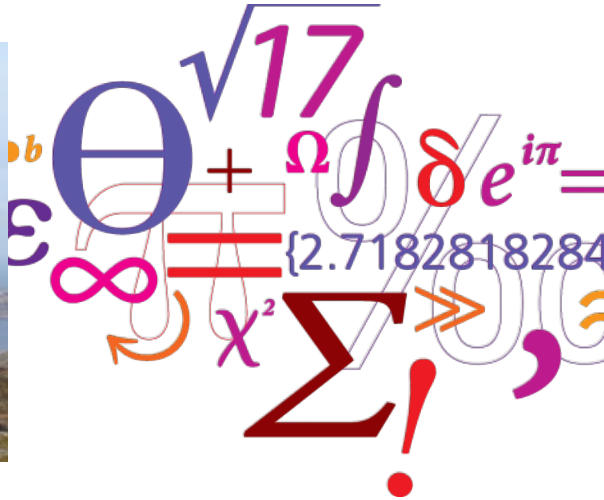
Center for Wind Turbine Aerodynamics and Atmospheric Turbulence

Vortex Breakdown and Instability in the Near Wake of a Wind Turbine

Jens N. Sørensen

Department of Wind Energy
Technical University of Denmark

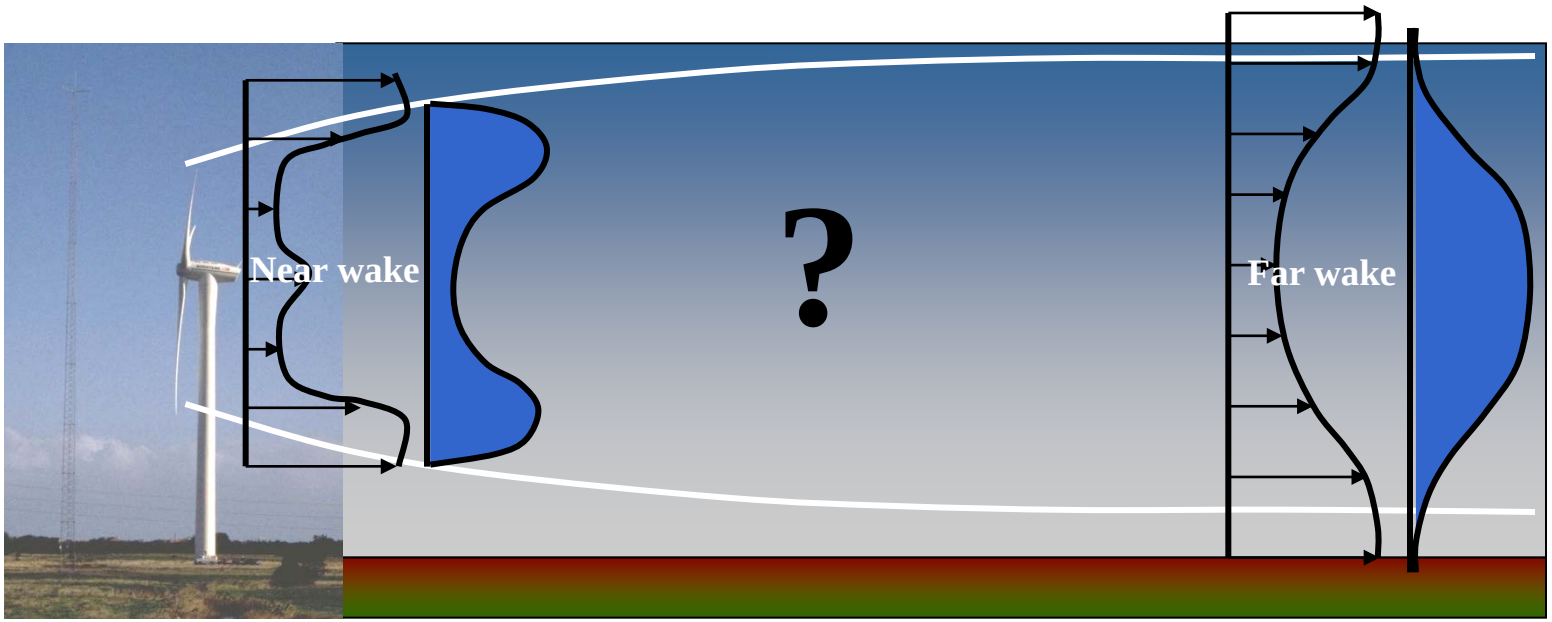


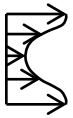
Collaborators:
Robert Mikkelsen, Valery Okulov (DTU)
Stefan Ivanell, Dan Henningson, Sasan Sarmast (KTH)

DTU Vindenergi
Institut for Vindenergi

Wake Aerodynamics

Wake development:




: Axial velocity


: Turbulence intensity

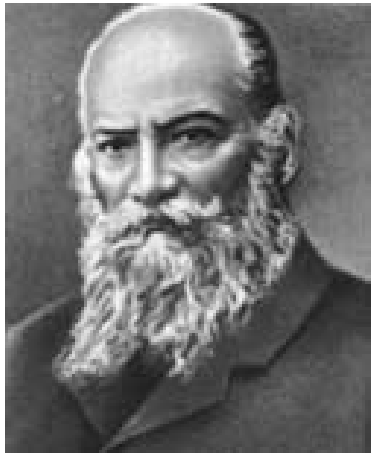
Development of the wake

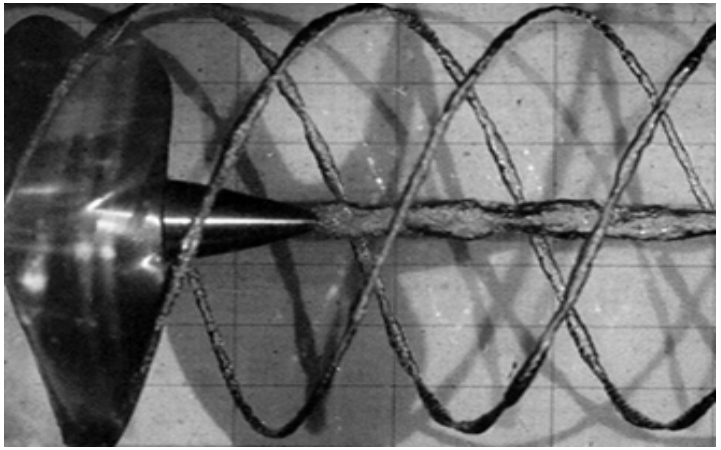
Scenario:

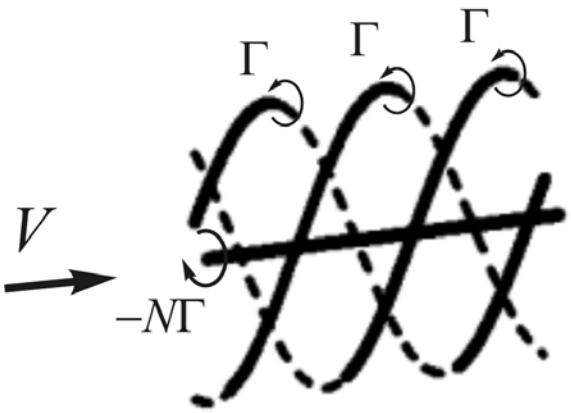
1. Vortex system formed from circulation
2. Roll-up into center vortex and distinct tip vortices
3. Destabilization of tip vortices
4. Break down into large-scale turbulence
5. Turbulent mixing
6. Interplay with meandering

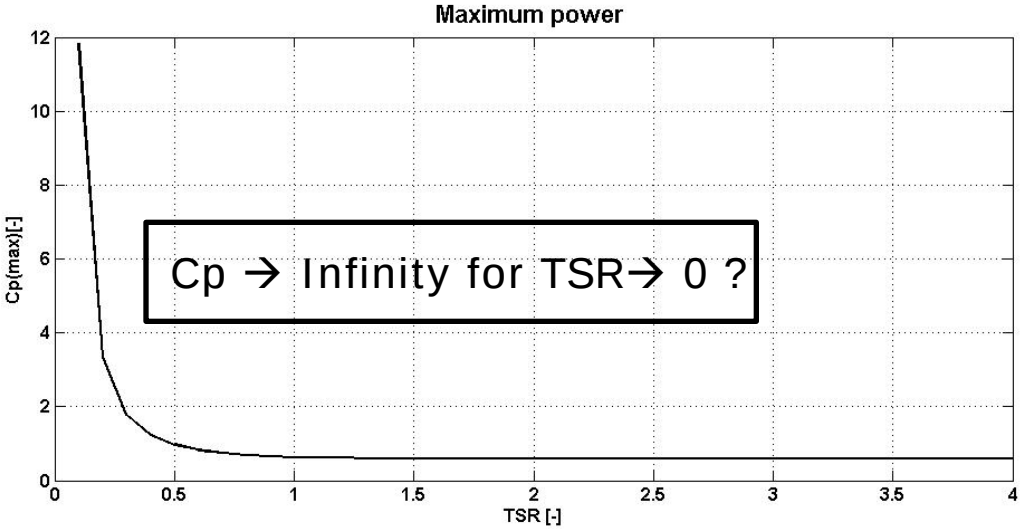
The enigma of the wake model of Joukowsky

The far wake model by Joukowsky (1912)









Can vortex breakdown the limiting factor?

Swirl number: $$S = \frac{\int_0^R \rho u u_\theta 2\pi r dr}{\int_0^R \rho u^2 2\pi R dr} = \frac{u_\theta(r=R)}{u_D} = \frac{q}{1-a}$$

Criterion for vortex breakdown (Squire (1960), Delery (1994)):

$$\frac{u_{\theta,\max}}{u_x} > 1.4 \Rightarrow \frac{q}{(1-a)} > 1.4\left(\frac{r_C}{R}\right)$$

Axisymmetric AD/NS analysis

The governing equations:

$$\frac{\partial \omega}{\partial t}+\frac{\partial u\omega}{\partial x}+\frac{\partial v\omega}{\partial r}-\frac{\partial}{\partial x}\left(\frac{w^2}{r}\right)=\varepsilon\left[\frac{\partial}{\partial x}\left(\frac{\partial \omega}{\partial x}\right)+\frac{\partial}{\partial r}\left(\frac{\partial \omega}{\partial r}\right)+\frac{1}{r}\frac{\partial \omega}{\partial r}-\frac{\omega}{r^2}\right]+\frac{1}{\rho}\frac{\partial f_r}{\partial x}-\frac{1}{\rho}\frac{\partial f_x}{\partial r}$$

$$\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial vw}{\partial r}+\frac{2vw}{r}=\qquad \varepsilon\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial r}\left(\frac{\partial w}{\partial r}\right)+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}\right]+\frac{f_\theta}{\rho}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right)+\frac{\partial}{\partial r}\left(\frac{\partial \psi}{\partial r}\right)-\frac{1}{r}\frac{\partial \psi}{\partial r}=r\omega$$

$$u=-\frac{1}{r}\frac{\partial \psi}{\partial r},\qquad v=\frac{1}{r}\frac{\partial \psi}{\partial x},\qquad \omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial r}$$

Actuator disc modelling and body forces

The body forces:

$$f_r = 0, \quad f_x = \rho u_\theta \left(\Omega r + \frac{1}{2} u_\theta \right), \quad f_\theta = \rho u_D u_\theta$$

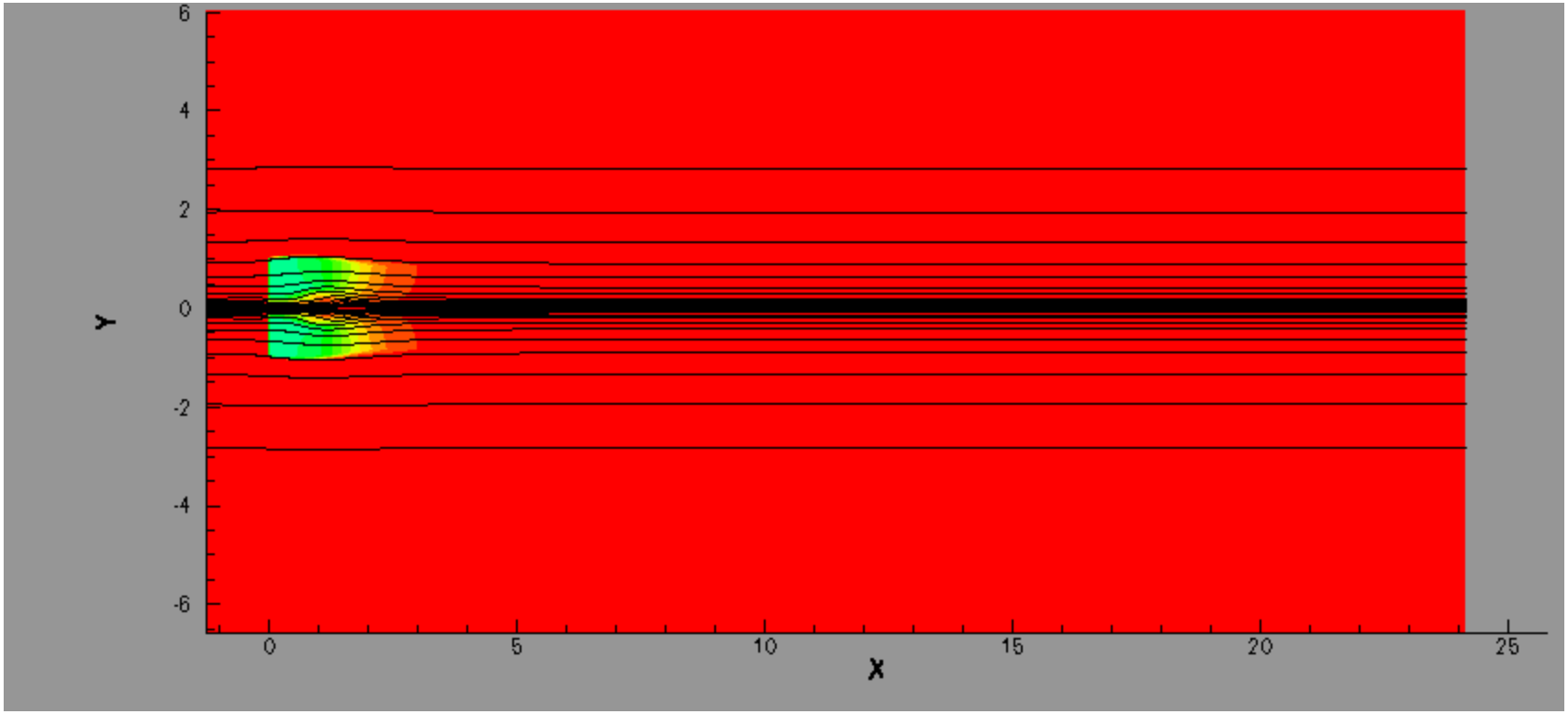
Assumed velocity profiles:

$$\frac{u_\theta}{U_0} = \frac{q}{r}\left[1 - \exp\left(-1.256\left(\frac{r}{\delta}\right)^2\right)\right], \quad u_D = U_0(1-a)$$

The resulting body forces:

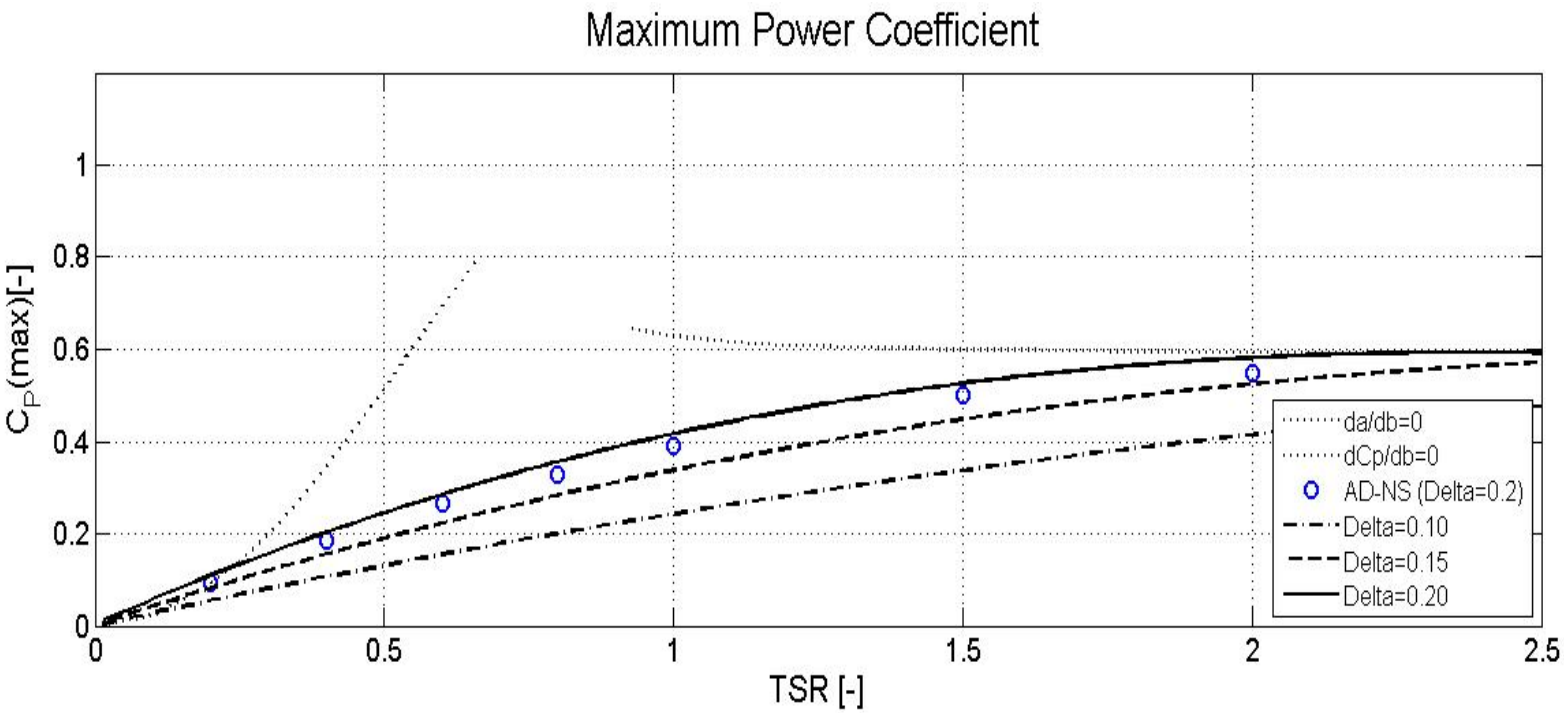
$$\frac{f_x}{\rho U_0^2} = \lambda q + \frac{q^2}{2r^2} \cdot g^2, \quad \frac{f_\theta}{\rho U_0^2} = \frac{q(1-a)}{r} \cdot g, \quad g = 1 - \exp\left(-1.256\left(\frac{r}{\delta}\right)^2\right)$$

The dynamics of the vortex breakdown ($\lambda = 1$)

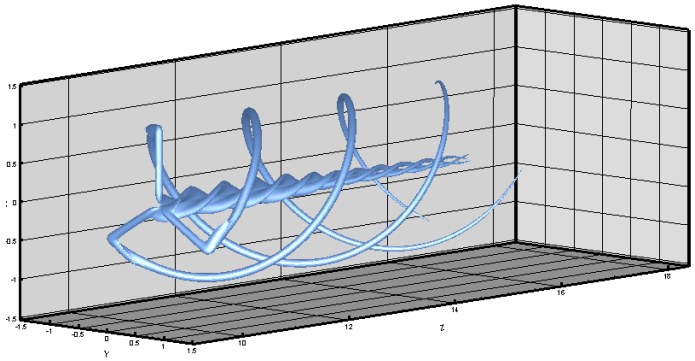


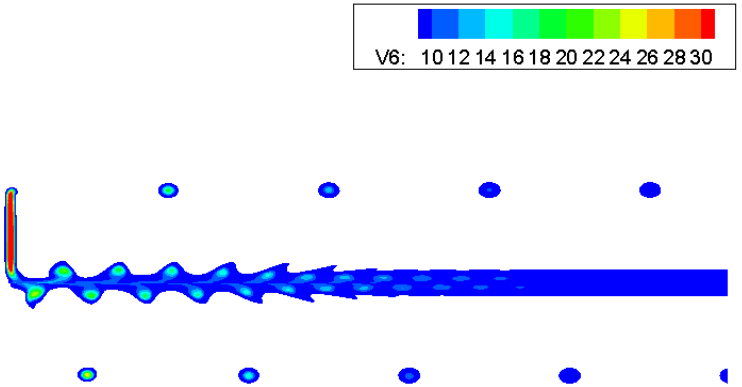
Sørensen and Mikkelsen (2013)

Comparison between AD-NS and momentum theory

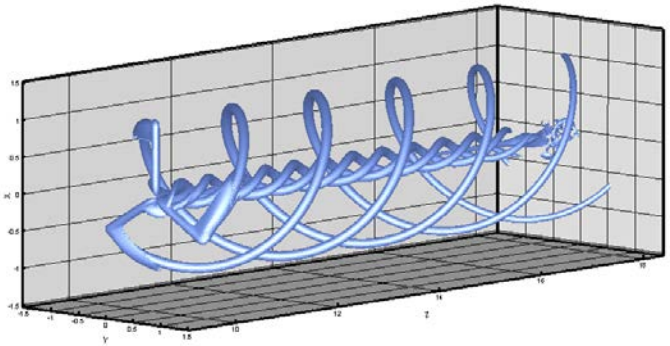


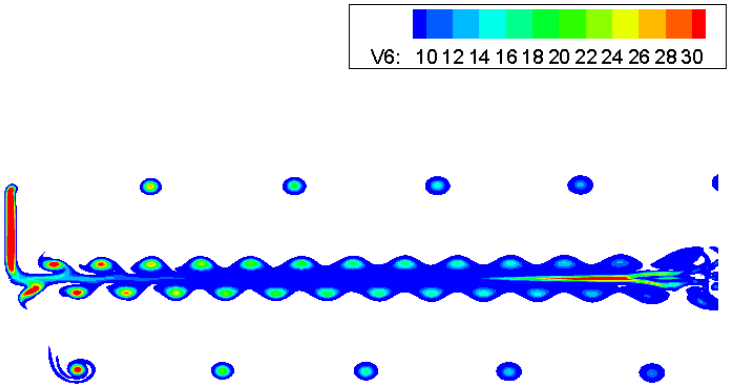
$\delta = \left(\frac{r_C}{R}\right)$ denotes the size of the inner viscous core



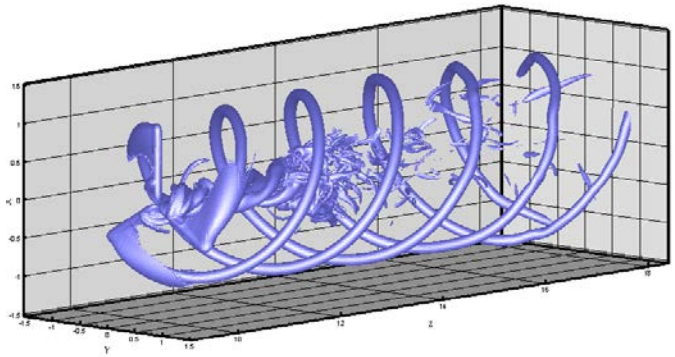


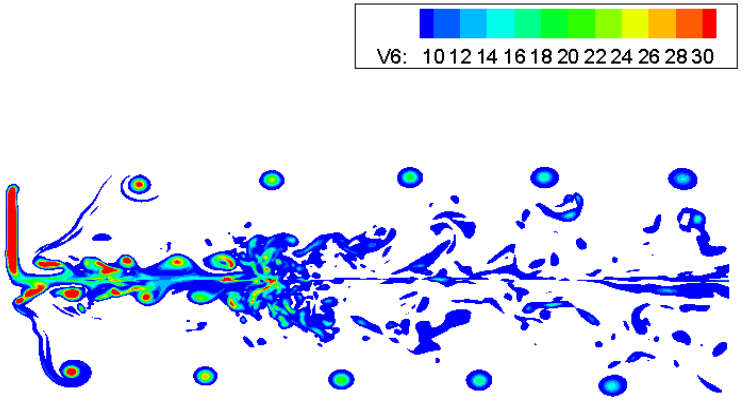
TSR=1, B=0.15 Cp=0.2238 , Ct=0.3095





TSR=1, B=0.25 Cp=0.3356 , Ct=0.5297

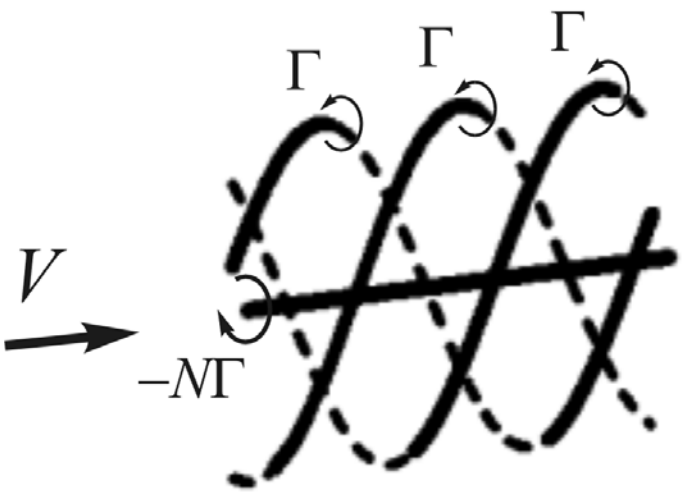




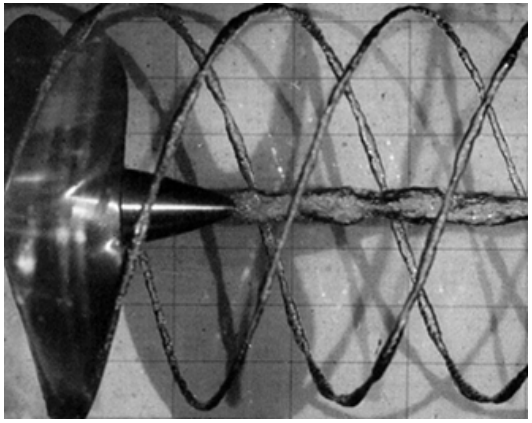
TSR=1, B=0.35 Cp=0.4173, Ct=0.7489

Modeling of vortex wake behind rotors

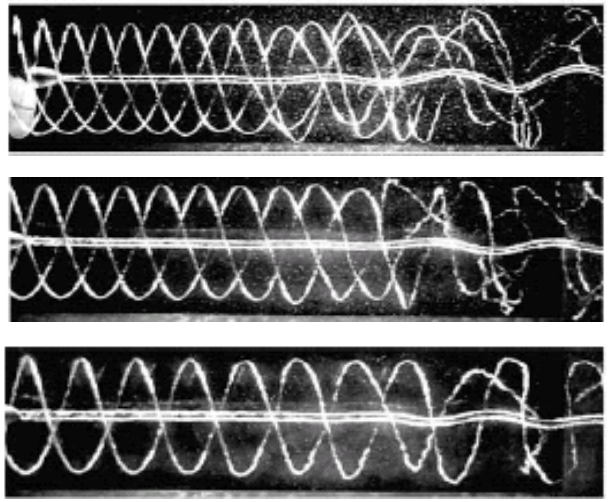
Far wake model of Joukowski (1912)



Observations



Felli et al. (2008)



Conclusion from stability analysis:

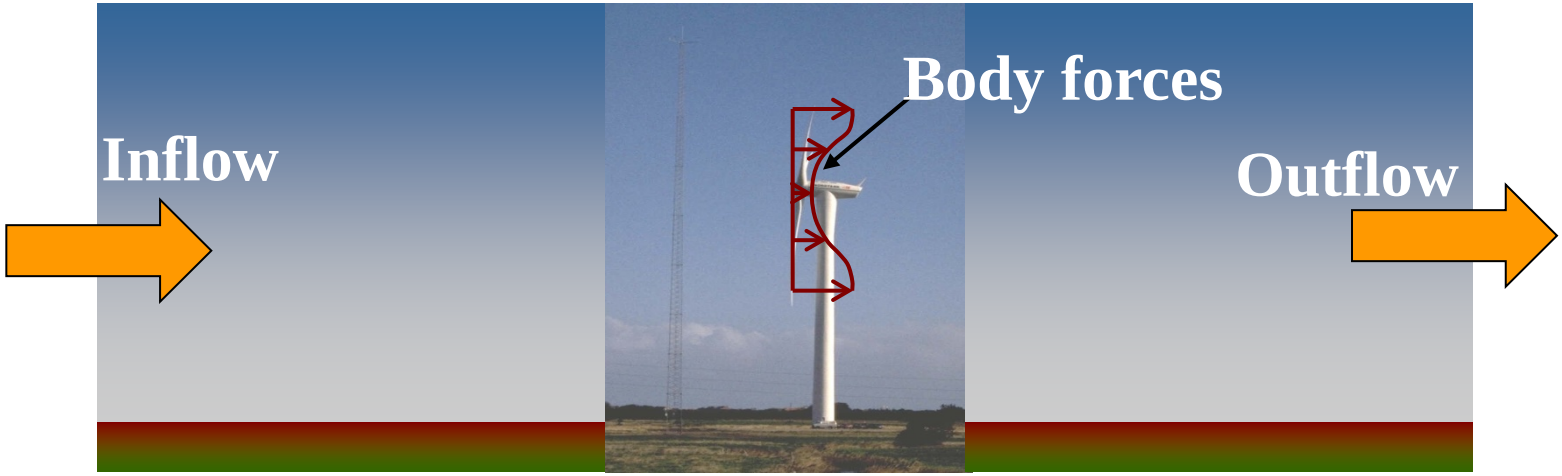
The vortex wake behind a wind turbine is <u>unconditional unstable</u>

Okulov and Sørensen JFM 576 (2007)

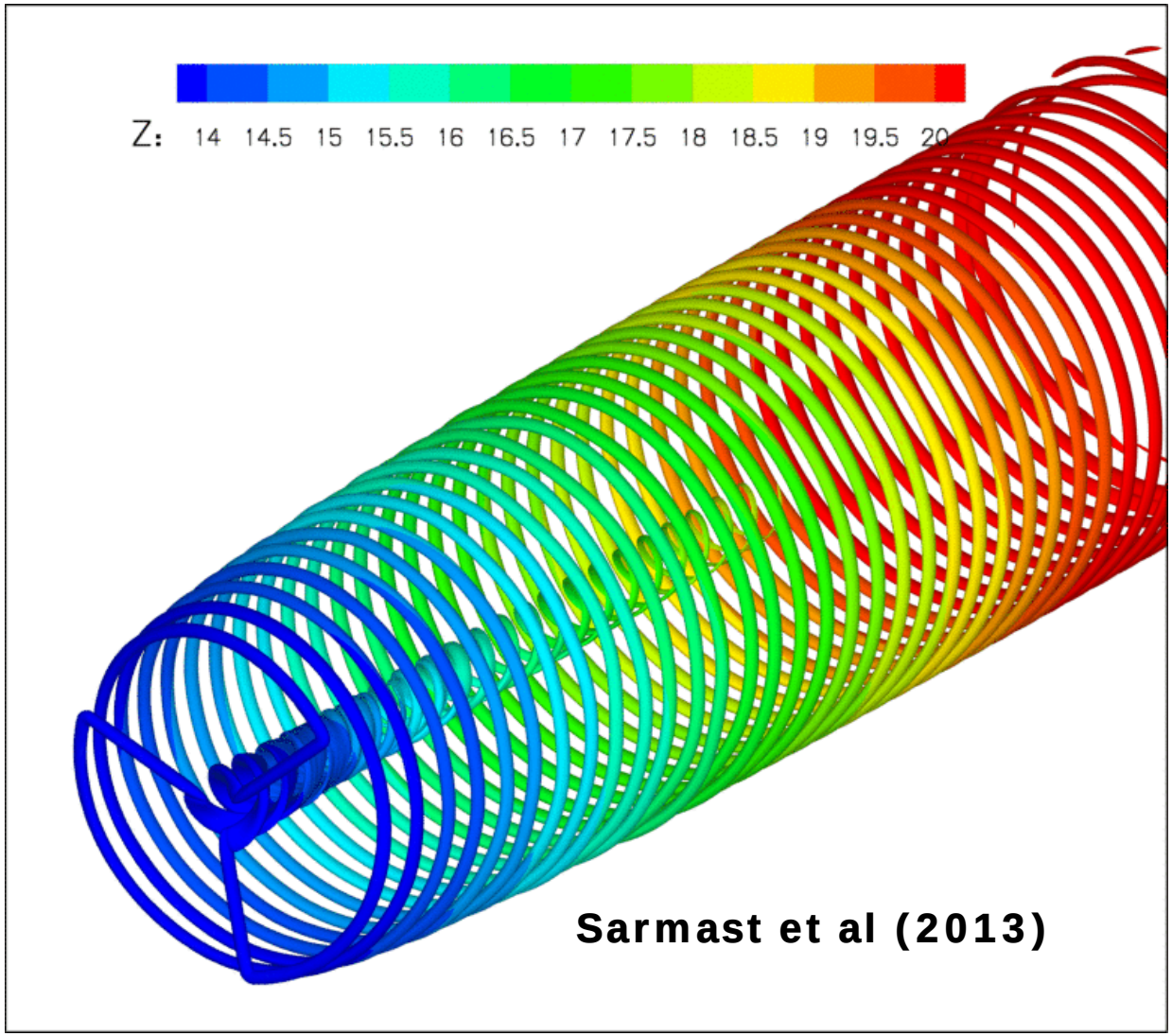
The actuator disc/line techniques

Basic idea:

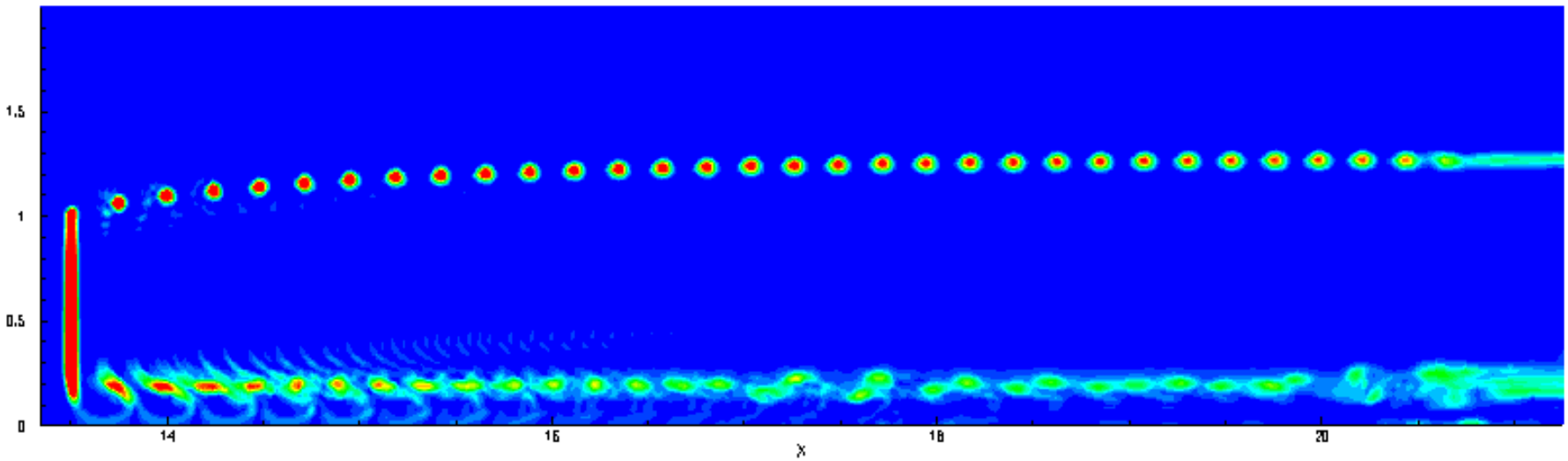
- **Replace rotor blades by body forces**
- **Determine body forces from aerofoil data**
- **Simulate flow domain using EllipSys/LES**



Wake breakdown due to presence of low upstream amplitude excitation

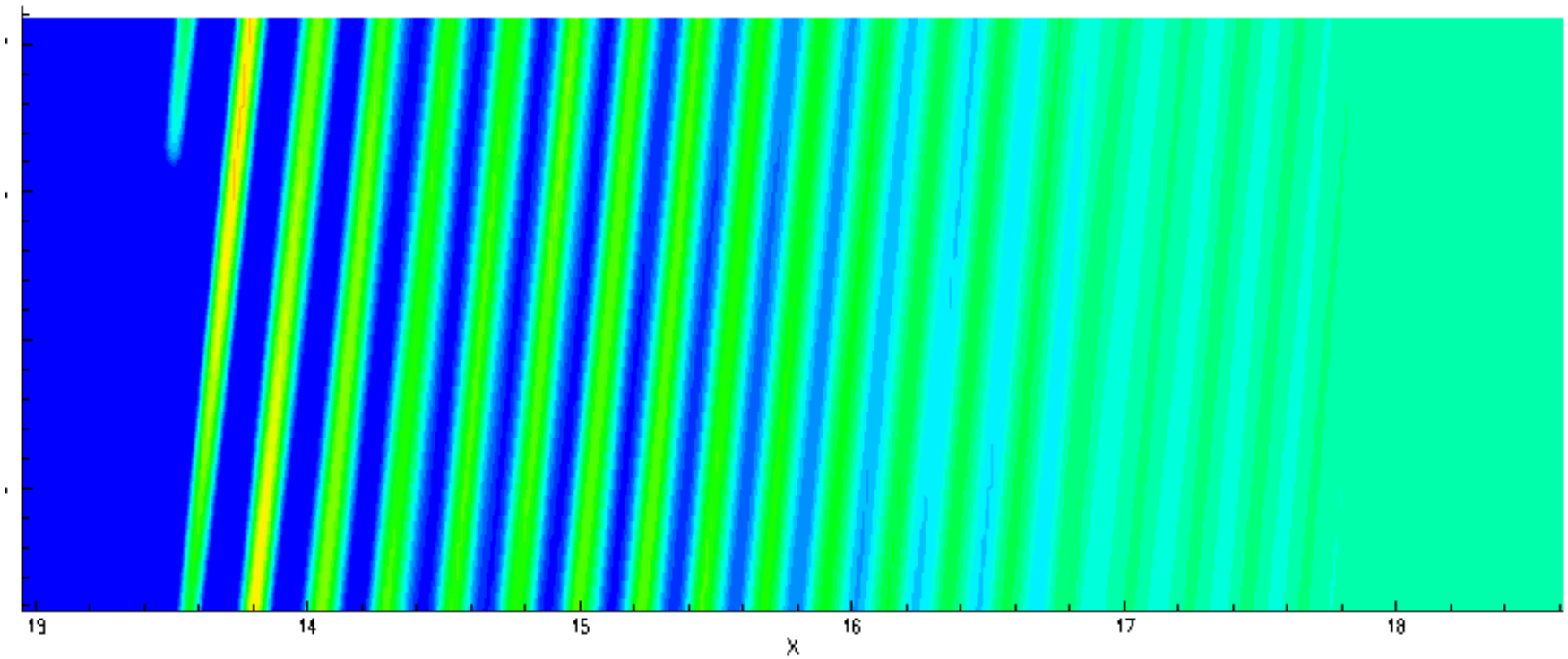


Mutual inductance instability of tip vortices



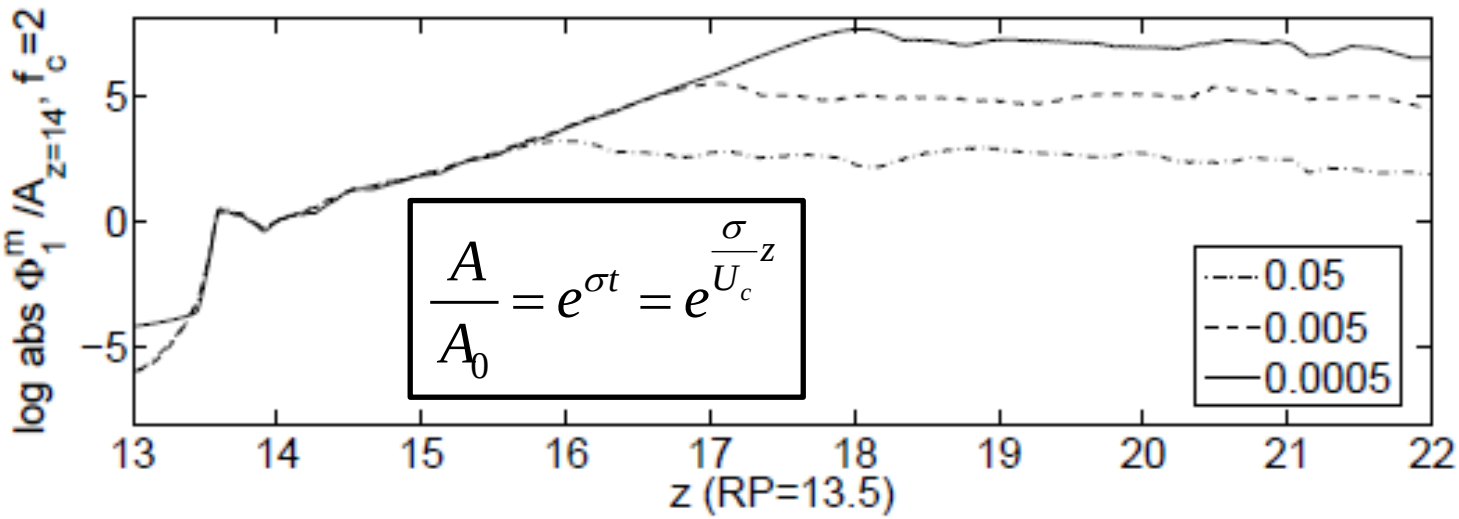
Ivanell et al (2010)

Mutual inductance instability of tip vortices



Ivanell et al (2010)

Mutual inductance instability of tip vortices



Amplification: $$\frac{\sigma \cdot 2h^2}{\Gamma} = \frac{\pi}{2}$$ (Lamb, 1932)

h : Distance between vortices

U_c: Convective velocity

Γ : Circulation

Relations between wake and rotor charateristics

Geometry of of wake: $$\frac{N_b h}{2\pi R} = \frac{U_c}{\Omega R} \Rightarrow h = \frac{2\pi R U_c}{N_b \lambda U_o}$$

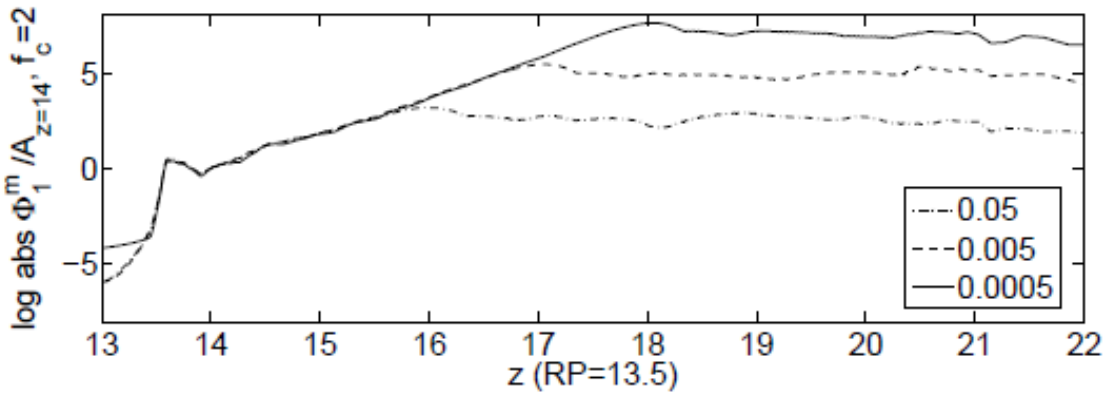
Assuming constant loading: $$\Gamma = \frac{\pi U_o^2 C_T}{N_b \Omega}$$

Roller bearing analogy: $$U_c = C_1 \cdot U_{wake} + \left(1 - C_1\right) \cdot U_o$$

From 1-D Momentum theory: $$U_{wake} = U_o \sqrt{1 - C_T}$$

Combining: $$\boxed{\frac{\sigma R}{U_c} = \frac{N_b \lambda C_T}{16\left[1 + C_1\left(\sqrt{1 - C_T} - 1\right)\right]^3}}$$

Expression for length of near wake



Amplitude amplification:

$$\ln\left(\frac{A_{\max}(t)}{A_o}\right) \simeq -\ln\left(\frac{u'}{U_o}\right)$$

Assuming that: $\dfrac{u'}{U_o} = C_2 \cdot Ti$

We get the following expression for the length of the near wake (defined as the stable wake of the tip vortices):

$$\boxed{\frac{l}{R} \simeq -\frac{16\left[1 + C_1\left(\sqrt{1 - C_T} - 1\right)\right]^3}{N_b C_T \lambda} \ln(C_2 \cdot Ti)}$$

Conclusions

- It has been shown both theoretically and numerically that the wake at small, i.e. TSR < 3, may be subject to Vortex Breakdown
- From stability analysis we have shown that the wake behind a wind turbine is unconditional unstable
- From linear stability theory it was shown that the normalized amplification factor is equal pi/2
- Defining the near wake as the linear area of amplitude amplification, a simple expression for determining the length of the near wake has been established
- The expression has been preliminary validated against LES/AL simulations and visualizations of a model rotor