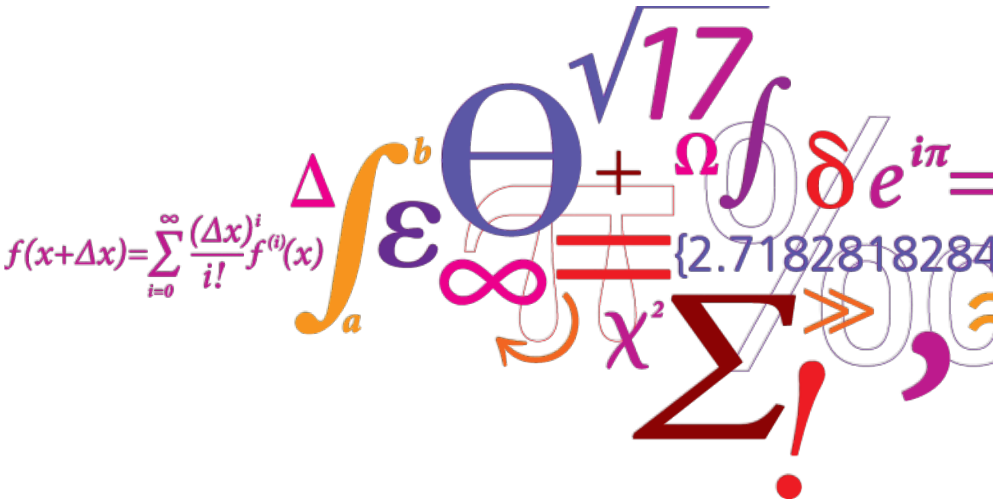
Cylindrical vortex wake models with application to tilted rotor

Emmanuel Branlard - Mac Gaunaa

Comflow Meeting 2013-11-01

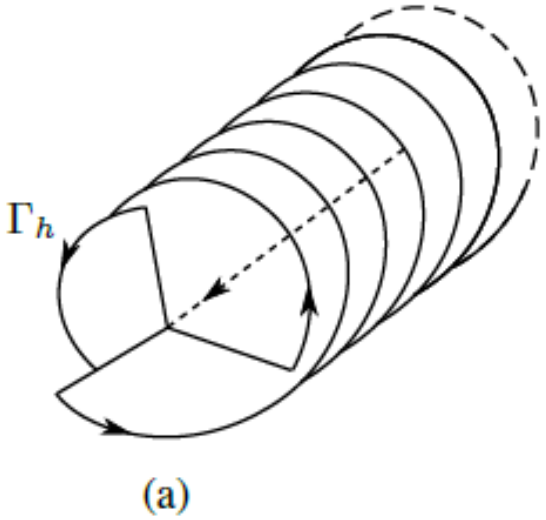


DTU Wind Energy
Department of Wind Energy

1. Presentation of the model

1. Presentation of the model

The wake fits into a cylinder



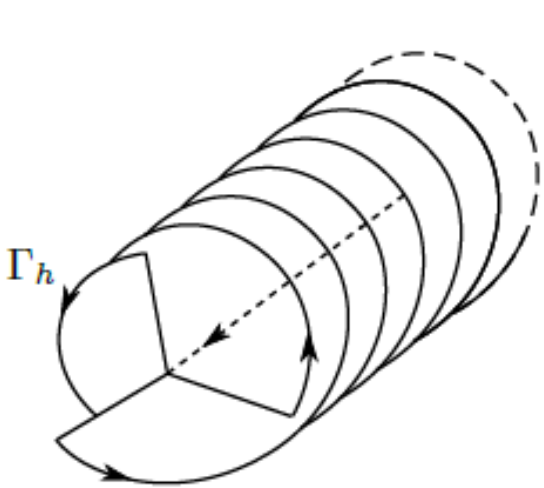
(a)

Lifting-line

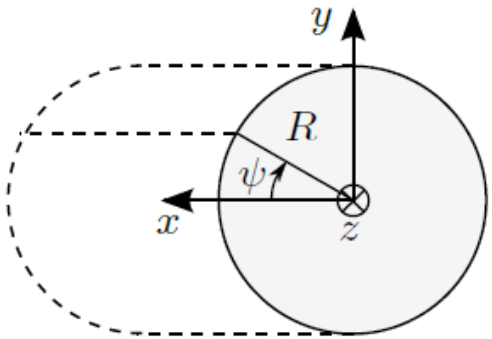
Constant Circulation

1. Presentation of the model

The wake fits into a cylinder



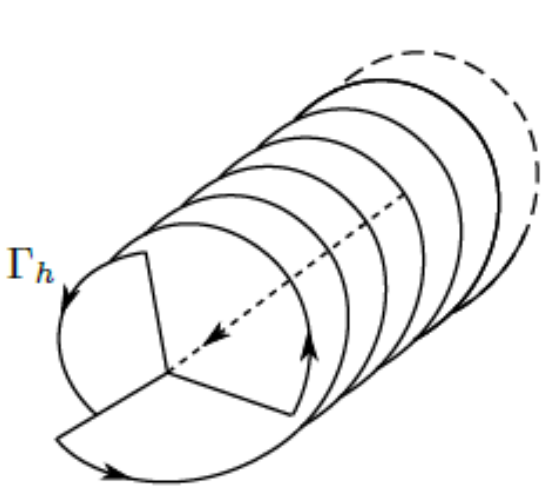
(a)



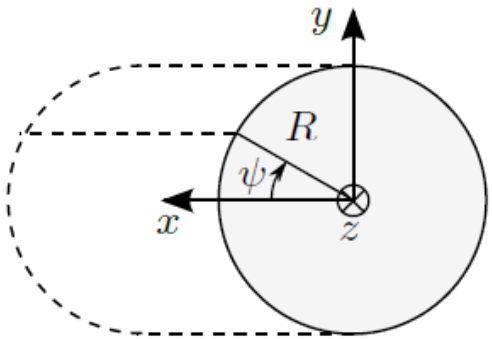
Front view

1. Presentation of the model

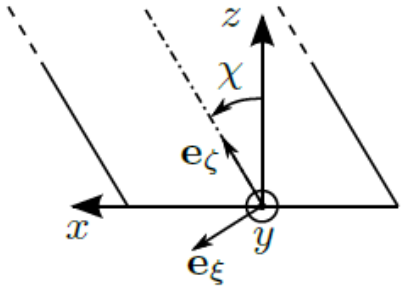
The wake fits in a cylinder



(a)



Front view

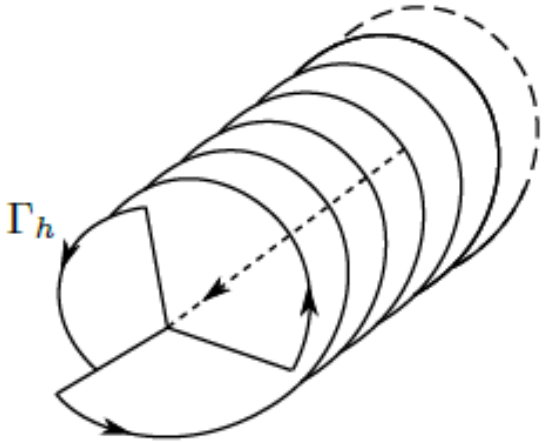


Top view

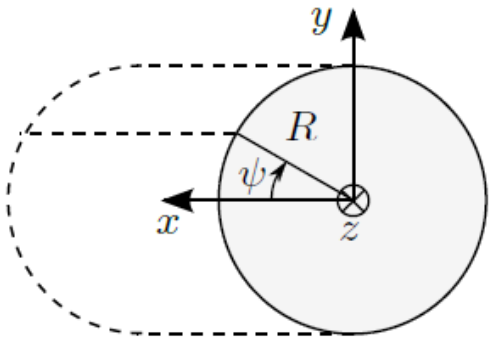
Incoming wind

1. Presentation of the model

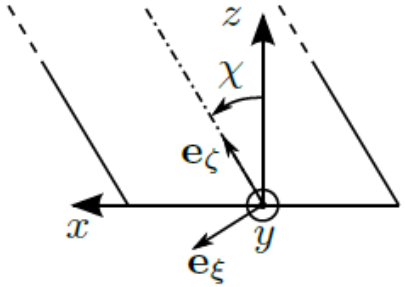
The wake fits in a cylinder



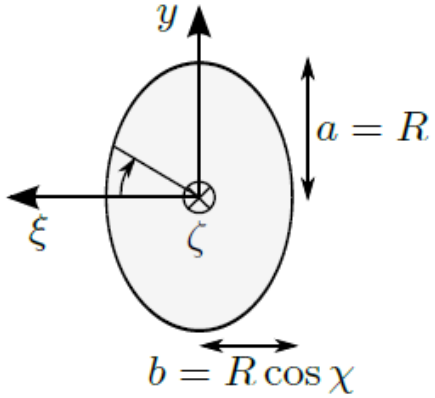
(a)



Front view



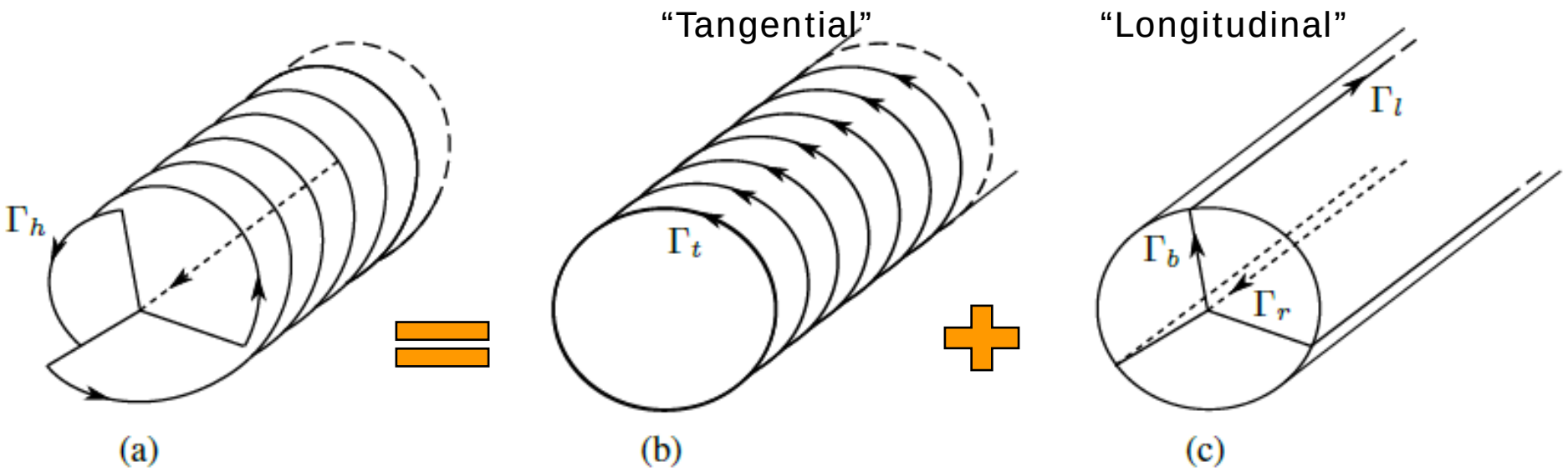
Top view



Skewed wake view

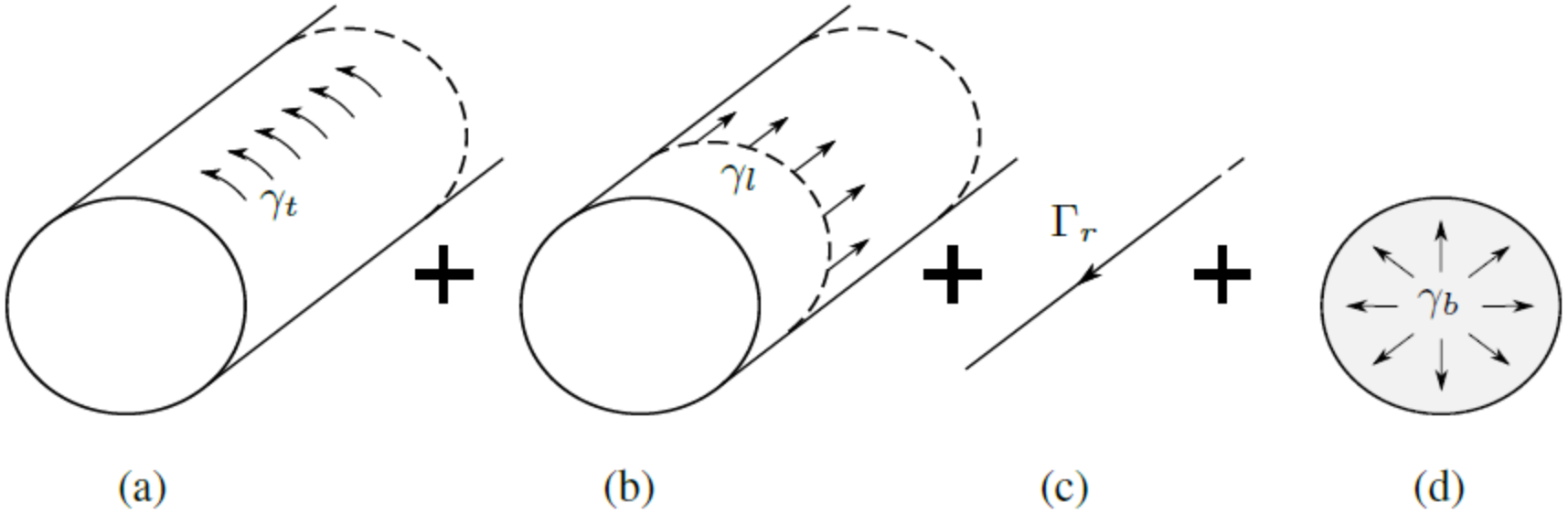
1. Presentation of the model

Decomposition of (skewed) helical wake



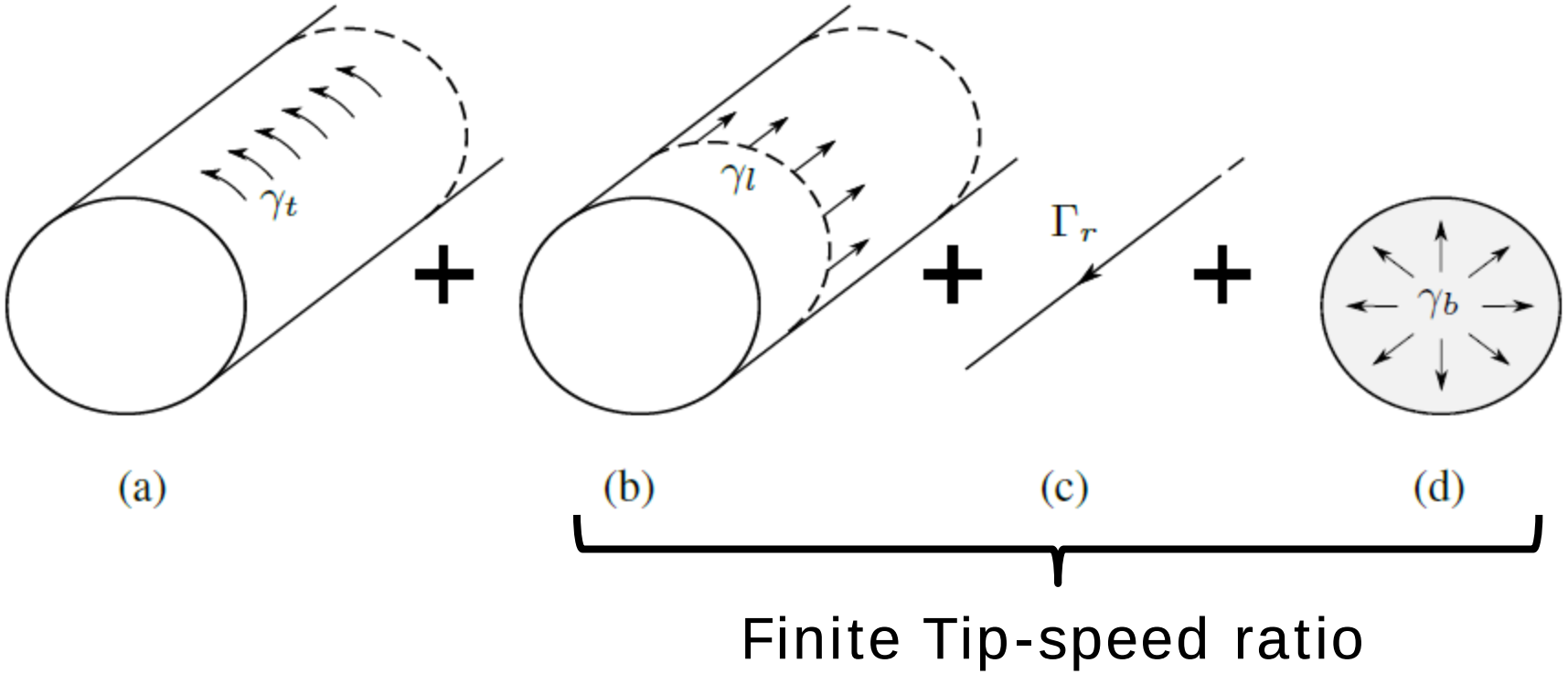
1. Presentation of the model

Viewed with infinite number of blades



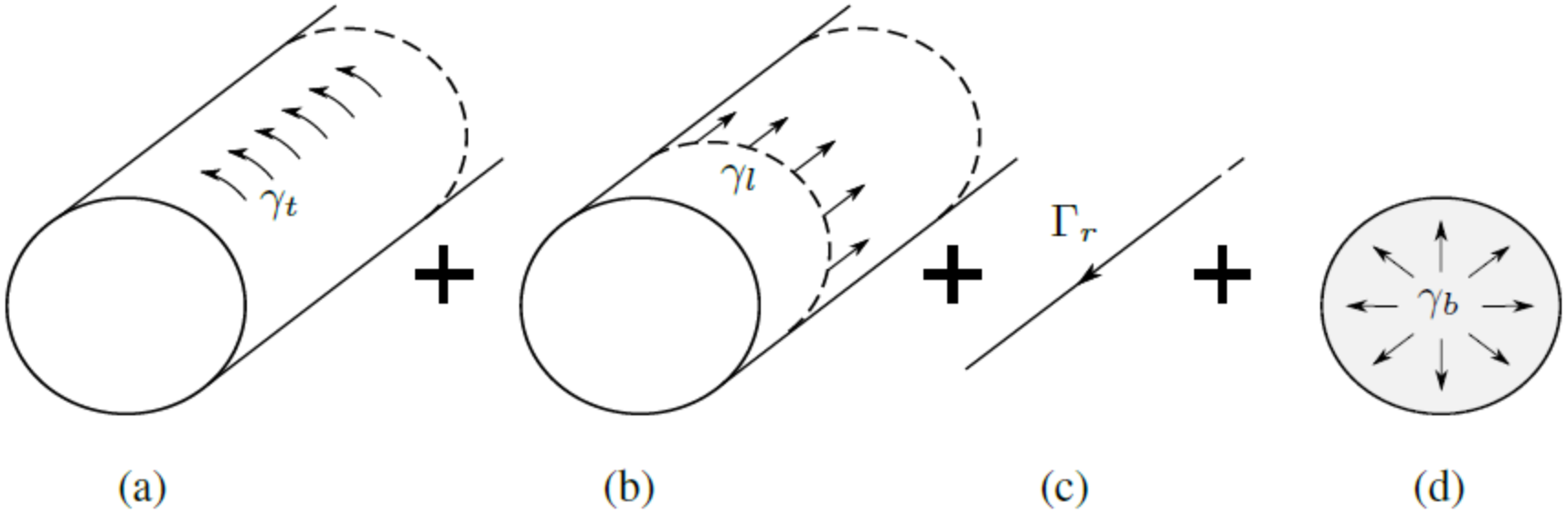
1. Presentation of the model

Viewed with infinite number of blades



1. Presentation of the model

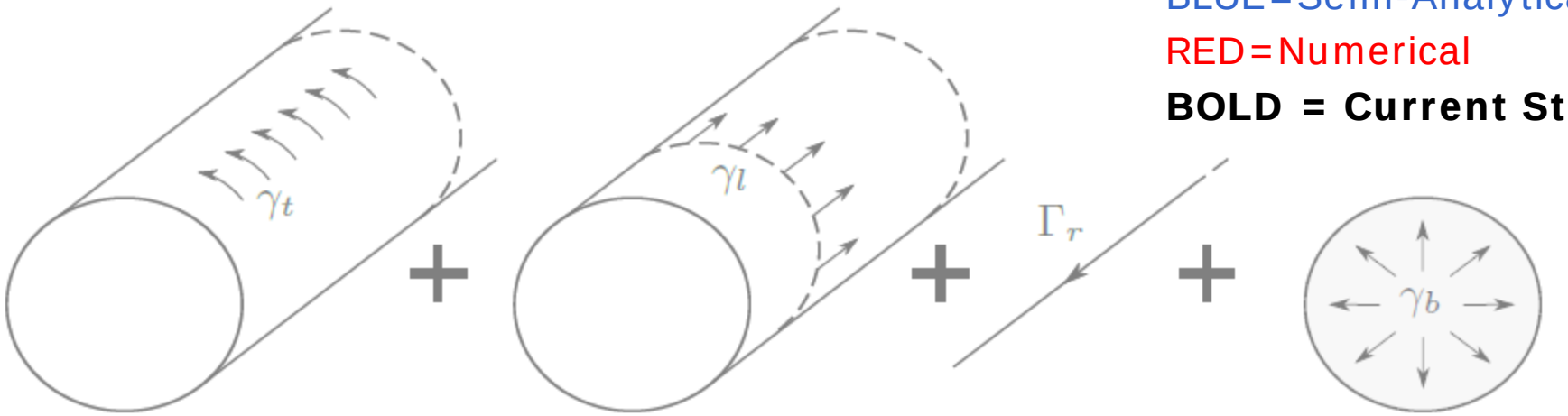
Straight Case (NO tilt)



1. Presentation of the model

Straight Case (NO tilt)

GREEN=Analytical
BLUE=Semi-Analytical
RED=Numerical
BOLD = Current Study



γ_t	γ_l	Γ_r	γ_b
Callaghan(1960)	Gibson(1974)	WEH	**Current study**
Gibson(1974)	**Current study**	**Current study**	
Current study	ECN(1995)	ECN(1995)	
WEH			

1. Presentation of the model

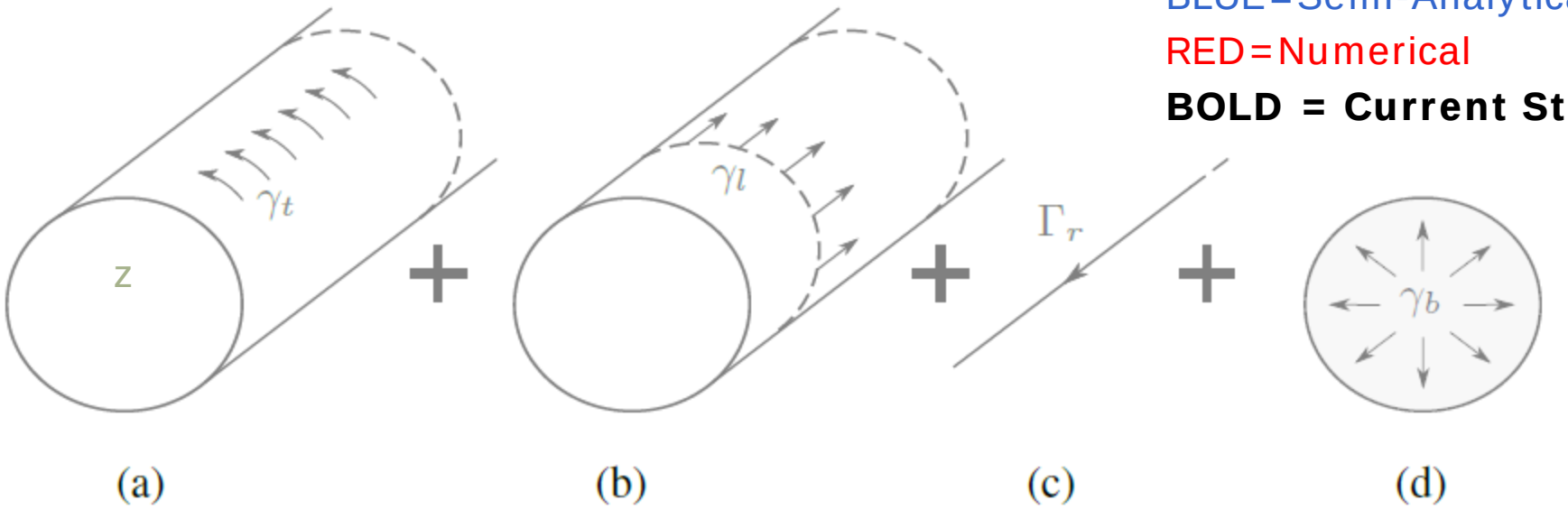
tilt

GREEN=Analytical

BLUE=Semi-Analytical

RED=Numerical

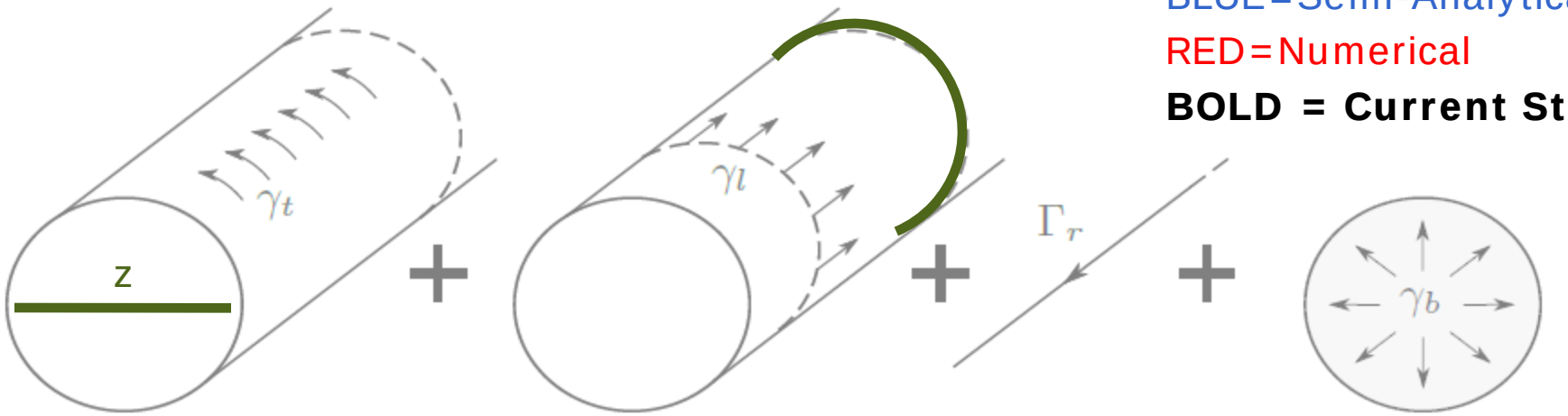
BOLD = Current Study



1. Presentation of the model

tilt

GREEN=Analytical
BLUE=Semi-Analytical
RED=Numerical
BOLD = Current Study



z Coleman (1945)
z Castles (1956)
xyz Current study
xyz WEH

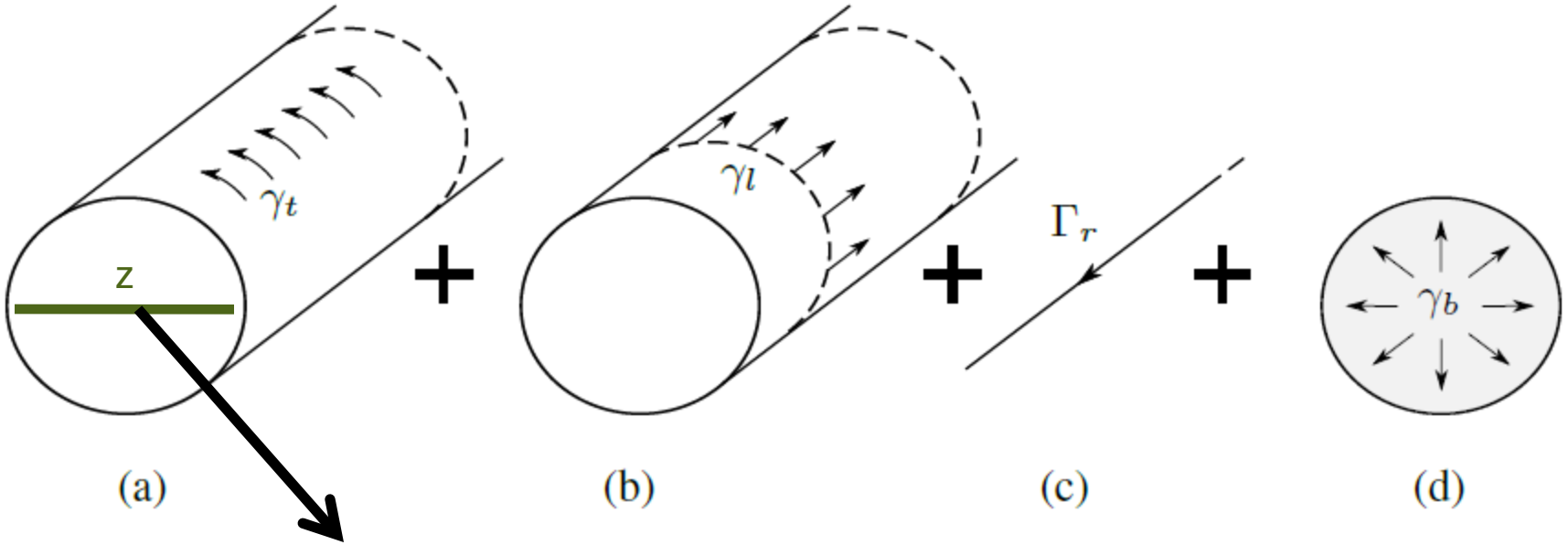
Far wake Current Study
xyz Current study
ECN (1995)

WEH
Current study
ECN(1995)

Current study

1. Presentation of the model

tilt - BEM

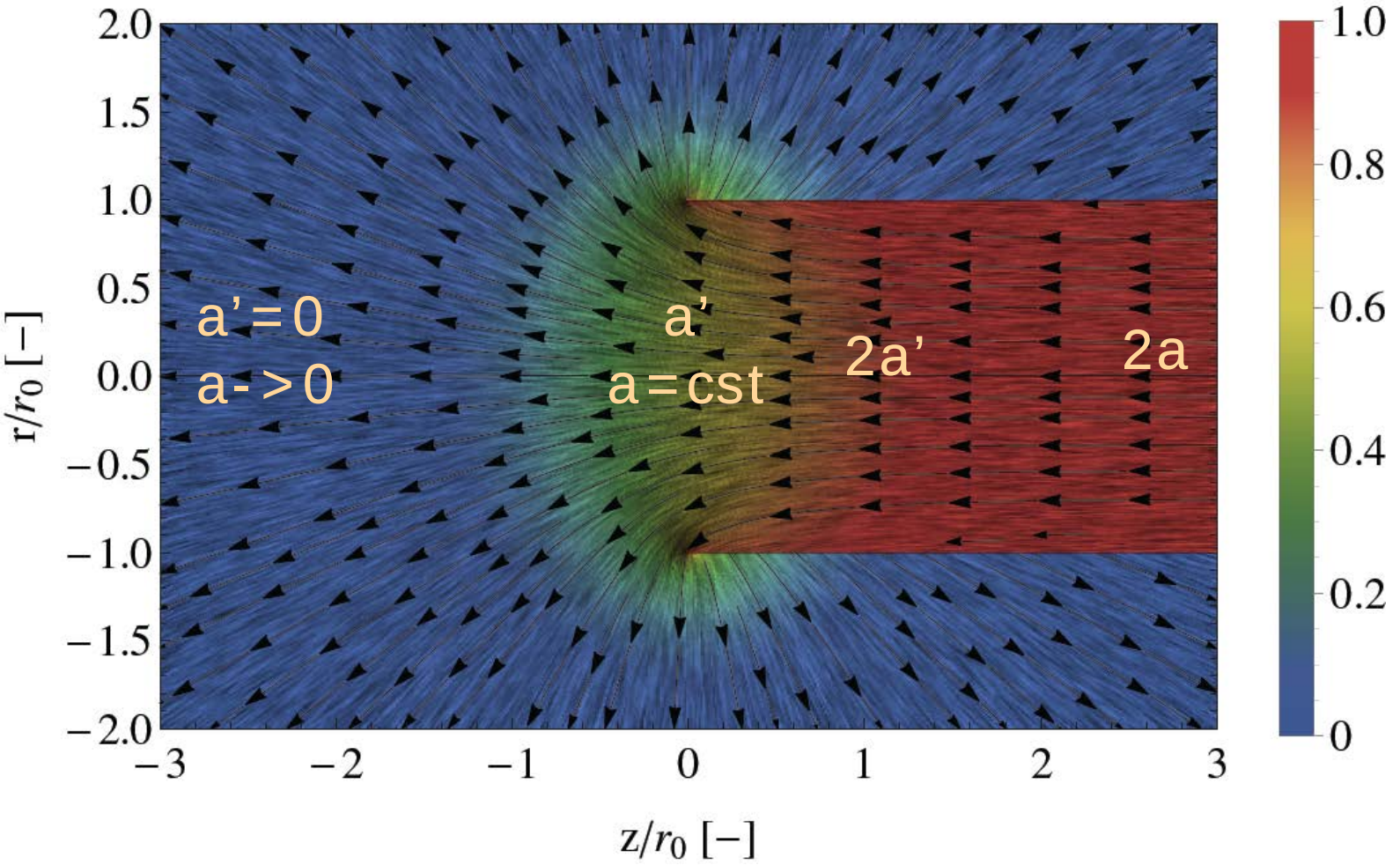


This is the basis of BEM tilt models.
Why not including the rest?

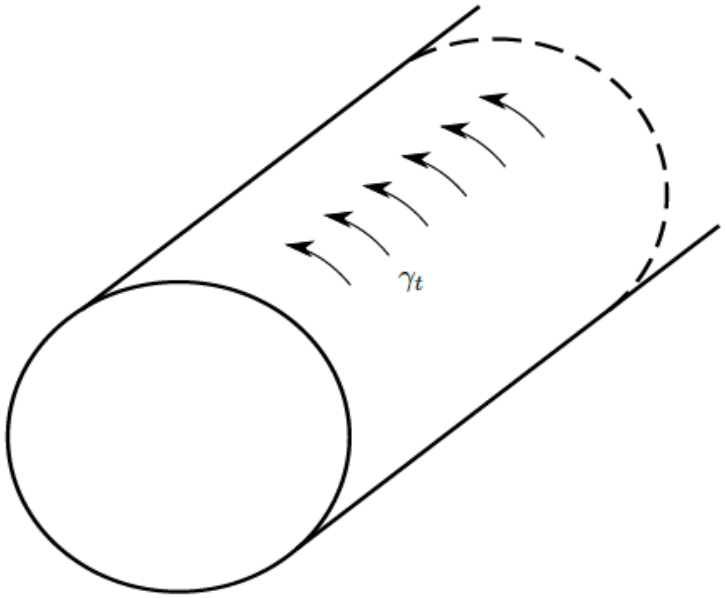
2. Few words on the straight cylinder case

2. Few words on the straight cylinder case

(tan.+longi.+root+bound)

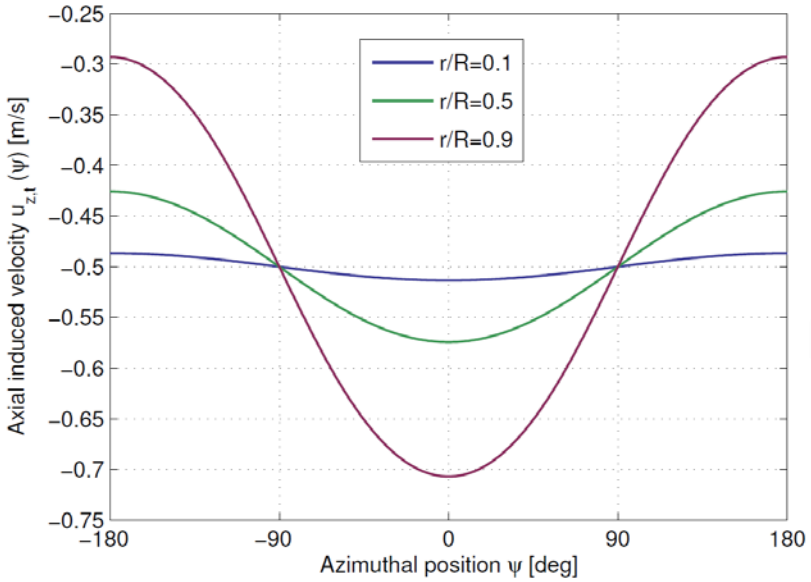


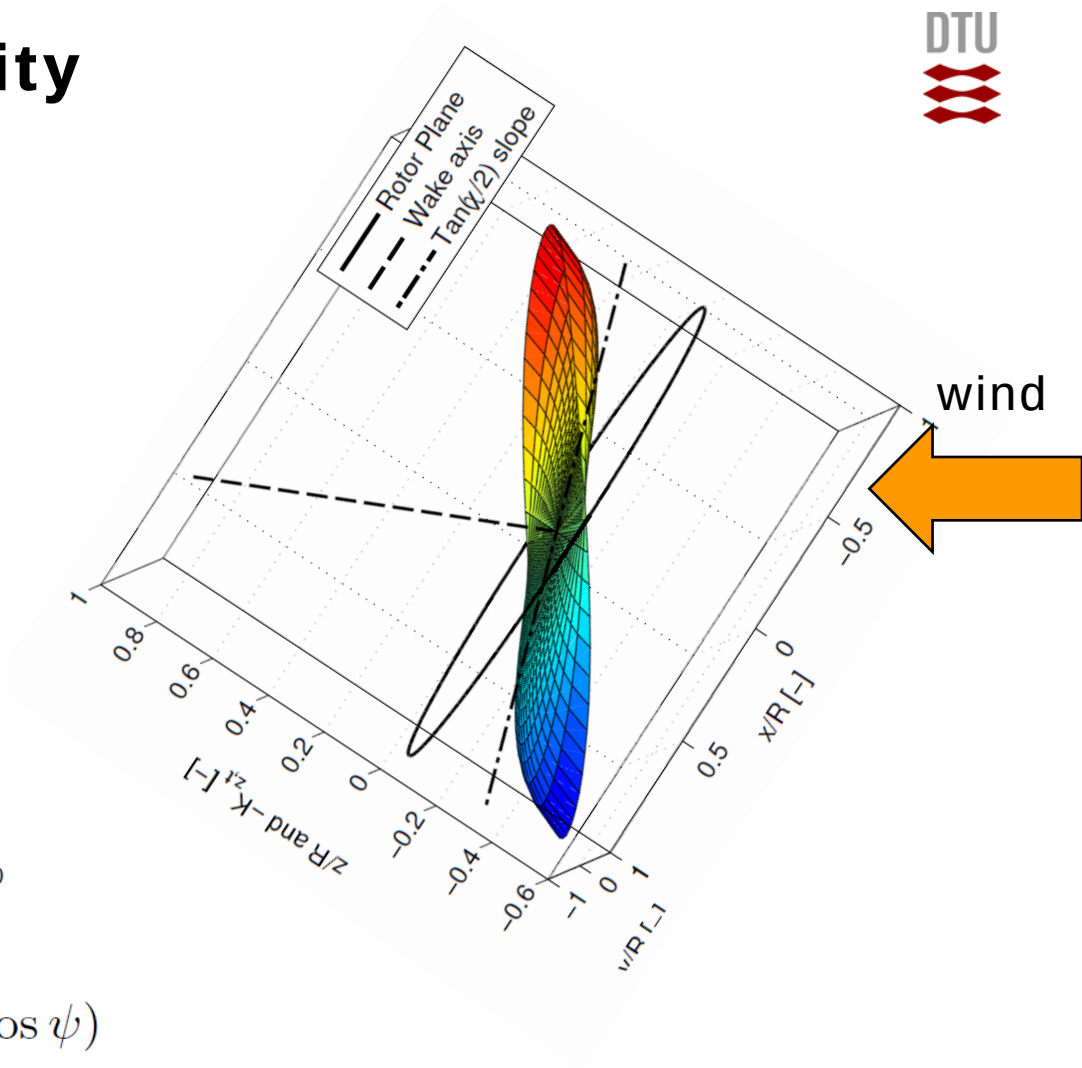
3. Tangential vorticity



3. Tangential vorticity

Axial component



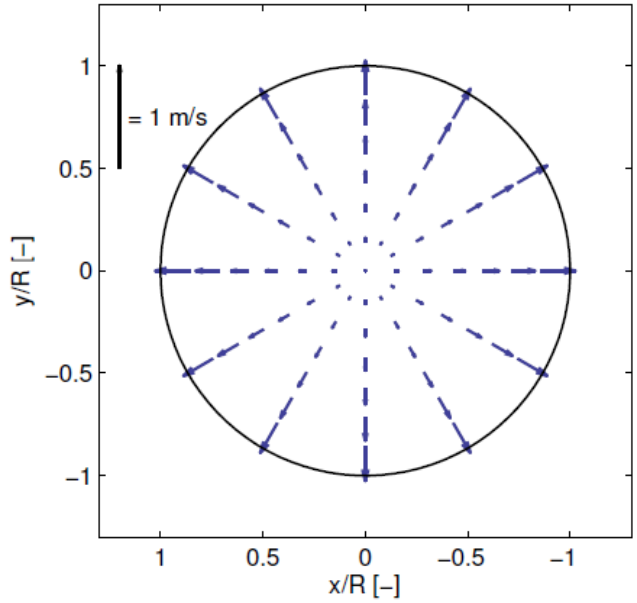


$$u_{z,t}(r,\psi,\chi) \approx u_{z,0}\left(1 + K_{z,t}(r,\chi)\cos\psi\right)$$

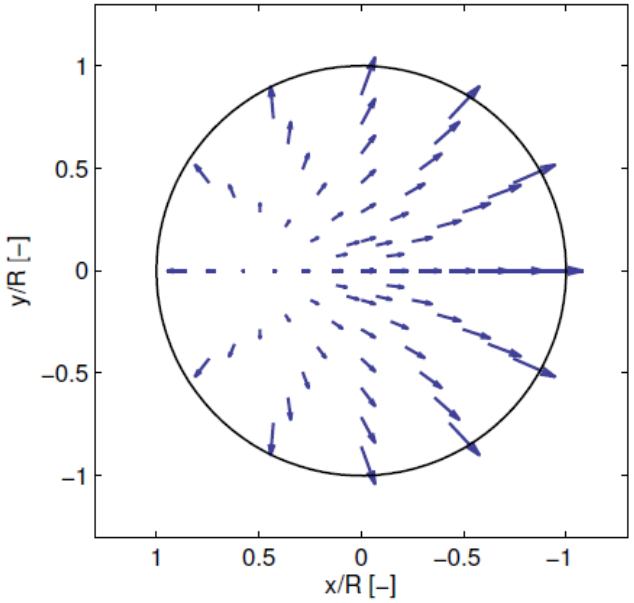
$$K_{z,t,\text{approx}} \approx \left.\frac{\partial K_{z,t}}{\partial r}\right|_{r=0} = \frac{r}{R}\tan\frac{\chi}{2}$$

3. Tangential vorticity

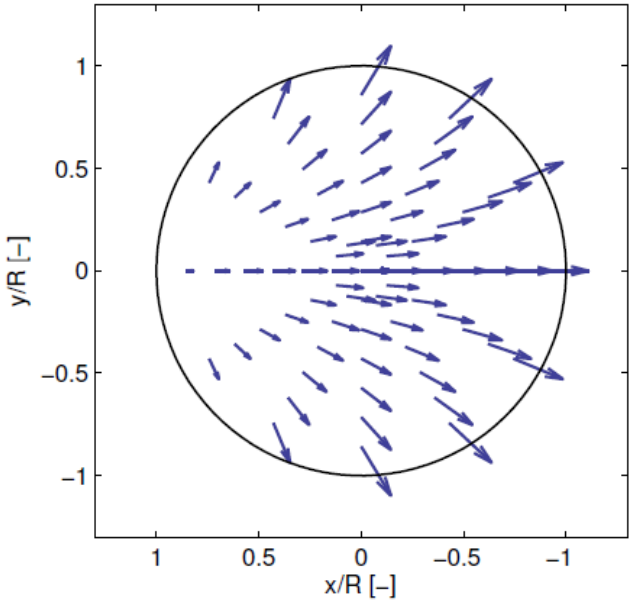
In-plane component for various skew angles



(a) $\chi = 0^\circ$



(b) $\chi = 30^\circ$



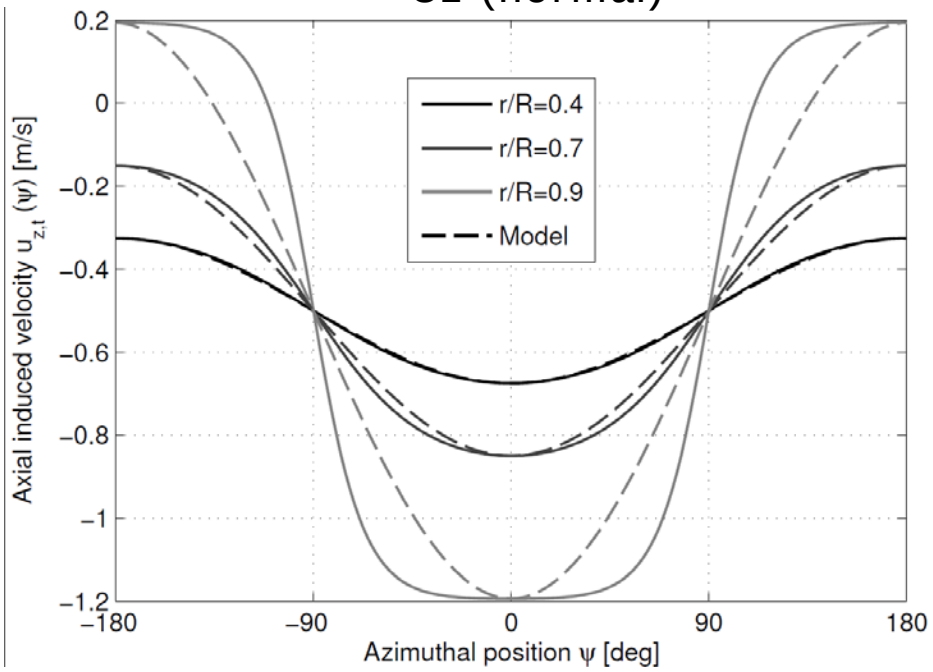
(c) $\chi = 60^\circ$

Engineering models provided using wake properties

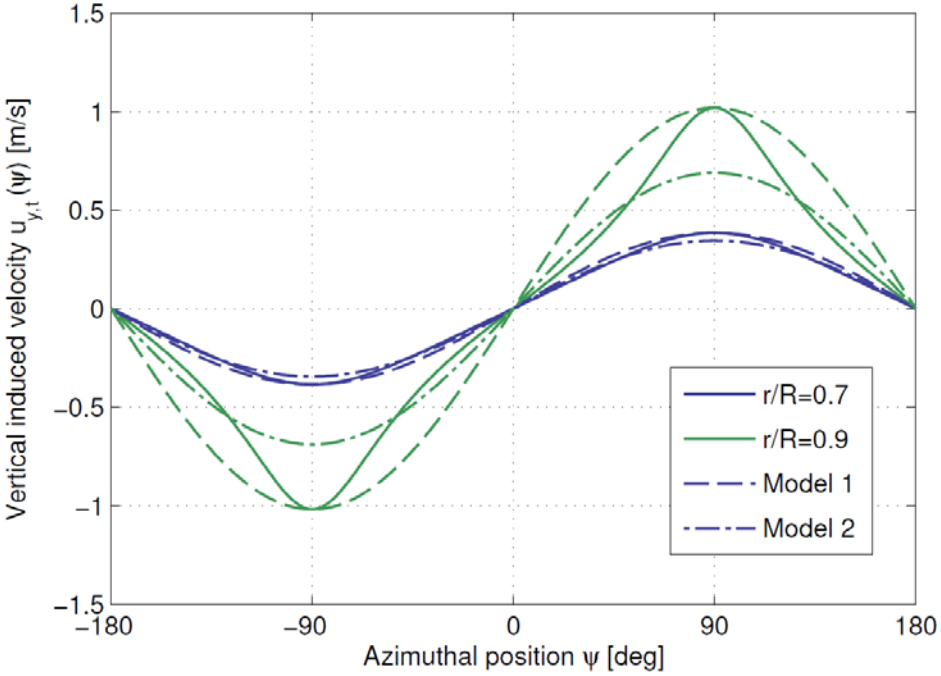
3. Tangential vorticity

Engineering models – limiting case

Uz (normal)

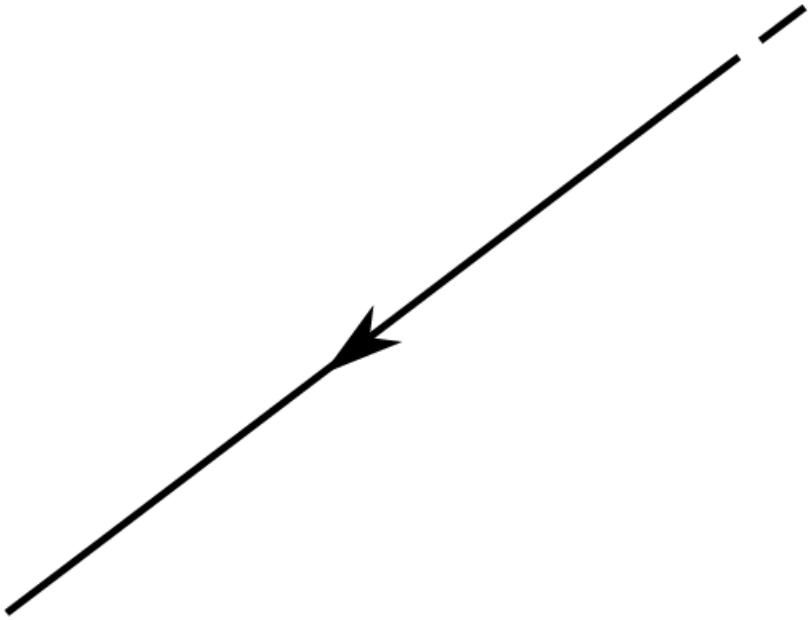


Uy (vertical)



4. Longitudinal vorticity

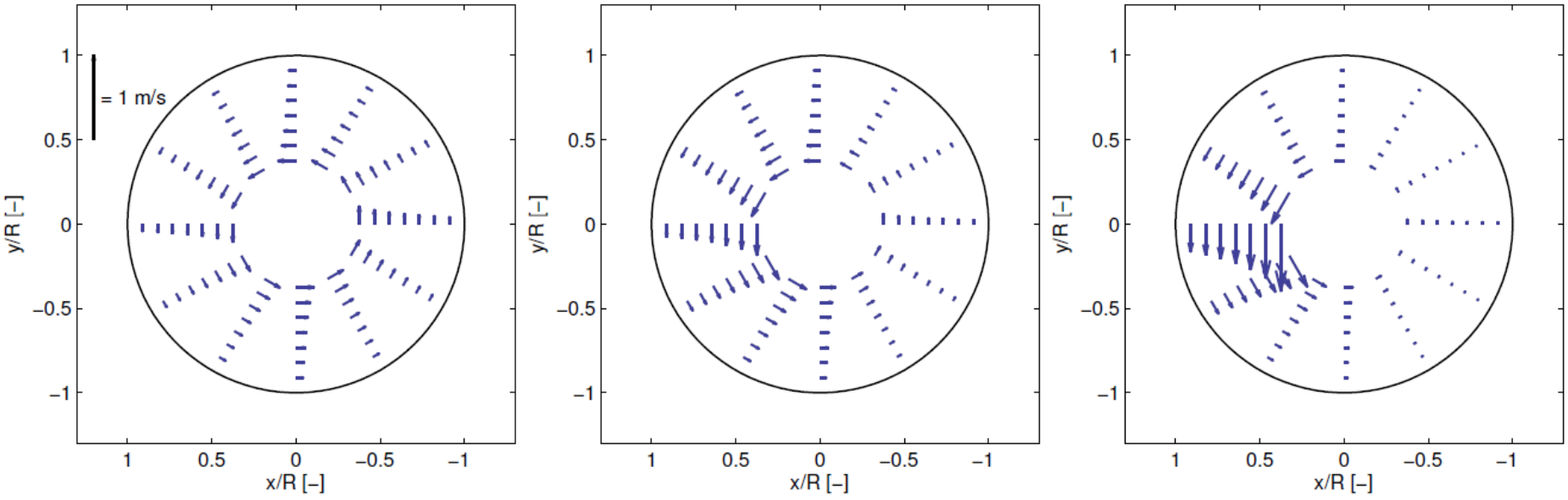
Root vortex



4. Longitudinal vorticity

Root vortex (in plane component)

$$u_{\psi,r} = \frac{\Gamma_r}{4\pi r(1 - \cos\psi \sin\chi)} \cos\chi$$

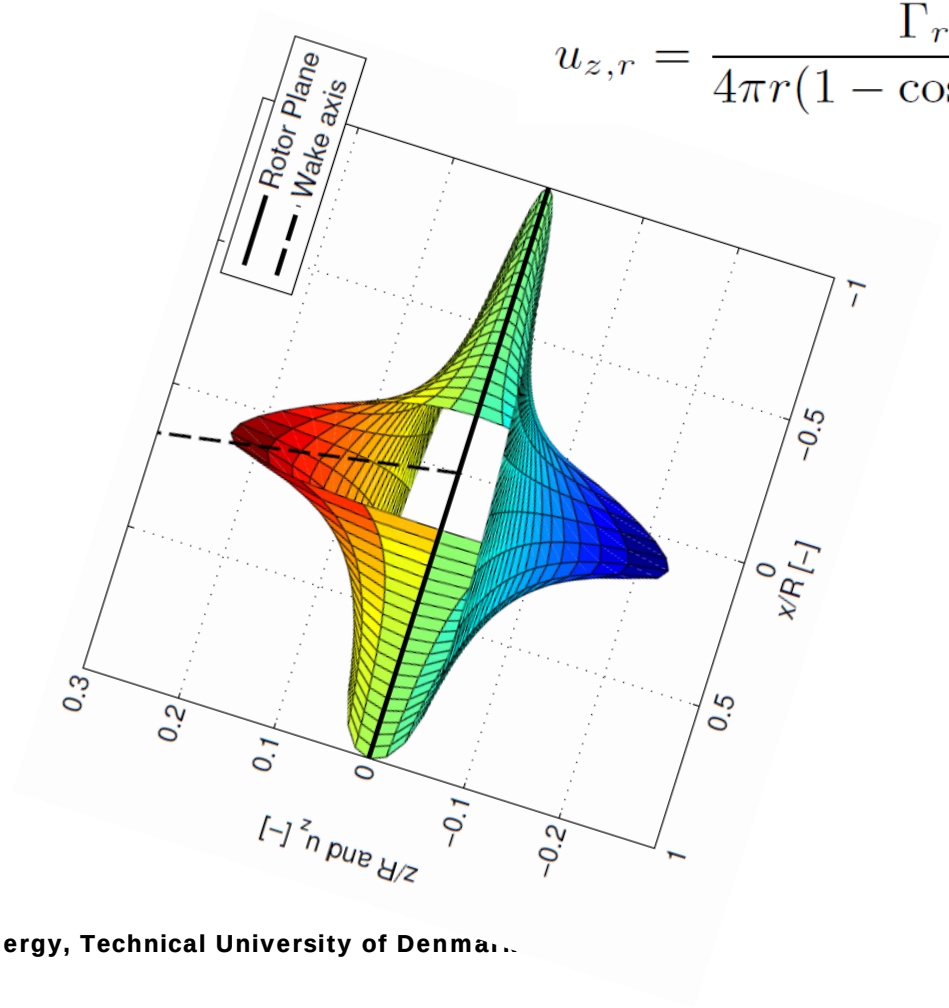


(a) $\chi = 0°$ (b) $\chi = 30°$ (c) $\chi = 60°$

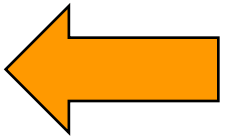
4. Longitudinal vorticity

Root vortex (normal component)

$$u_{z,r} = \frac{\Gamma_r}{4\pi r(1-\cos\psi\sin\chi)}\sin\psi\sin\chi$$

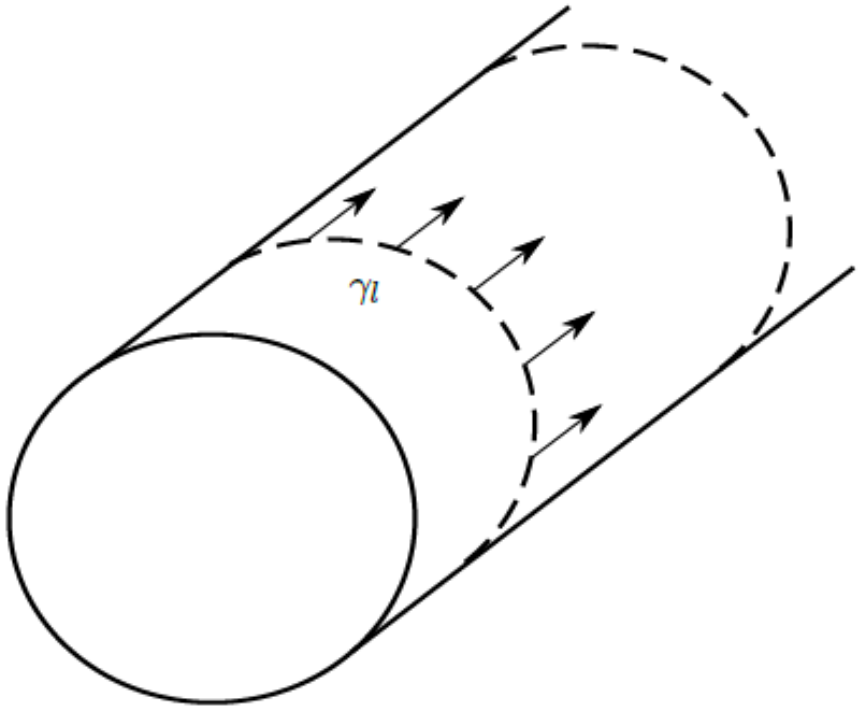


wind



4. Longitudinal vorticity

Tip-vortices

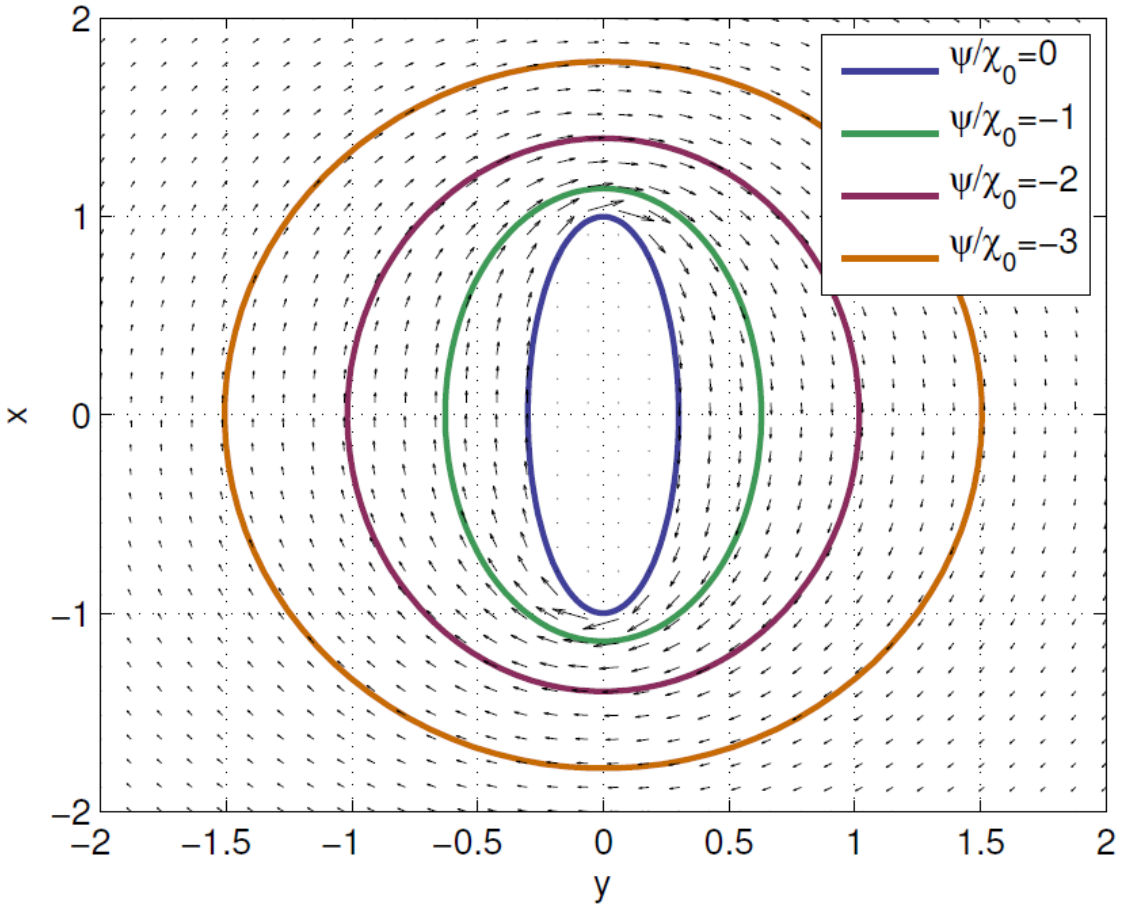


4. Longitudinal vorticity

Tip-vortices – Far Wake

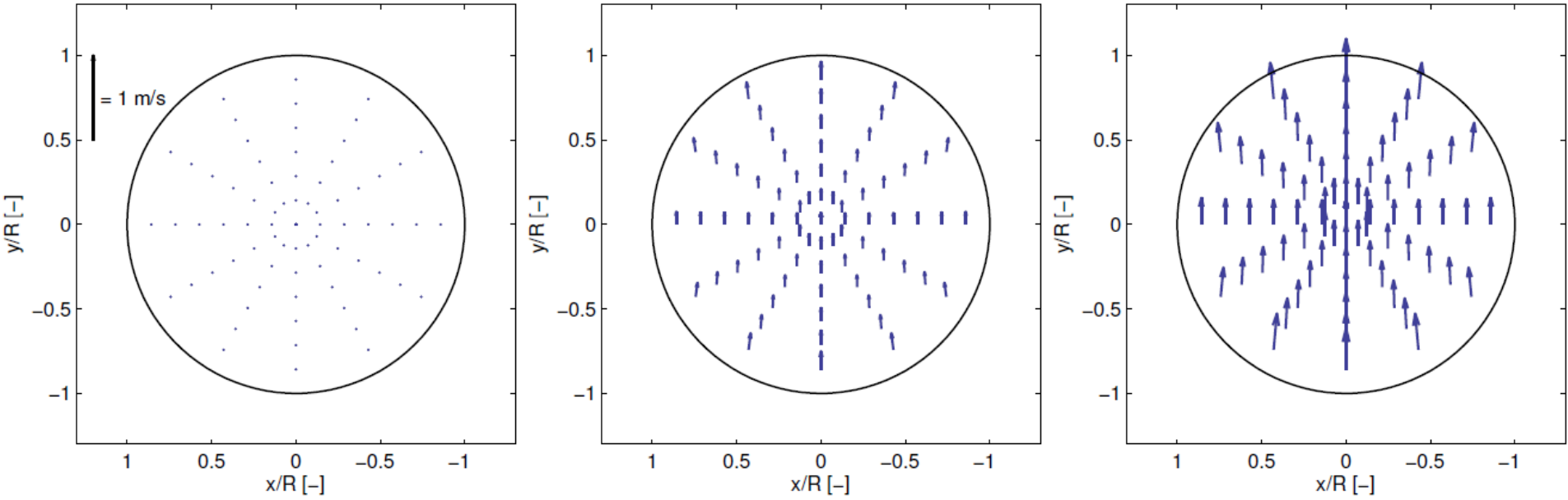
$$\frac{\partial^2 \psi}{\partial \chi^2} = -\delta(\chi - \chi_0).$$

$$\psi = -H(\chi - \chi_0) \cdot (\chi - \chi_0)$$



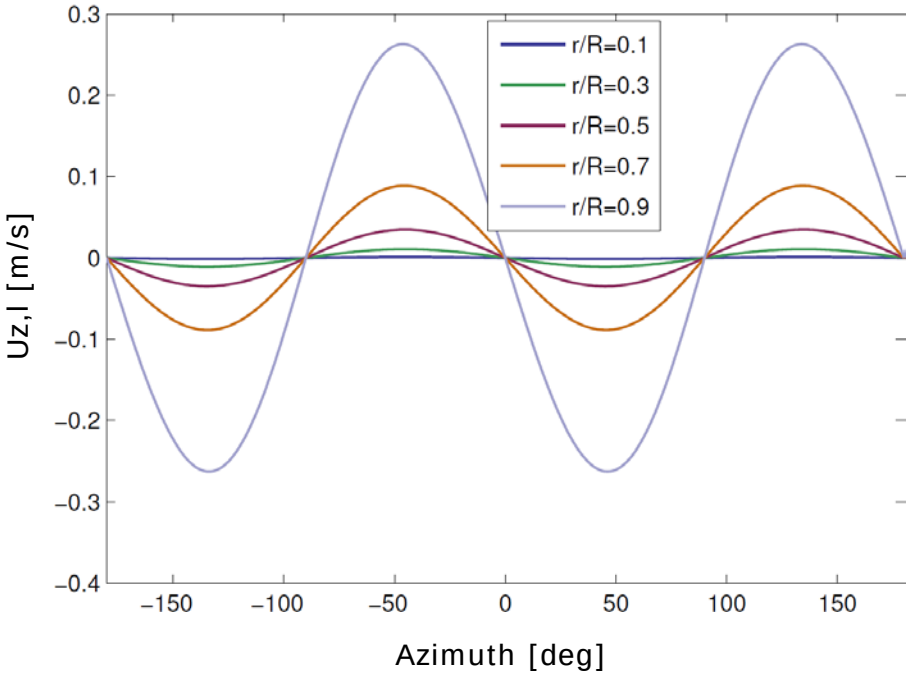
4. Longitudinal vorticity

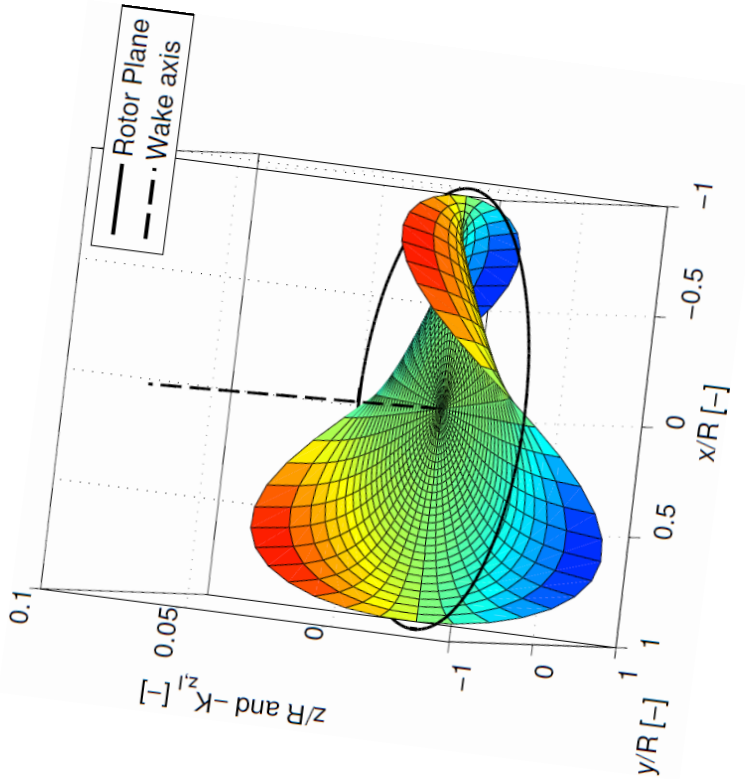
Tip-vortices – In plane component



4. Longitudinal vorticity

Tip-vortices – normal component





wind

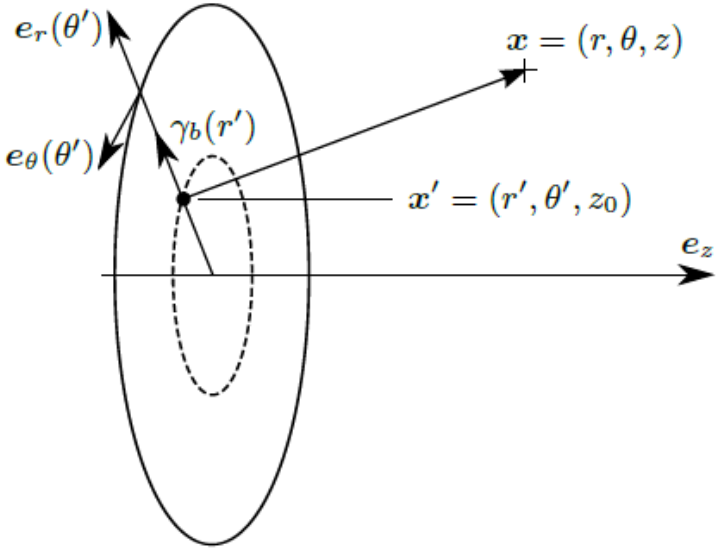
Engineering models provided

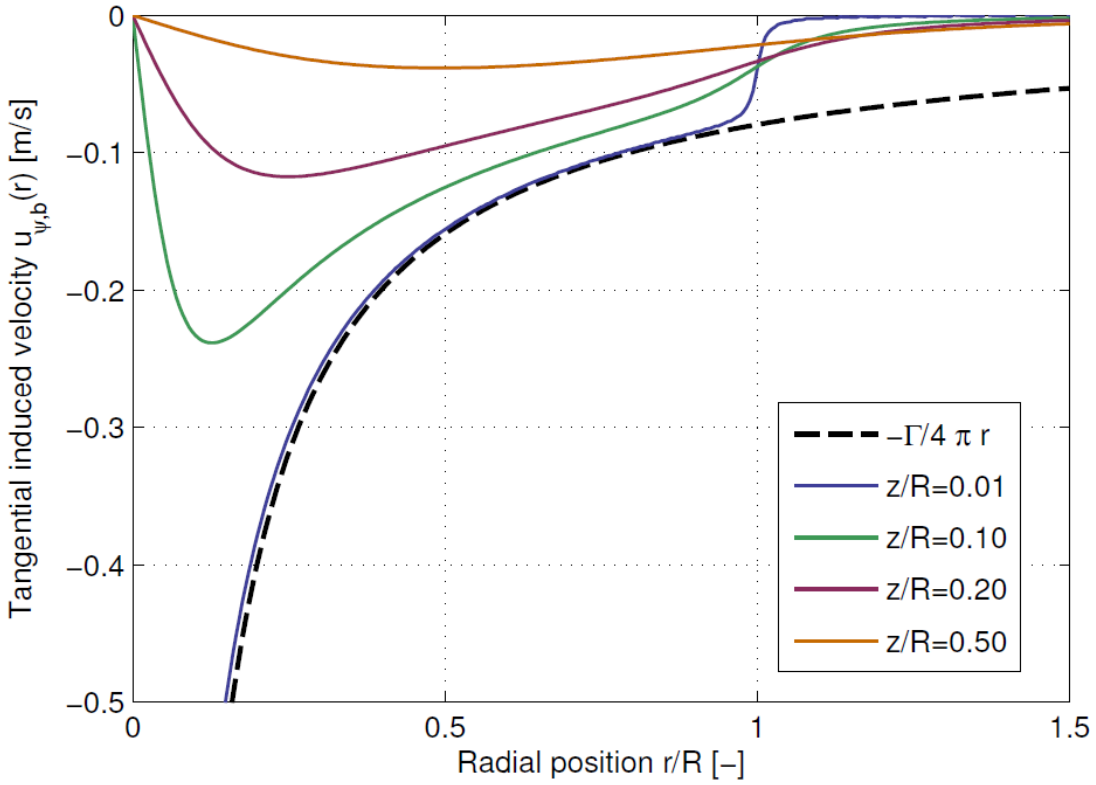
5. Bound vortex actuator disk

$$u_\theta^{r'}(\boldsymbol{x}) = -\frac{\Gamma_{\text{tot}}}{4\pi^2 z}\sqrt{\frac{r}{r'}}\frac{z^2}{r^2}\sqrt{m}\left[K(m) + T_1\,\Pi(n_1,m) - T_2\,\Pi(n_2,m)\right]$$

$$T1 = \frac{\left(\sqrt{r^2+z^2} - r\right)(r+r') - z^2}{2z^2}$$

$$T2 = \frac{\left(\sqrt{r^2+z^2} + r\right)\left(\sqrt{r^2+z^2} + r'\right)}{2z^2}$$



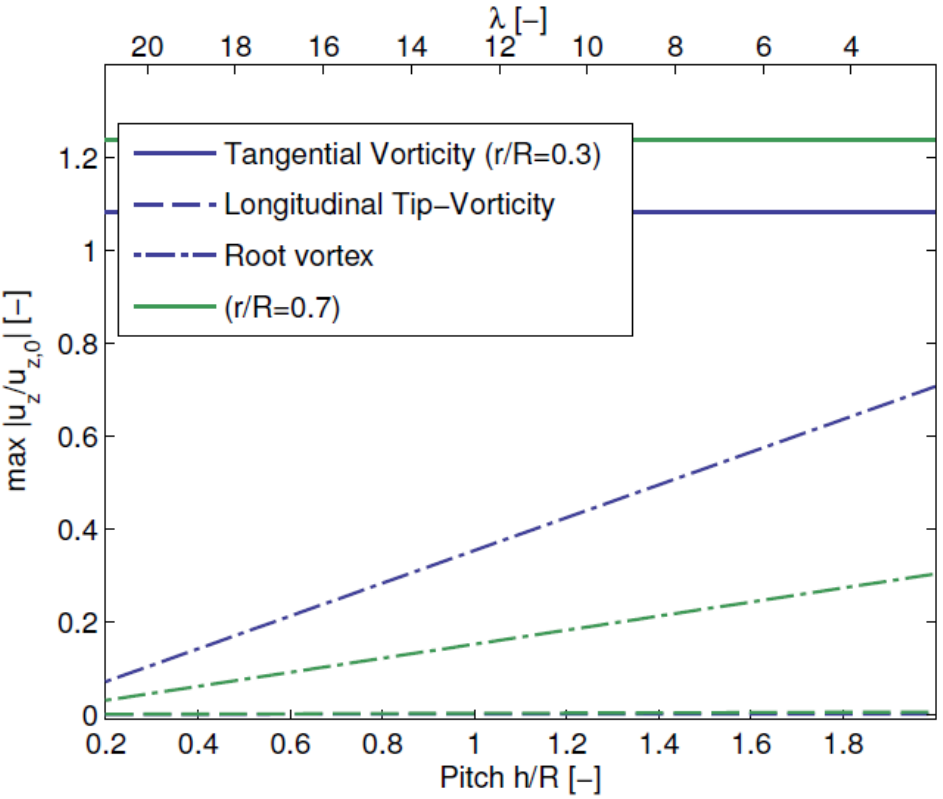


6. Putting pieces together

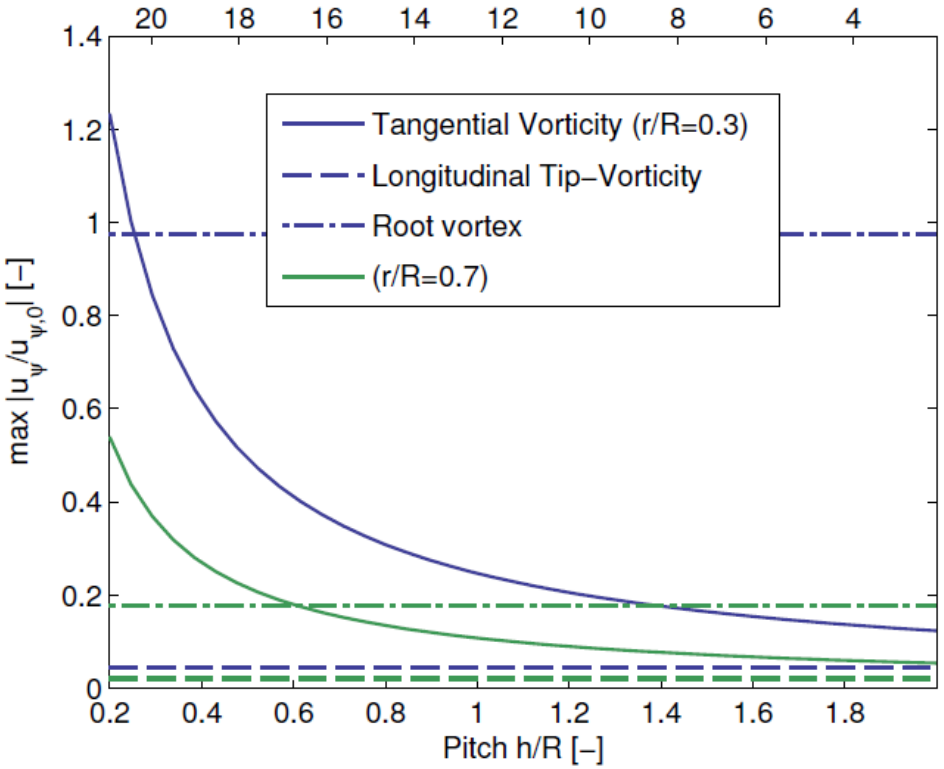
6. Putting pieces together

The most influent component

Axial

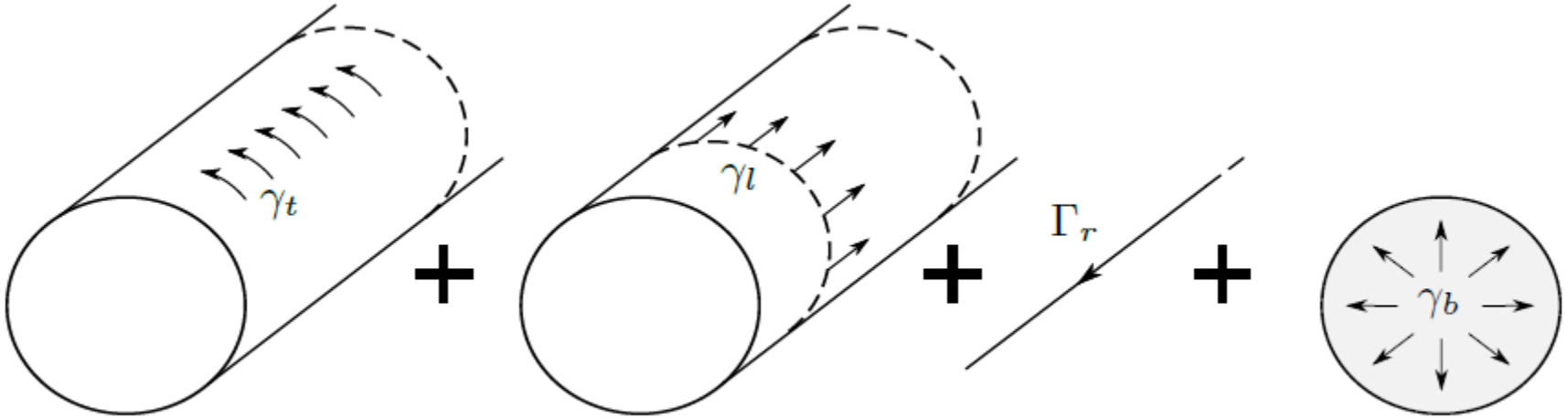


Tangential



6. Putting pieces together

Should we consider it for BEM codes?



Yes

z>psi

Partially

psi

Yes

Psi>z

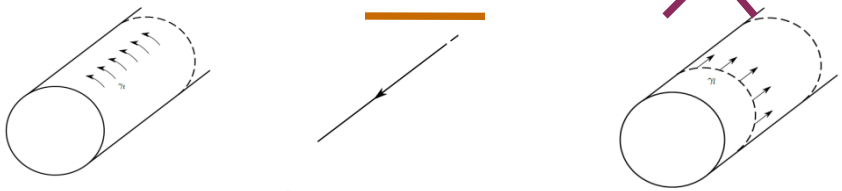
No

6. Putting pieces together

BEM implementation

$$aU_0 R_z \qquad R_z = 1 + 2F_t(r,\chi)\tan\frac{\chi}{2}\cos\psi$$

$$u_z = u_{z,t} + u_{z,r} + u_{z,l}$$



$$u_\psi = u_{\psi,t} + u_{\psi,r} + u_{\psi,l}$$

$$a'\Omega r R_\psi \qquad R_\psi = \frac{1}{1-\cos\psi\sin\chi}\cos\chi$$

Conclusions

- A new semi-analytical Yaw/Tilt model that accounts for finite tip-speed ratio
- Full velocity field from longitudinal and tangential vorticity obtained with combined analytical and numerical integration
- Simple approximations or empirical formulae can be derived for implementation in BEM codes
- Influence of longitudinal tip-vorticity is small compared to other components

Future work

- Implementation in BEM
- Comparison with free-wake vortex code and experiments
- Relaxing infinite number of blade assumption (tip-losses)
- Relaxing the constant circulation hypothesis


Thank you for your attention