A modified k-ε model applied to wind farms [1]

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Department of Wind Energy

Outline

1. Introduction
2. The **k**-ε-$\mathbf{f_P}$ model
3. Wind farms
4. Conclusions and future work

Wind turbine wakes in an off-shore wind farm

Figure : Horns Rev off-shore wind farm. Photographer: Christian Steiness

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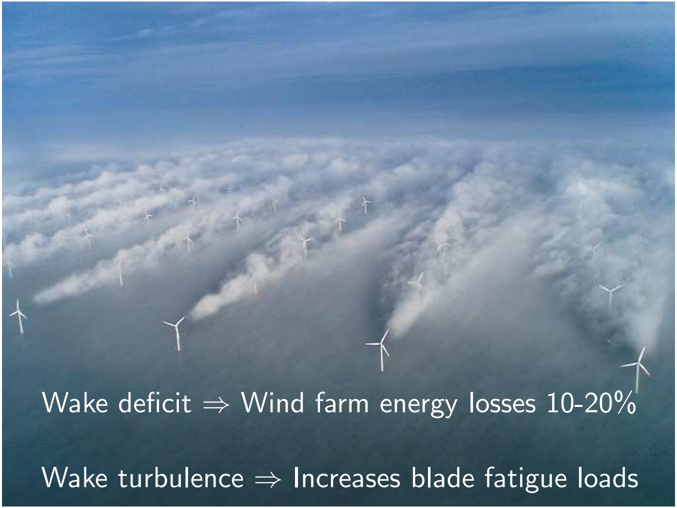
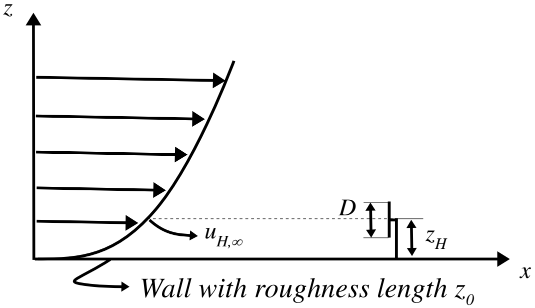


Figure : Horns Rev off-shore wind farm. Photographer: Christian Steiness

Wakes in CFD



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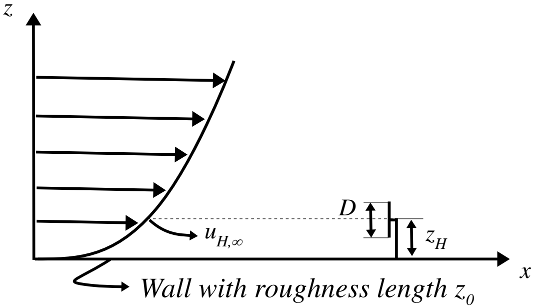

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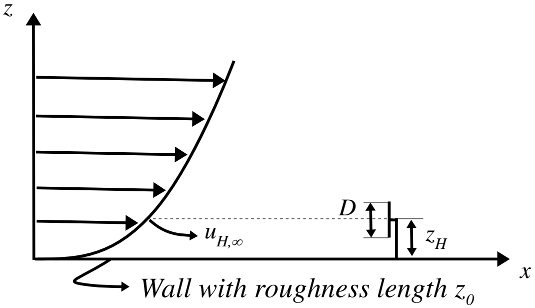

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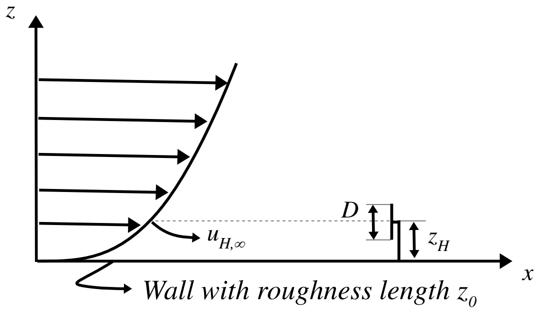
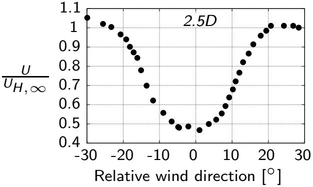


- The rotor forces are modeled with an actuator disk.
- The flow is resolved with EllipSys3D.
- The flow is driven by turbulence since $Re \sim 10^7$.

Wakes in CFD

Velocity deficit



- (•) Field measurements of Nibe wind turbine (D=40 m).

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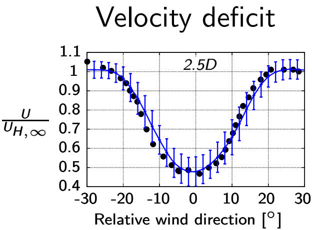

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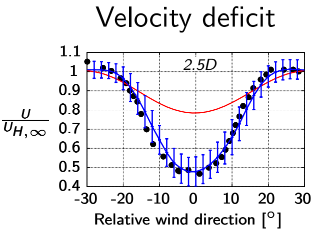

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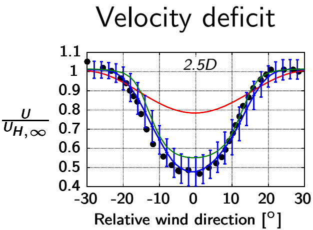

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    - (—) k-ε-f_P model.

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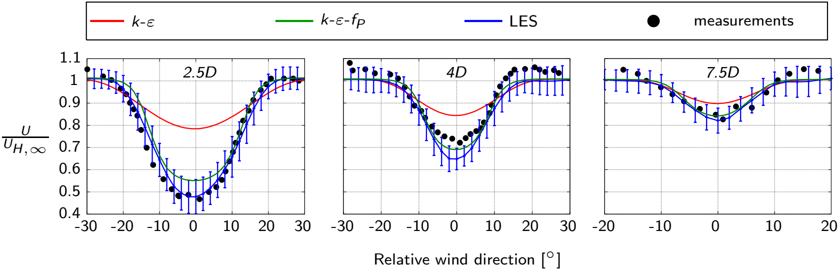
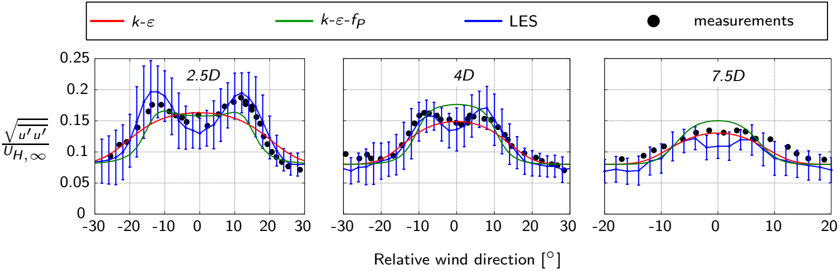


Wakes in CFD

Streamwise Reynolds-stress



The $\mathbf{k}$-ε-$\mathbf{f_P}$ model

The **k**-ε-**f$_P$** model [3].

$$\overline{u_i' u_j'} = \frac{2}{3} k \delta_{ij} - \nu_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \tag{1}$$

$$\nu_T = C_\mu f_P \frac{k^2}{\varepsilon} \tag{2}$$

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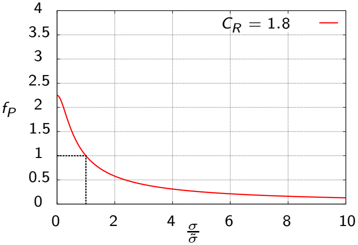
- An isotropic eddy-viscosity model with a variable C_μ.
- f_P is derived from Reynolds-stress modelling and it is based on the work of Apsley and Leschziner [2].

The k-ε-f_P model.



- σ as the shear parameter $\sigma \equiv \frac{k}{\varepsilon}\sqrt{\left(\frac{\partial U_i}{\partial x_j}\right)^2}$ and $\tilde{\sigma}$ is the shear parameter in the log law region.

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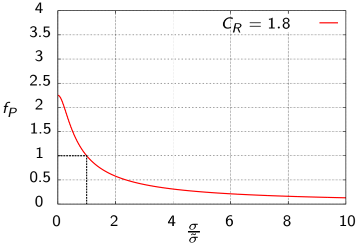

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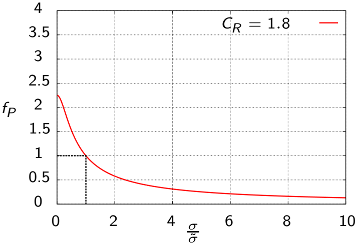

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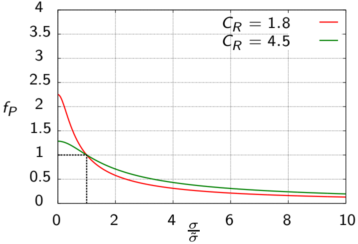

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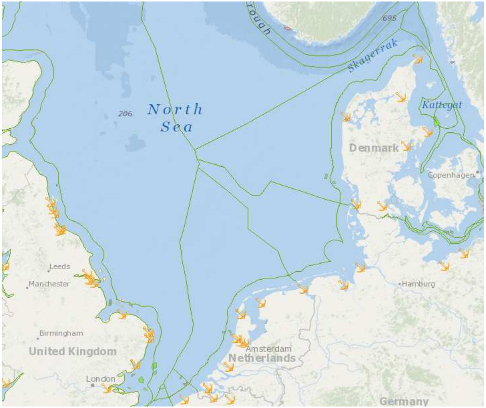
- C_R controls the behavior of f_P. From calibration with LES: $C_R = 4.5$.

Wind Farms

Test cases: two wind farms.



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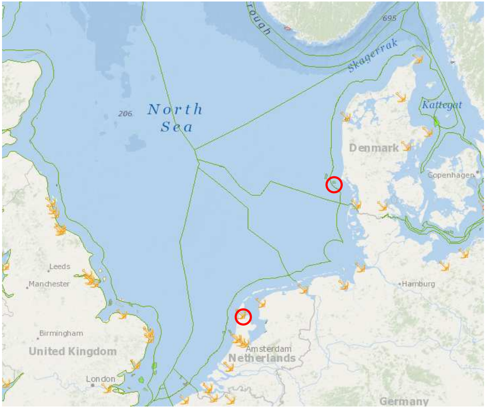
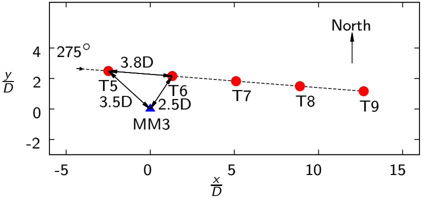
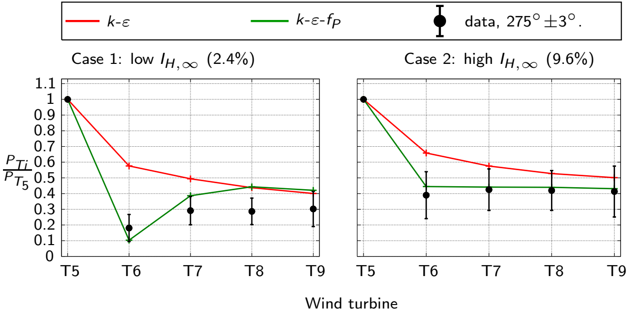


- Wieringermeer: Dutch on-shore wind farm, 5×2.5 MW.
- Horns Rev: Danish off-shore wind farm, 80×2 MW.

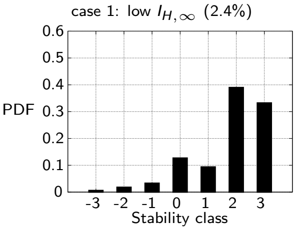
Wieringermeer wind farm: layout.

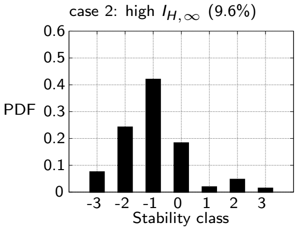


Wieringermeer wind farm: power deficit.



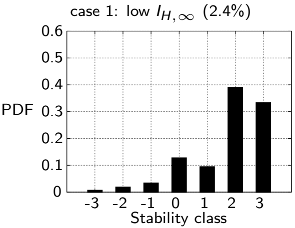
Wieringermeer wind farm: atmospheric stability.

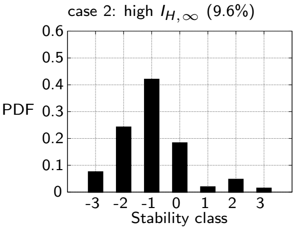




Class	Description
-3	Very unstable
	$\updownarrow$
0	Neutral
	$\updownarrow$
3	Very stable

Wieringermeer wind farm: atmospheric stability.





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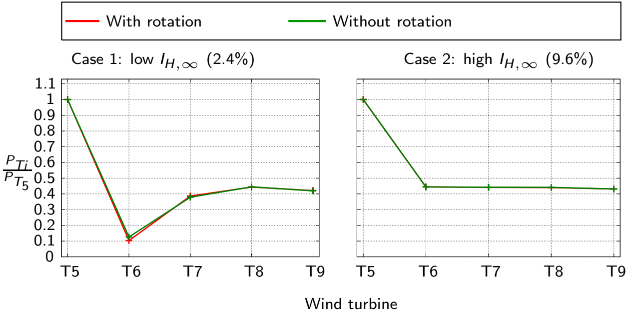
- Stable atmospheric stability cannot be modelled by simply lowering the turbulence intensity!

Wieringermeer wind farm: effect of wake rotation.



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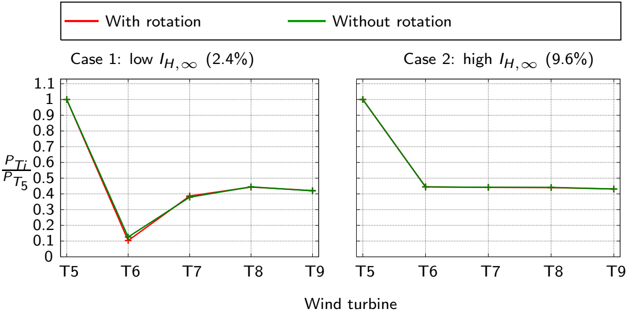

- Effect of wake rotation on power deficit is negligible.

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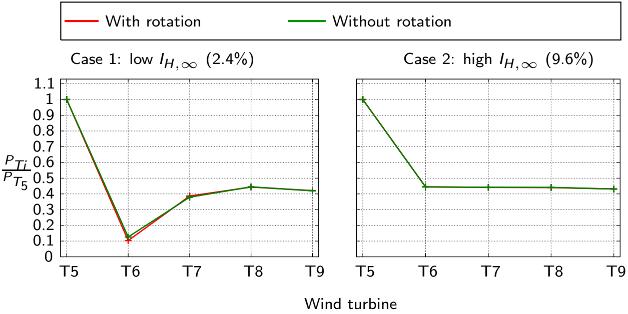
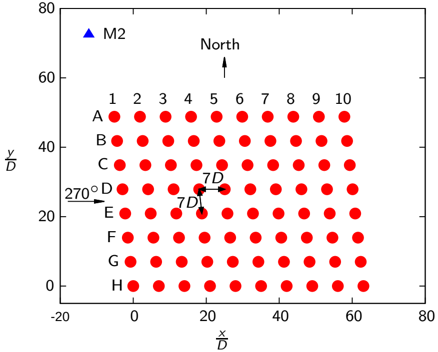
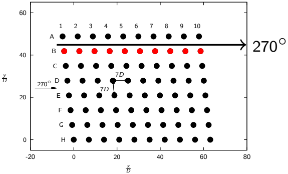


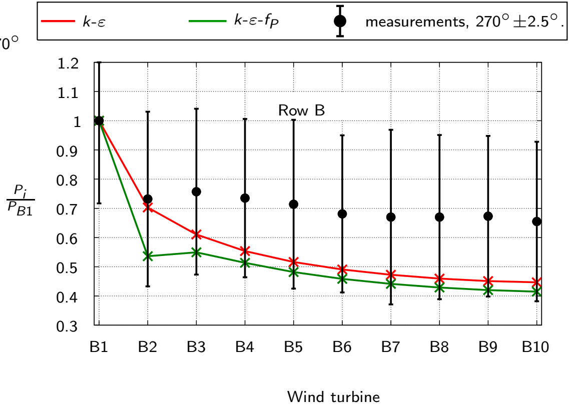
- Effect of wake rotation on power deficit is negligible.
- In contradiction with Wu and Porté-Agel [4], but they used a different AD force methods, for the wind farm simulations with and without wake rotation.

Horns Rev wind farm: layout.

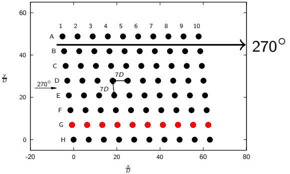


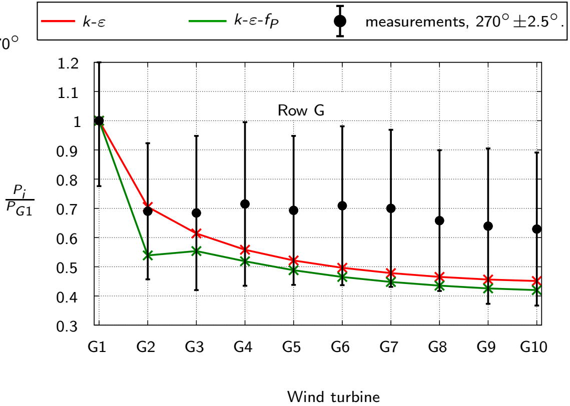
Horns Rev wind farm: power deficit in row B.





Horns Rev wind farm: power deficit in row G.





What is going on in the Horns Rev wind farm?

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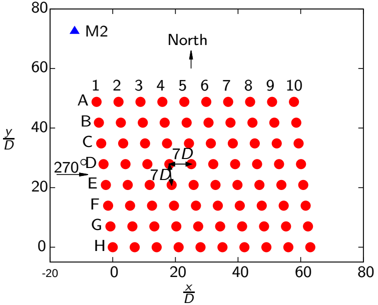
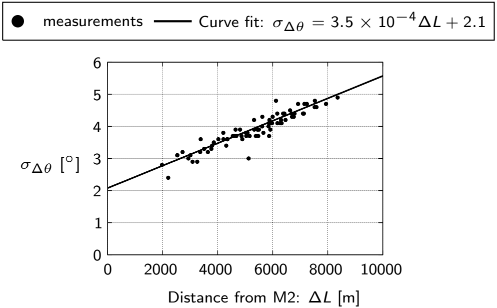
Answer $\Rightarrow$ wind direction uncertainty of the measurements caused by:

- Spatial decorrelation of the reference wind direction measurement and power measurements.
- Using wind turbine yaw sensors to determine as wind direction measurements.
- The change in wind direction due to large scale turbulence that is statistically not well represented within ten minute averages.

Wind direction uncertainty in the Horns Rev wind farm.

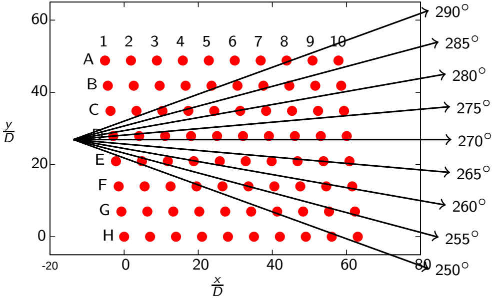




$$\Delta\theta_{\mathrm{i}} = \theta_{\mathrm{yaw,i}} - \theta_{\mathrm{M2,i}} \tag{3}$$

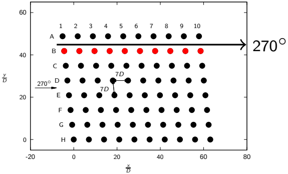
Including the wind direction uncertainty as a Gaussian average.

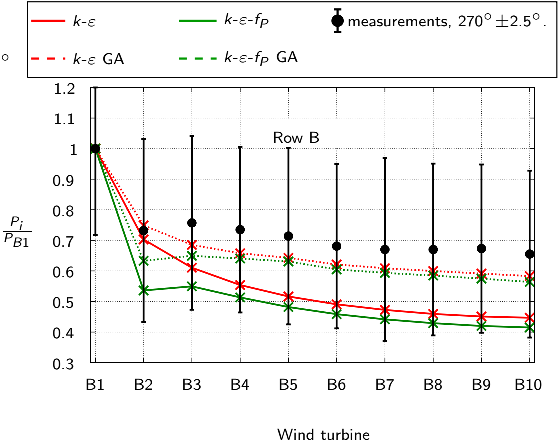
Simulate multiple wind directions on an interval $\pm 3\sigma$:



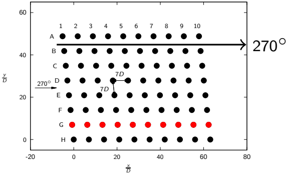
$$\text{Power} = \sum_{i=1}^{N} \text{Power}(\theta_i) \times \text{probability}(\theta_i) \quad (4)$$

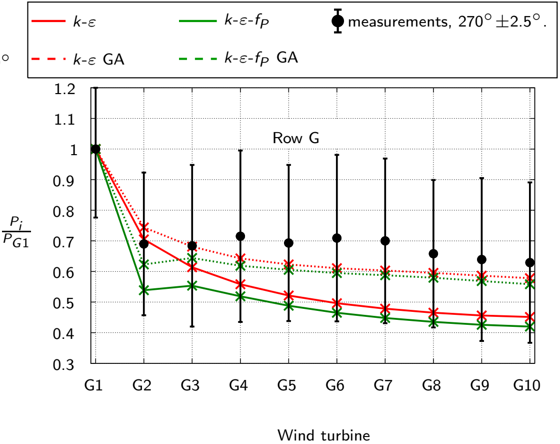
Horns Rev wind farm: power deficit in row B including wind direction uncertainty.





Horns Rev wind farm: power deficit in row G including wind direction uncertainty.





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  - The improvements are realized by introducing a variable C_μ that decreases the eddy-viscosity behind the wind turbine and delays the wake recovery.
  - The k-ε-f_P model does not improve the wake turbulence significantly because it is an isotropic turbulence model.

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  - The k-ε-f_P model compares better with the measurements than the standard k-ε model, especially for closely spaced wind turbines.
  - Both RANS models calculate similar power deficits at the third/fourth wind turbine in a row. Hence, the standard k-ε model may not be a bad model choice if the anual energy production of large wind farms needs to be calculated.

Future work

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- Further investigate measurements uncertainty in wind farms.

Bibliography I

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