Topology Optimization

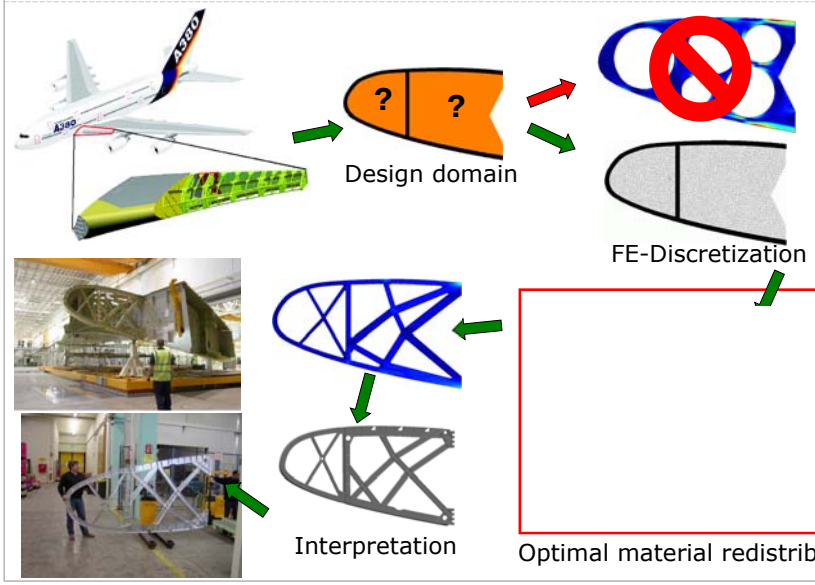
Ole Sigmund

Mechanical Engineering
Technical University of Denmark (DTU)

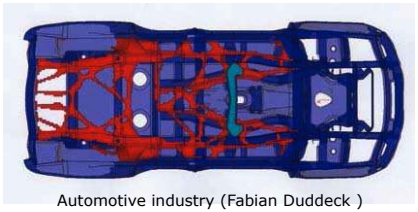
TopOpt-group: Niels Aage, Joe Alexandersen, Casper S. Andreasen, Erik Andreassen, Anders Clausen, Asger N. Christiansen, Rasmus E. Christiansen, Villads Egede Johansen, Boyan Lazarov, Morten Nobel-Jørgensen, Fengwen Wang, Mingdong Zhou

DTU Mekanik
Institut for Mekanisk Teknologi

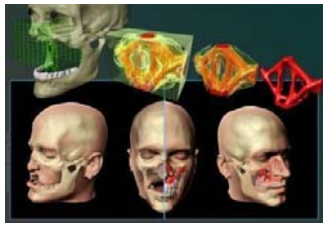
Topology Optimization in Aerospace

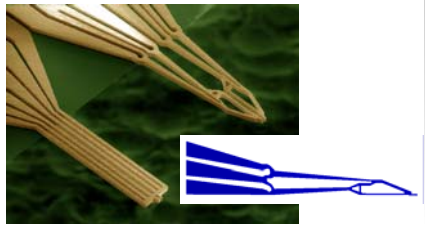


Topology Optimization Applications

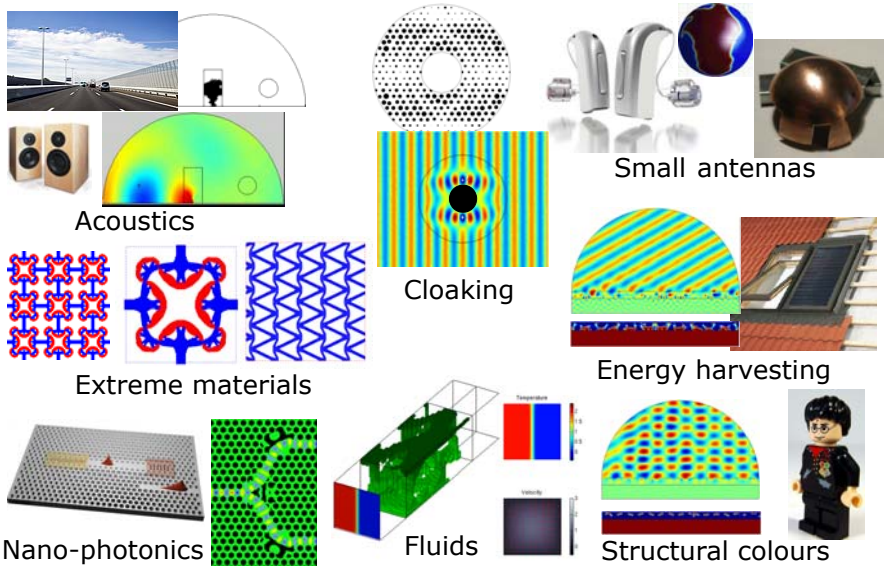

Automotive industry (Fabian Duddeck)


Wind turbines (SUZLON and FE-Design GmbH)


Reconstructive surgery (Paulino/Sinn-Hanlon)

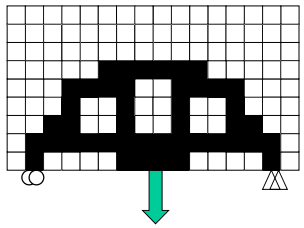

Micromachines (DTU Nanotech)

Topology Optimization Applications



Applications in Architecture / Design

Discrete topopt formulation



$$\begin{aligned}
\min_{\boldsymbol{\rho}} :\ & \Phi(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \\
s.t. :\ & \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \boldsymbol{\rho} \le V^* \\
: \ & g_i(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \le g_i^*, \quad i = 1, ..., M \\
: \ & \rho_e = \begin{cases} 0 \text{ (void)} \\ 1 \text{ (material)} \end{cases}, \quad e = 1, ..., N \\
: \ & \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F}
\end{aligned}$$

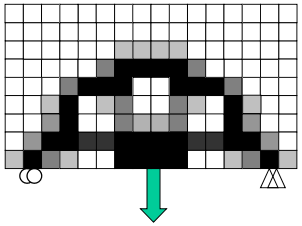
0/1 Integer problem

- Combinations: $\frac{N!}{(N-M)!\,M!}$

N=10, M=5 => 252
N=20, M=10 => 185.000
N=40, M=20 => $1.4 \cdot 10^9$
N=100, M=50 => 10^{29}

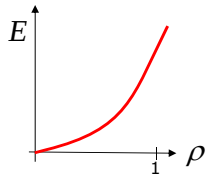
SIMP-approach

Bendsøe (1989), Zhou and Rozvany (1991), Mlejnek (1992)



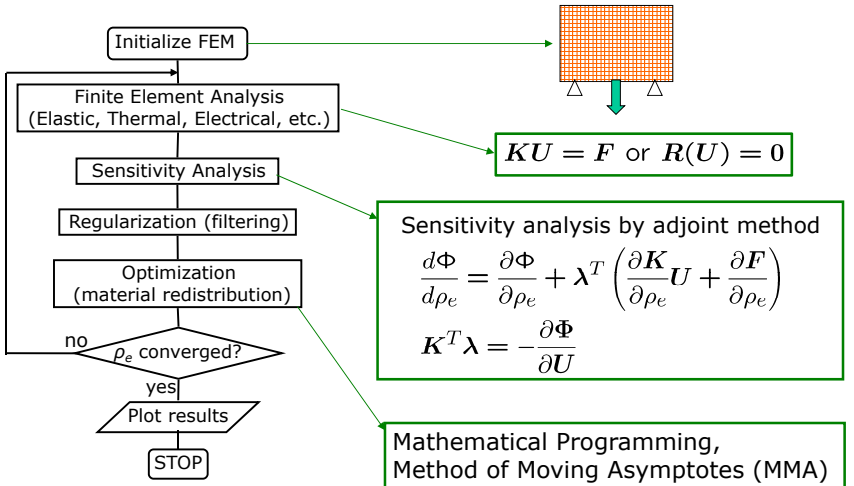
$$\begin{aligned}
\min_{\boldsymbol{\rho}} :\ & \Phi(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \\
s.t. :\ & \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \boldsymbol{\rho} \le V^* \\
: \ & g_i(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \le g_i^*, \quad i = 1, ..., M \\
: \ & 0 < \rho_{\min} \le \rho \le 1 \\
: \ & \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F}
\end{aligned}$$

Stiffness interpolation:



$$E(\rho_e) = \rho_e^p E_0$$
$$p > 1$$

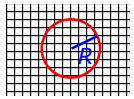
The Topology Optimization Process



Regularization by low-pass filtering

Neighborhood:

$$N_e = \{i \mid ||\mathbf{x}_i - \mathbf{x}_e|| \leq R\}$$



Checkerboards

Sensitivity filtering (Sigmund 1997, Sigmund&Maute 2012)

$$\frac{\widetilde{\partial \Phi}}{\partial \rho_e} = \frac{\sum_{i \in N_e} H(\mathbf{x}_i)\rho_i \frac{\partial \Phi}{\partial \rho_i}}{\rho_e \sum_{i \in N_e} H(\mathbf{x}_i)}$$

Density filtering (Bruns/Bourdin 2001)

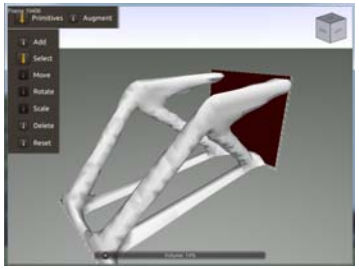
$$E_e(\boldsymbol{\rho}) = \tilde{\rho}_e^p E_0, \quad \tilde{\rho}_e = \frac{\sum_{i \in N_e} H(\mathbf{x}_i)\rho_i}{\sum_{i \in N_e} H(\mathbf{x}_i)}$$

Mesh refinement

PDE-based filtering (Lazarov&Sigmund 2011)

$$-r^2 \Delta \tilde{\rho} + \tilde{\rho} = \rho$$

The "TopOpt App"



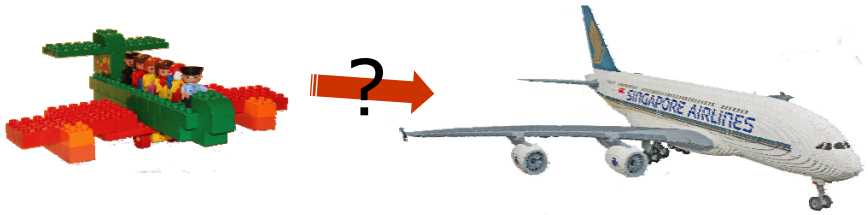
3D version coming soon!

- The "TopOpt App": AppStore (iOS)
  Google Play (Android)
  Web-version: www.topopt.dtu.dk

Stats 17/3-2014:
Android: 2800, iOS: 6860

See **www.topopt.dtu.dk** for more

The "Duplo" problem

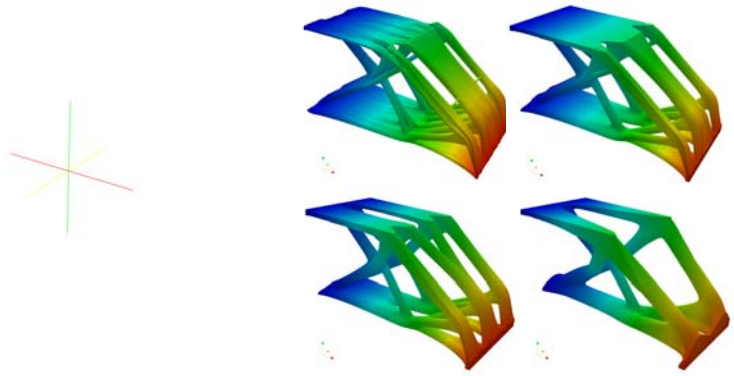


NextTop-project (Villum foundation)

- Algorithms
- Solvers
- Multiscale
- Parameterizations
- Robustness
- Parallel computing

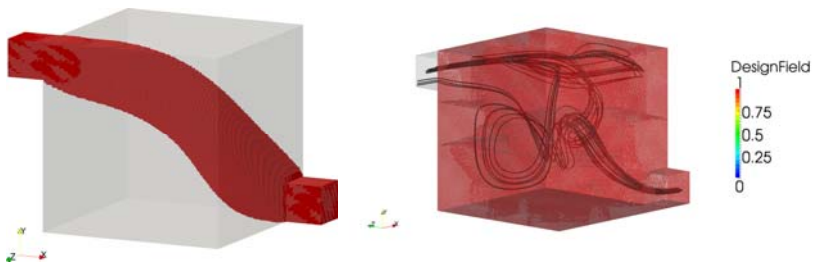
Large scale topology optimization

- Topology optimization using PETSc
  - Solver: F-GMRES with Galerkin projection MG preconditioner
  - Optimizer: Parallel MMA



83Mdof and 27M design variables
Solution time: 60s - 30s per design cycle depending on filter radius on 144 cores

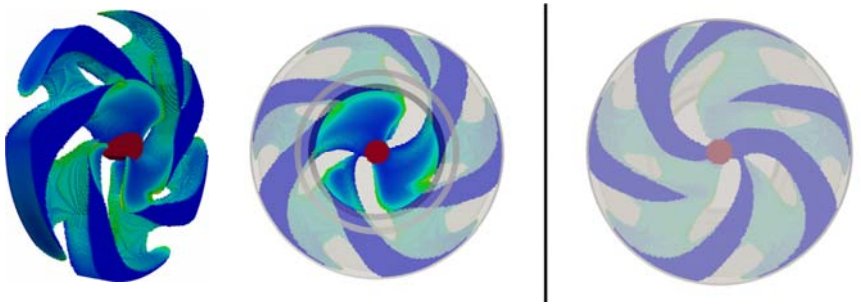
Manifold Moderate Reynolds number (Navier-Stokes)



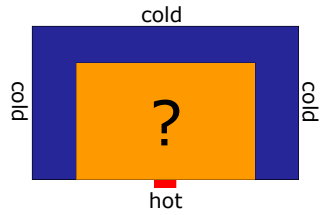
Linear solver: F-GMRES using GMG-GMRES pre-conditioner
2.16Mdof and 0.5M design variables
Solution time: ~60s per linear solve on 24 cores, 200 design iterations

Alexandersen & Aage

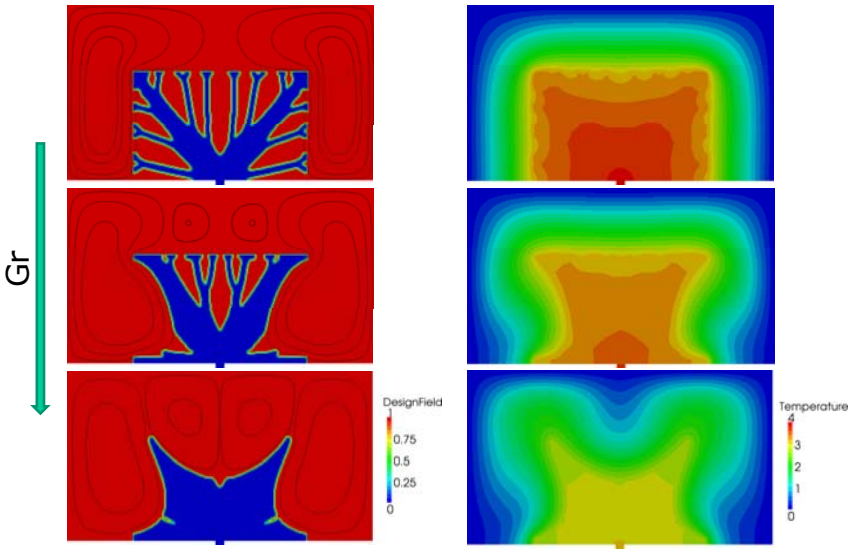
Impeller / radial pump



Free convection problem (liquid cooler)

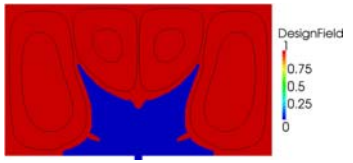


Natural convection – cavity problem



Coupled multiphysics: 2D

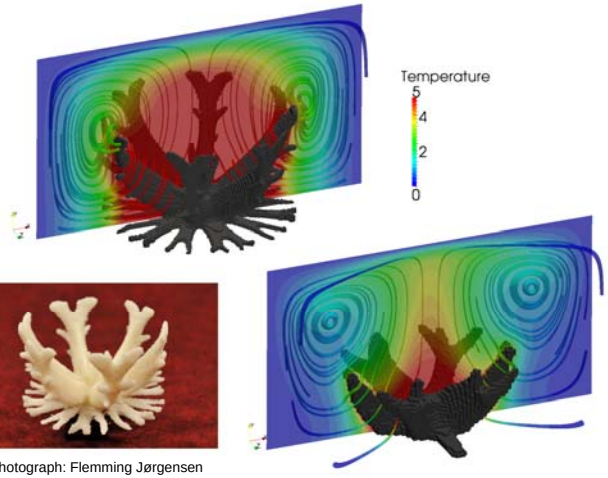
Optimised design for steady state conditions, Gr = 6,400:



Gr = 6,400 Gr = 640,000,000:

Alexandersen et al.

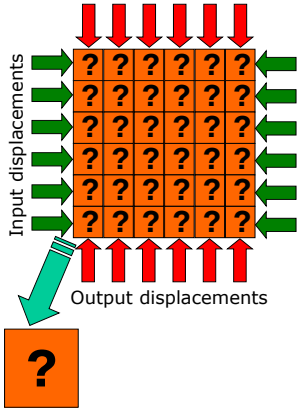
Coupled multiphysics: 3D

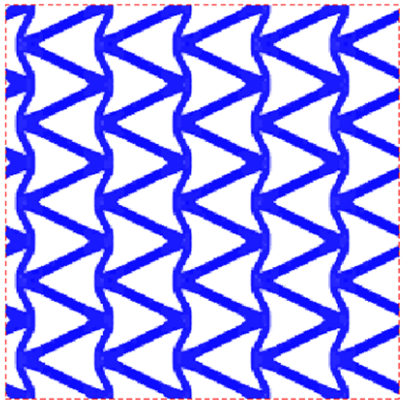


Photograph: Flemming Jørgensen

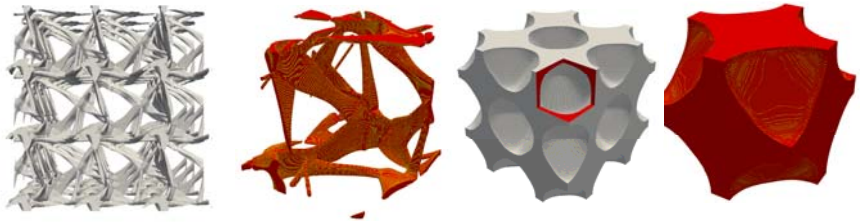
Alexandersen et al.

Extremal material design





Material design

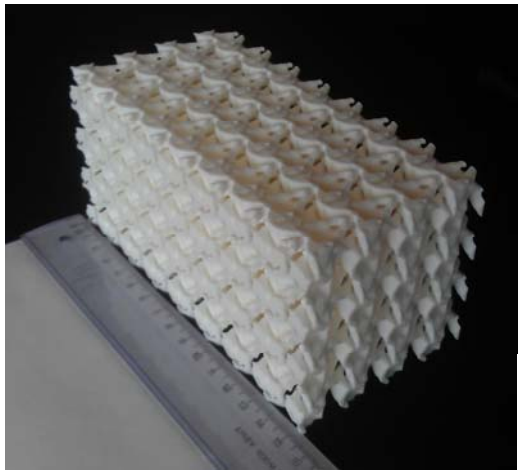


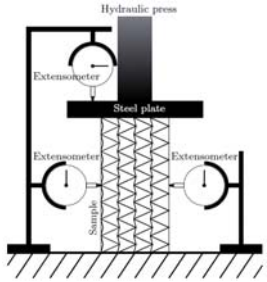
Minimum **Poisson's ratio**: v=-0.8
24 Mdofs and 8M design variables

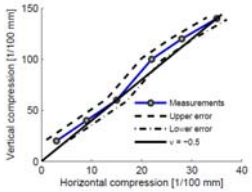
Maximum **bulk modulus**:
72 Mdofs and 24M design variables

Solution time: 60s per cycle including 6 linear solves on 240 cores

3D Manufacturing and testing

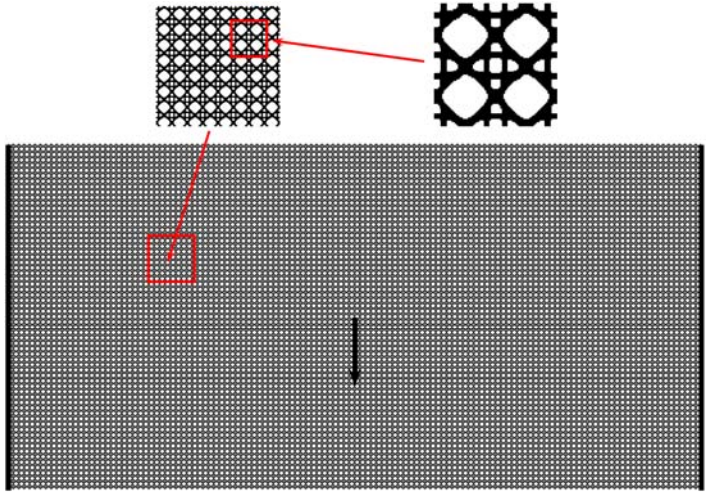






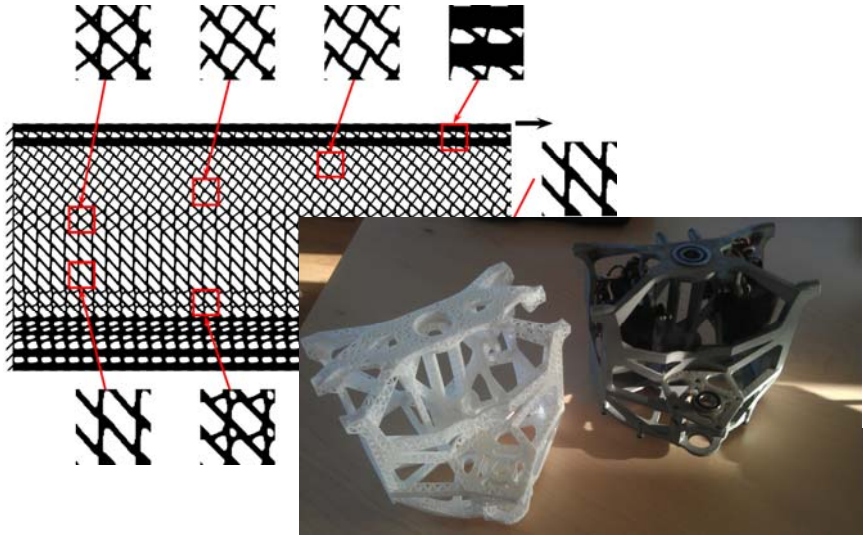
- Andreassen, Lazarov & *OS, MofM,* **2014**, *69*, 1-10

Multiscale design



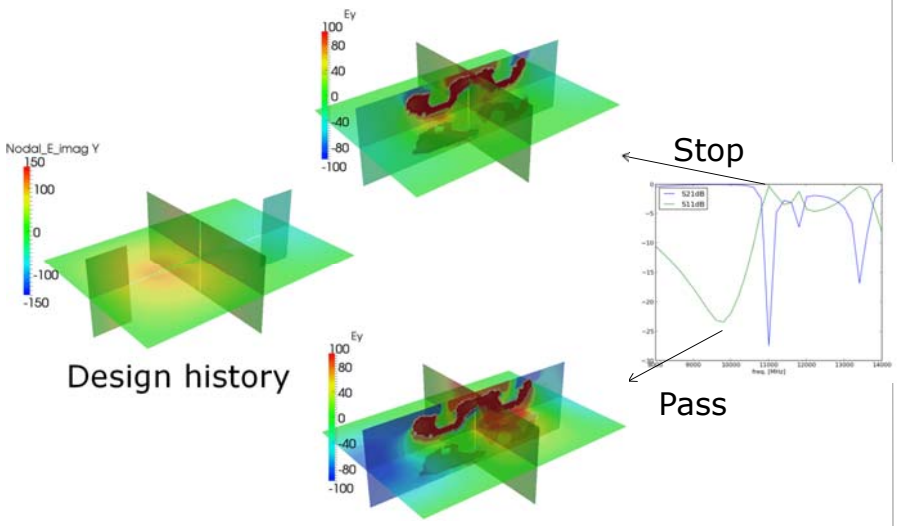
Periodic structure, non-separable length-scales, MsFEM-GMRES
26Mdof, 300 design iterations. 24 hours on one-core Matlab

Multiscale design

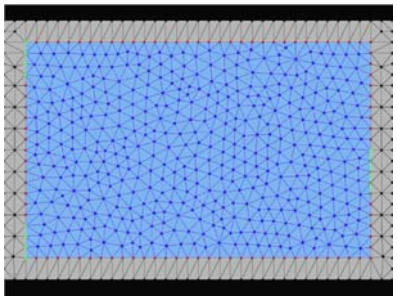


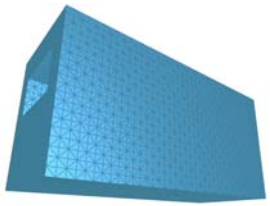
Klaus Højbjerre, DTI, Århus

Microwave applications



Alternative parameterizations





Discrete Simplicial Complexes (DSC) by Bærentzen et al., DTU Compute

Conclusions

Topology optimization has a huge potential (and need) for large scale computing

New cluster eagerly awaited by the **TopOpt**-group and **DTU Mechanical Engineering**

Further reading

TopOpt background

- Bendsøe, M.P. & *OS*, Topology Optimization - Theory, Methods and Applications, *Springer Verlag,* **2004**
- *OS* & Maute, K., Topology optimization approaches: A comparative review, **2013**, *48*, 1031-1055
- *OS*, On the usefulness of non-gradient approaches in topology optimization, *SMO,* **2011**, *43*, 589-596
- Andreassen, E.; Clausen, A.; Schevenels, M.; Lazarov, B. & *OS*, Efficient topology optimization in MATLAB using 88 lines of code, *SMO,* **2011**, *43*, 1-16
- Wang, F., Lazarov, B. & *OS*, On projection methods, convergence and robust formulations in topology optimization, *SMO*, **2011,** *43*, 767-784

Material Design

- Andreassen, E.; Lazarov, B. & *OS*, Design of manufacturable 3D extr. elastic microstr., *MofM,* **2014**, *69*, 1-10

Fluids

- Aage, N.; Poulsen, T. H.; Gersborg-Hansen, A. & Sigmund, O., Topology Optimization of Large Scale Stokes Flow Problems, *SMO,* **2008**, *35*, 175-180
- Alexandersen, J; Aage, N., Andreasen, C.S. & Sigmund, O. Design of natural convection dominated problems using topopt, *submitted*, **2014**

Alternative parameterizations

- Christiansen, A.; Nobel-Jørgensen, M.; Aage, N.; *OS* & Bærentzen, J. Topology optimization using an explicit interface representation, *SMO,* **2014**, *49*, 387-399

Large-scale TopOpt

- Aage, N. & Lazarov, B., Parallel framework for topology optimization using MMA, *SMO,* **2013**, *47*, 493-505

See **www.topopt.dtu.dk** for more

"TopOpt App"
(iOS, Android,web)