 \text{ (and } L 0 39 \text{ m)}$ and, consequently, the turbulence is very close to being isotropic.

To simulate atmospheric turbulence we shall not rely solely on the two atmospheric experiments mentioned above. In Mann (1998) the spectral tensor model is compared to commonly used spectra and coherences. Below we shall take a closer look on comparisons with off-shore data.

2.3 Fitting spectra to observations

Uncertainties on estimated spectra have several sources. These are either variations in atmospheric stability, which persists even at high wind speeds ($16 \text{ m/s}$) over water, or statistical variations. First, the measured neutral spectra are fitted to the spectral tensor model. Based on this fit the coherences are predicted and compared to the measurements.

In order to conduct simultaneous measurements of spectra and coherence over the sea a 70 m high mast was erected 40 m from an existing mast on the easterly spit of Sprogø, an island in the midst of the Great Belt separating the two Danish islands Funen and Zealand. A 15 m long horizontal boom was mounted symmetrically at the top of the new mast so that the whole construction has the form of a letter “T”. A Kaijo-Denki DAT-300 omni-directional sonic...
Predicted coherences

Figure 4: The dots are measured coherences from the same set of data as used for figure 3 for various horizontal separations \( \Delta y \) and for all three velocity components. The lines are the coherences predicted by the model.

More details about the experiment may be found in Mann et al. (1991).

Figure 2 shows the result of an analysis of 14 two-hour time series from the Great Belt. The series have mean speeds \( U \) between 16 and 20 m/s and the mean directions are within a narrow range around south where there is an uninterrupted fetch over water for at least 20 km.

The observed variations in the spectra cannot be explained by statistical variations alone. Most noticeably, there are spectra with only 10% of the spectral density of the others. This variation is due to the stability of the atmosphere not being neutral. The case with suppressed turbulence is slightly stable and has \( U = 16 \) m/s. Unstable stratification also alters the spectrum.

Though none of the spectra from the Great Belt are obtained under very unstable situations, an analysis of unstable, high-wind spectra on the west coast of Norway indicates that the spectra are mainly enhanced (by more than 100%) at very low frequencies \( (f \approx 0.02 \text{ Hz}) \).

The measured spectra shown in figure 3 are an average of 16 neutral two-hour runs with wind speeds between 16 and 20 m/s. The smooth curves are model spectra derived from the spectral tensor model with the parameters \( \Gamma = 3, L = 61 \text{ m}, \) and \( \alpha \varepsilon \approx 2 \), which are taken from Mann (1994b), who used fewer two-hour runs but slightly higher wind speeds. These parameters are in turn used to predict the coherences as shown in figure 4. As seen from this figure the predictions agree well with the measurements except for the \( w \) coherence, especially at the largest separation.

3 TURBULENCE IN COMPLEX TERRAIN

As indicated in section 2 a lot is known about turbulence over flat terrain. The purpose of this part of the paper is to sketch the development of models that take into account the influence of roughness changes and gentle hills on the turbulence statistics. The turbulence model used input from a mean flow model called LINCOM:

3.1 The linear flow model LINCOM

Within the concept of linearized flow models originally introduced by Jackson and Hunt (1975), Troen and de Baas (1986) developed a relatively simple model for neutrally stable flow over hilly terrain. The model was later named LINCOM, an acronym for LINearized COMputation. The base of this version of the code, giving the influence of the topography on the flow of a...
Site and measurements at Høvsøre

- 20 Hz Sonics at 10, 20, 40, 60, 80, 100 and 160 m
- 10-min time series collected for ~1 year
### Results: diabatic observations

<table>
<thead>
<tr>
<th>Obukhov length interval [m]</th>
<th>Atmospheric stability class</th>
<th>$L$ [m]</th>
<th>$u_{*o}$ [m s$^{-1}$]</th>
<th>$z_o$ [m]</th>
<th>$z_i$ [m]</th>
<th>No. of 10-min data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-100 \leq L \leq -50$</td>
<td>Very unstable (vu)</td>
<td>$-74$</td>
<td>$0.35$</td>
<td>$0.013$</td>
<td>$600$</td>
<td>$397$</td>
</tr>
<tr>
<td>$-200 \leq L \leq -100$</td>
<td>Unstable (u)</td>
<td>$-142$</td>
<td>$0.41$</td>
<td>$0.012$</td>
<td>$600$</td>
<td>$459$</td>
</tr>
<tr>
<td>$-500 \leq L \leq -200$</td>
<td>Near unstable (nu)</td>
<td>$-314$</td>
<td>$0.40$</td>
<td>$0.012$</td>
<td>$550$</td>
<td>$292$</td>
</tr>
<tr>
<td>$</td>
<td>L</td>
<td>\geq 500$</td>
<td>Neutral (n)</td>
<td>$5336$</td>
<td>$0.39$</td>
<td>$0.013$</td>
</tr>
<tr>
<td>$200 \leq L \leq 500$</td>
<td>Near stable (ns)</td>
<td>$318$</td>
<td>$0.36$</td>
<td>$0.012$</td>
<td>$451$</td>
<td>$439$</td>
</tr>
<tr>
<td>$50 \leq L \leq 200$</td>
<td>Stable (s)</td>
<td>$104$</td>
<td>$0.26$</td>
<td>$0.008$</td>
<td>$257$</td>
<td>$1144$</td>
</tr>
<tr>
<td>$10 \leq L \leq 50$</td>
<td>Very stable (vs)</td>
<td>$28$</td>
<td>$0.16$</td>
<td>$0.002$</td>
<td>$135$</td>
<td>$704$</td>
</tr>
</tbody>
</table>

- $z_i = C \frac{u_{*o}}{|f_c|}$ for neutral and stable conditions
- $z_o$ from

$$U = \frac{u_{*o}}{\kappa} \left[ \ln \left( \frac{Z}{z_o} \right) - \psi_m \right]$$  \hspace{1cm} (3)
Results: Mean wind profiles for various atmospheric stabilities
Very unstable spectra

For wind turbine load modeling:
Three dimensional turbulence structure
Parameter variations

Høvsøre site
Diabatic observations
Extensions to complex terrain

Turbulence and Spectra for wind field simulation

Jakob Mann
Unstable spectra

![Graphs showing unstable spectra.](image-url)
Near unstable spectra
Neutral spectra

Turbulence for wind turbine load modeling
Three dimensional turbulence structure
Parameter variations

Høvsøre site
Ombolic observations
Extensions to complex terrain

Jakob Mann
Turbulence and Spectra for wind field simulation
Near stable spectra

![Graphs showing near stable spectra](image_url)
Stable spectra
Very stable spectra
Mann (1994) model parameters
Turbulence in complex terrain

Two complications compared to flat terrain:

- varying roughness
- orography

These are incorporated in WAsP Engineering, but only for moderately complex terrain: The basic limitation:

*If there are extended areas within a radius of 3 to 4 km from the site of interest with slopes of more than 20° to 25°, then turbulence may be much larger than calculated. In this situation measurements at the site may be required.*
Really complex terrain
A table mountain in Spain

2 masts and 54 turbines.
Stretched a factor of 2
Mast 1: Ratio of between wind speed at lower tip and hub height

- $5 < U_{42.3} < 12$
- $12 < U_{42.3}$
Mast 2: Ratio of wind speed at lower tip and hub height

- $5 < U_{42.3} < 12$
- $12 < U_{42.3}$
Mast 2: Diff. in wind direction at hub and lower tip height
Mast measurements at $z = 19$, 31, and 42 m (hub). Direction diff. up to 90°.
Outline of Part II: Turbulence simulation

4 Recapitulation of Part I
Outline of Part II: Turbulence simulation

4 Recapitulation of Part I

5 Simulation methods
   - The Sandia method
   - The spectral tensor method
Outline of Part II: Turbulence simulation

4 Recapitulation of Part I

5 Simulation methods
   - The Sandia method
   - The spectral tensor method

6 Constrained simulation
   - Mathematical basis
   - Examples
   - Discussion and overall conclusion
What aspects of the wind are important for a turbine?

- $D \sim 50–150$ m
What aspects of the wind are important for a turbine?

- $D \sim 50$–$150$ m
- $z_{\text{hub}} \sim 50$–$150$ m
What aspects of the wind are important for a turbine?

- $D \sim 50–150$ m
- $z_{\text{hub}} \sim 50–150$ m
- $0.05 \lesssim f \lesssim 1$ Hz and even higher.
What aspects of the wind are important for a turbine?

- $D \sim 50–150 \text{ m}$
- $z_{\text{hub}} \sim 50–150 \text{ m}$
- $0.05 \lesssim f \lesssim 1 \text{ Hz}$ and even higher.
- time history in one point not enough
What aspects of the wind are important for a turbine?

- $D \sim 50–150$ m
- $z_{hub} \sim 50–150$ m
- $0.05 \lesssim f \lesssim 1$ Hz and even higher.
- Time history in one point not enough
- $u$, but also $v$ and $w$. 
Wind statistics over flat terrain

- Gaussianity often OK, despite $\langle \delta v_{\parallel}(r)^3 \rangle = -\frac{4}{5} \varepsilon r$. 
Wind statistics over flat terrain

- Gaussianity often OK, despite \( \langle \delta v_\parallel (r)^3 \rangle = -\frac{4}{5} \varepsilon r \).
- Stationarity often OK
Wind statistics over flat terrain

- Gaussianity often OK, despite $\langle \delta v_\parallel (r)^3 \rangle = -\frac{4}{5} \varepsilon r$.
- Stationarity often OK
- Homogeneity often OK, at least in the horizontal.
  \[
dU/dz = u_*/\kappa z
\]
Wind statistics over flat terrain

- Gaussianity often OK, despite $\langle \delta v_{\|}(r)^3 \rangle = -\frac{4}{5} \varepsilon r$.
- Stationarity often OK
- Homogeneity often OK, at least in the horizontal.
  $dU/dz = u_*/\kappa z$
- Taylor’s hypothesis OK for most practical purposes
Gauss OK $\Rightarrow$ 2. order statistics is everything

Taylor’s frozen turbulence hypothesis

$$\tilde{u}(x, y, z, t) = \tilde{u}(x - Ut, y, z, 0)$$

Need to know: covariance tensor $R_{ij}(r) = \langle u_i(x)u_j(x + r) \rangle$, the spectral tensor $\Phi_{ij}(k)$

or

One-dimensional spectrum $F_i(k_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{ij}(x, 0, 0)e^{-ik_1x}dx$, and cross-spectra $\chi_{ij}(k_1, \Delta y, \Delta z)$ or the coherence $\text{coh}_{ij}(k_1, \Delta y, \Delta x)$
Sandia/Veers simulation method

- Uses spectra and coherences to simulate a mesh of correlated time series
Sandia/Veers simulation method

- Uses spectra and coherences to simulate a mesh of correlated time series
- Choleski decomposition
Recapitulation of Part I
Simulation methods
Constrained simulation

Sandia/Veers simulation method

- Uses spectra and coherences to simulate a mesh of correlated time series
- Choleski decomposition
- Homogeneity not necessary, but often assumed.
Sandia/Veers simulation method

- Uses spectra and coherences to simulate a mesh of correlated time series
- Choleski decomposition
- Homogeneity not necessary, but often assumed.
- Phases ignored, incompressibility ignored.
The Mann model

The spectral tensor $\Phi_{ij}(k)$ is modelled based on

- Incompressibility $k_i \Phi_{ij}(k) = 0$
The Mann model

The spectral tensor $\Phi_{ij}(k)$ is modelled based on

- Incompressibility $k_i \Phi_{ij}(k) = 0$
- Linearized Navier-Stokes equations close with “eddy life time” considerations (Mann, 1994)
The Mann model

The spectral tensor $\Phi_{ij}(k)$ is modelled based on

- Incompressibility $k_i \Phi_{ij}(k) = 0$
- Linearized Navier-Stokes equations close with “eddy life time” considerations (Mann, 1994)
- Small scale isotropy $\Phi_{ij}(k) = \frac{E(k)}{4\pi k^4} (k^2 \delta_{ij} - k_i k_j)$, $E(k) = \alpha \varepsilon^{2/3} k^{-5/3}$ for $k \rightarrow \infty$. 
The Mann model

The spectral tensor $\Phi_{ij}(k)$ is modelled based on

- Incompressibility $k_i \Phi_{ij}(k) = 0$
- Linearized Navier-Stokes equations close with “eddy life time” considerations (Mann, 1994)
- Small scale isotropy $\Phi_{ij}(k) = \frac{E(k)}{4\pi k^4} (k^2 \delta_{ij} - k_i k_j)$, $E(k) = \alpha \varepsilon^{2/3} k^{-5/3}$ for $k \to \infty$.
- Large scale anisotropy, $\sigma_u > \sigma_v > \sigma_w$, $\langle uv \rangle < 0$, $L_u > L_v > L_w$
The Mann model

The spectral tensor $\Phi_{ij}(k)$ is modelled based on

- Incompressibility $k_i \Phi_{ij}(k) = 0$
- Linearized Navier-Stokes equations close with “eddy life time” considerations (Mann, 1994)
- Small scale isotropy $\Phi_{ij}(k) = \frac{E(k)}{4\pi k^4} \left( k^2 \delta_{ij} - k_i k_j \right)$, $E(k) = \alpha \varepsilon^{2/3} k^{-5/3}$ for $k \to \infty$.
- Large scale anisotropy, $\sigma_u > \sigma_v > \sigma_w$, $\langle uw \rangle < 0$, $L_u > L_v > L_w$
- Notice: Only three parameters: $L$, $\alpha \varepsilon^{2/3}$, and $\Gamma$. 

Jakob Mann
Turbulence and Spectra for wind field simulation
Fourier simulation

\[ u(x) = \sum e^{i\mathbf{k} \cdot \mathbf{x}} u(k) \]

where

\[ \langle u_i(k) u_j^*(k') \rangle \propto \Phi_{ij}(k) \delta(k - k') \]
Recapitulation of Part I
Simulation methods
Constrained simulation

Wind simulation

The Sandia method
The spectral tensor method

Turbulence and Spectra for wind field simulation

Jakob Mann
Constrained simulation

Simulate $\mathbf{v} = \{v_1, v_2, \ldots\}$ provided that $\sum_{i=1}^{n} \varphi_i v_i = \varphi \cdot \mathbf{v} = f_c$.

**Examples:**

1. **Gust in the middle of the time series:** $\varphi_i = 1$ for $i = n/2$ and zero elsewhere. I.e. the condition is $v_{n/2} = f_c$. 

2. **Velocity jump over 5 time steps:** $\varphi_j = -1$ and $\varphi_{j+5} = 1$. The condition is $u_{j+5} - u_j = f_c$.

3. **Choose $\varphi$ to match time scales in the control system.**
Constrained simulation

Simulate $\mathbf{v} = \{v_1, v_2, \ldots\}$ provided that $\sum_{i=1}^{n} \varphi_i v_i = \varphi \cdot \mathbf{v} = f_c$.

**Examples:**

1. Gust in the middle of the time series: $\varphi_i = 1$ for $i = n/2$ and zero elsewhere. I.e. the condition is $v_{n/2} = f_c$.

2. Velocity jump over 5 time steps: $\varphi_j = -1$ and $\varphi_{j+5} = 1$. The condition is $u_{j+5} - u_j = f_c$. 
Constrained simulation

Simulate \( \mathbf{v} = \{v_1, v_2, \ldots \} \) provided that \( \sum_{i=1}^{n} \varphi_i v_i = \varphi \cdot \mathbf{v} = f_c \).

**Examples:**

1. **Gust in the middle of the time series:** \( \varphi_i = 1 \) for \( i = n/2 \) and zero elsewhere. I.e. the condition is \( v_{n/2} = f_c \).

2. **Velocity jump over 5 time steps:** \( \varphi_j = -1 \) and \( \varphi_{j+5} = 1 \). The condition is \( u_{j+5} - u_j = f_c \).

3. **Choose \( \varphi \) to match time scales in the control system.**
Most probable $v$:

Minimize \[
\frac{1}{2} v \cdot Qv
\]
subject to \[
\varphi \cdot v = f_c
\]

The solution is the “average gust” $v_{\text{ave}}$:

\[
v_{\text{ave}} = \frac{f_c R\varphi}{\varphi \cdot R\varphi}
\]

**Ex:** In the continuum limit suppose $\varphi(t) = \delta(t)$. Then

\[
v_{\text{ave}}(t) = \langle v(t) | v(0) = f_c \rangle = \frac{f_c}{\sigma^2} R(t)
\]
Constrained Gaussian simulation

To simulate \( \mathbf{v} \) under the constraint \( \varphi \cdot \mathbf{v} = f_c \):

1. Simulate stationary \( \mathbf{v}_s = \{v_{s1}, v_{s2}, \ldots\} \) with no constraints.
2. Calculate

\[
\mathbf{v} = \mathbf{v}_s + \frac{f_c - \varphi \cdot \mathbf{v}_s}{\varphi \cdot R\varphi} R\varphi
\]

\( \mathbf{v} \) fulfills the constraint \( \varphi \cdot \mathbf{v} = f_c \) and is generated at the right rate.
Example: “Velocity gust”

The stationary gaussian turbulence $v_s$ is simulated assuming a von Kármán spectrum. The correlation $R$ is derived from this spectrum.

$$\varphi(t) = \delta(t - 60s).$$

$f_c$ is 5 times the standard deviation.
Larger velocity gust

\[ \varphi(t) = \delta(t - 60\text{s}). \]

\( f_c \) is 20 times the standard deviation.
Example: “Velocity jump”

Jump of size $\Delta u(t) \equiv u(t + \Delta t/2) - u(t - \Delta t/2) = 5\sigma_u$ and with $\Delta t = 3$ s
Generalizations of constrained simulation

1. More than one component of the velocity.
Generalizations of constrained simulation

1. More than one component of the velocity.
2. More than one constraint
Generalizations of constrained simulation

1. More than one component of the velocity.
2. More than one constraint
3. Time series $\rightarrow$ spatial fields
Example: "Anisotropic 3D jump" (1:2)

*u*-turbulence simulation of velocity jump based on the spectral tensor by Mann
Example: "Strong velocity shear" (1:2)

Shear increases

- Tilt moment on rotor
- Dynamic loads on blades
Example: ”Strong velocity shear” (2:2)

The difference in $u$ at the two black point is 10 m/s.
Example: "Measured time series" (1:3)

Mast measurements at $z = 19, 31, \text{ and } 42 \text{ m (hub)}$. 
Example: "Measured time series" (2:3)

Result of the constrained simulation at $z = 19, 31, \text{ and } 42$ m,...
Example: "Measured time series" (3:3)

... and the entire $u$-field.
Discussion of simulation

- The nature of extreme wind gusts is probably radically different from Gaussian simulation
The nature of extreme wind gusts is probably radically different from Gaussian simulation.

Constrained simulation is also Gaussian, but probabilities of extreme events can be “controlled”.
Discussion of simulation

- The nature of extreme wind gusts is probably radically different from Gaussian simulation.
- Constrained simulation is also Gaussian, but probabilities of extreme events can be “controlled”.
- There is a myriad of other ways to simulate extreme events...
Recapitulation of Part I
Simulation methods
Constrained simulation
Mathematical basis
Examples
Discussion and overall conclusion

Discussion of simulation

- The nature of extreme wind gusts is probably radically different from Gaussian simulation.
- Constrained simulation is also Gaussian, but probabilities of extreme events can be “controlled”.
- There is a myriad of other ways to simulate extreme events...
- for example Rosales & Meneveau (2006, 2008)
Conclusion

- The spatial structure is important for loads on turbines
- Turbulence is quite well described over flat homogeneous terrain/sea under neutral atmospheric stratification.
- Turbulence is very dependent on stability and on the terrain
- There is no simple connection between the terrain and the turbulence.
- Constrained simulation can incorporate extreme events in stochastic fields
Exercise

1. Download and install the *IEC turbulence simulator*
   [http://www.wasp.dk/Products/weng/IECturbulenceSimulator.htm](http://www.wasp.dk/Products/weng/IECturbulenceSimulator.htm)

2. Simulate and display a turbulence field

3. Investigate the meaning of the many parameters