Vortex methods for wind turbines aerodynamics and aeroelasticity

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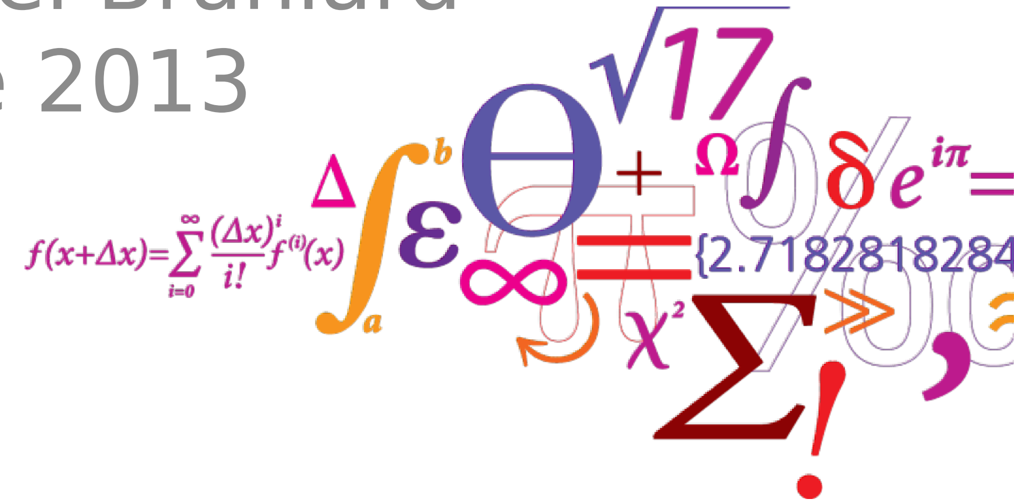
June 2013



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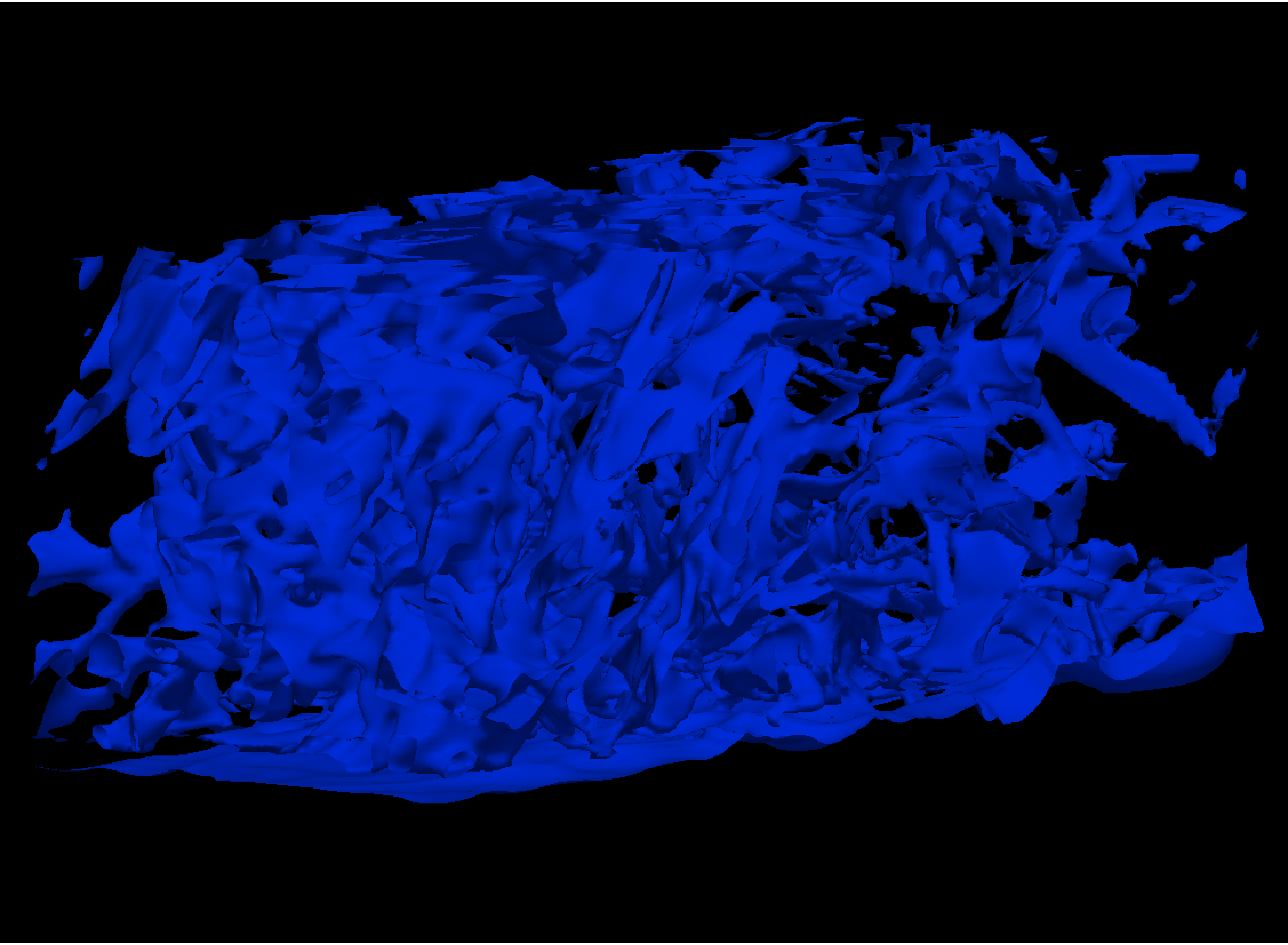
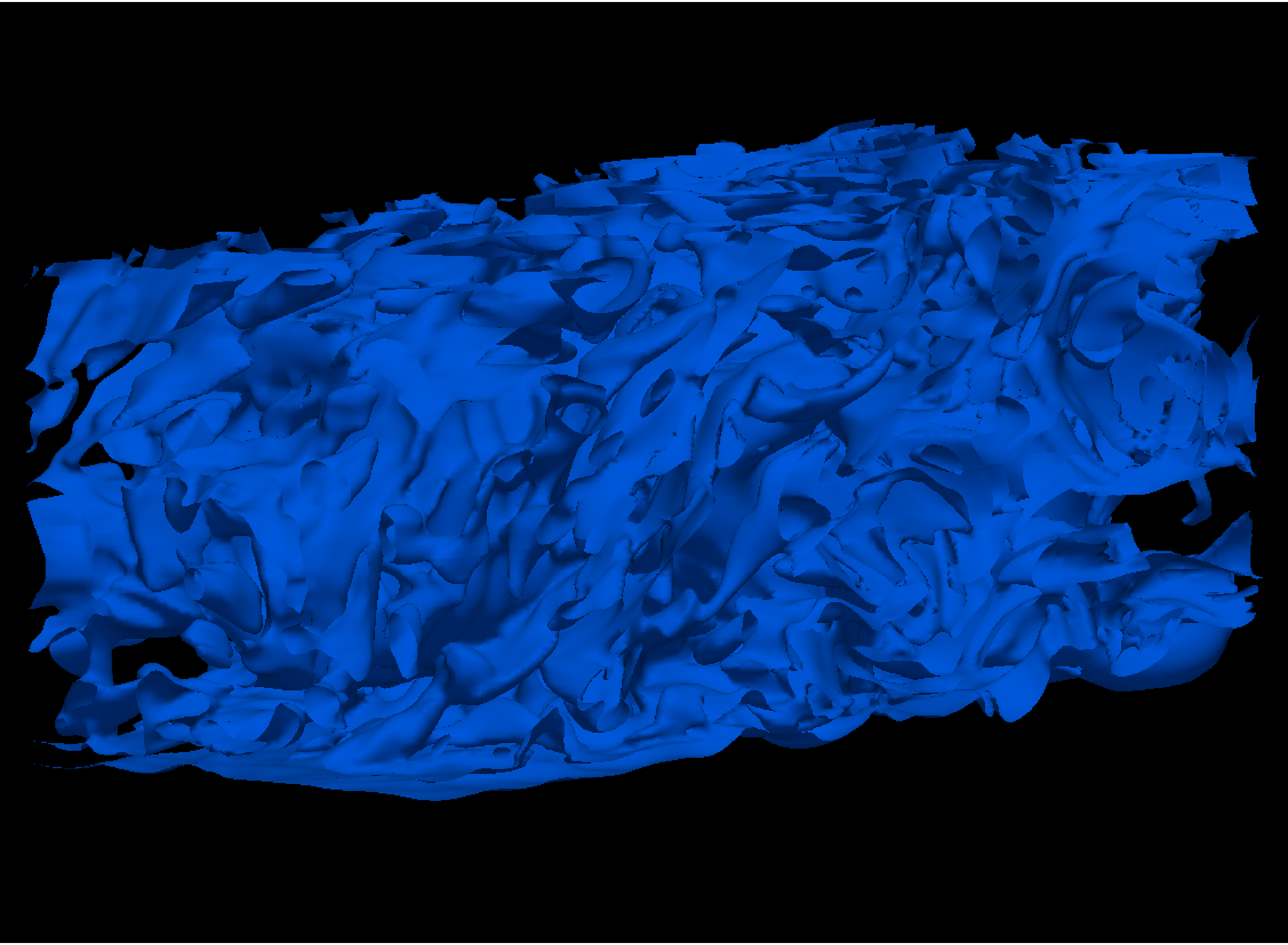
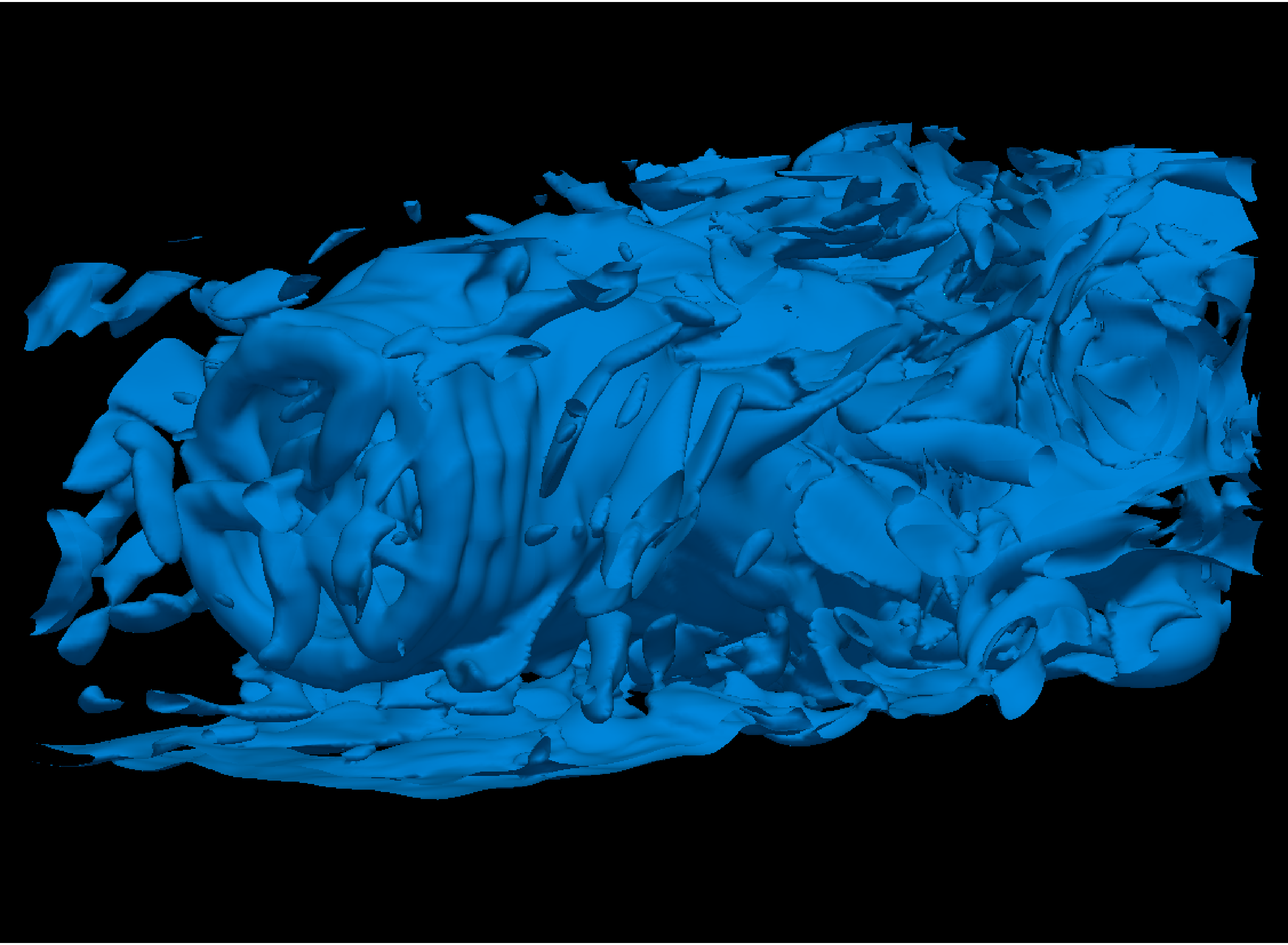
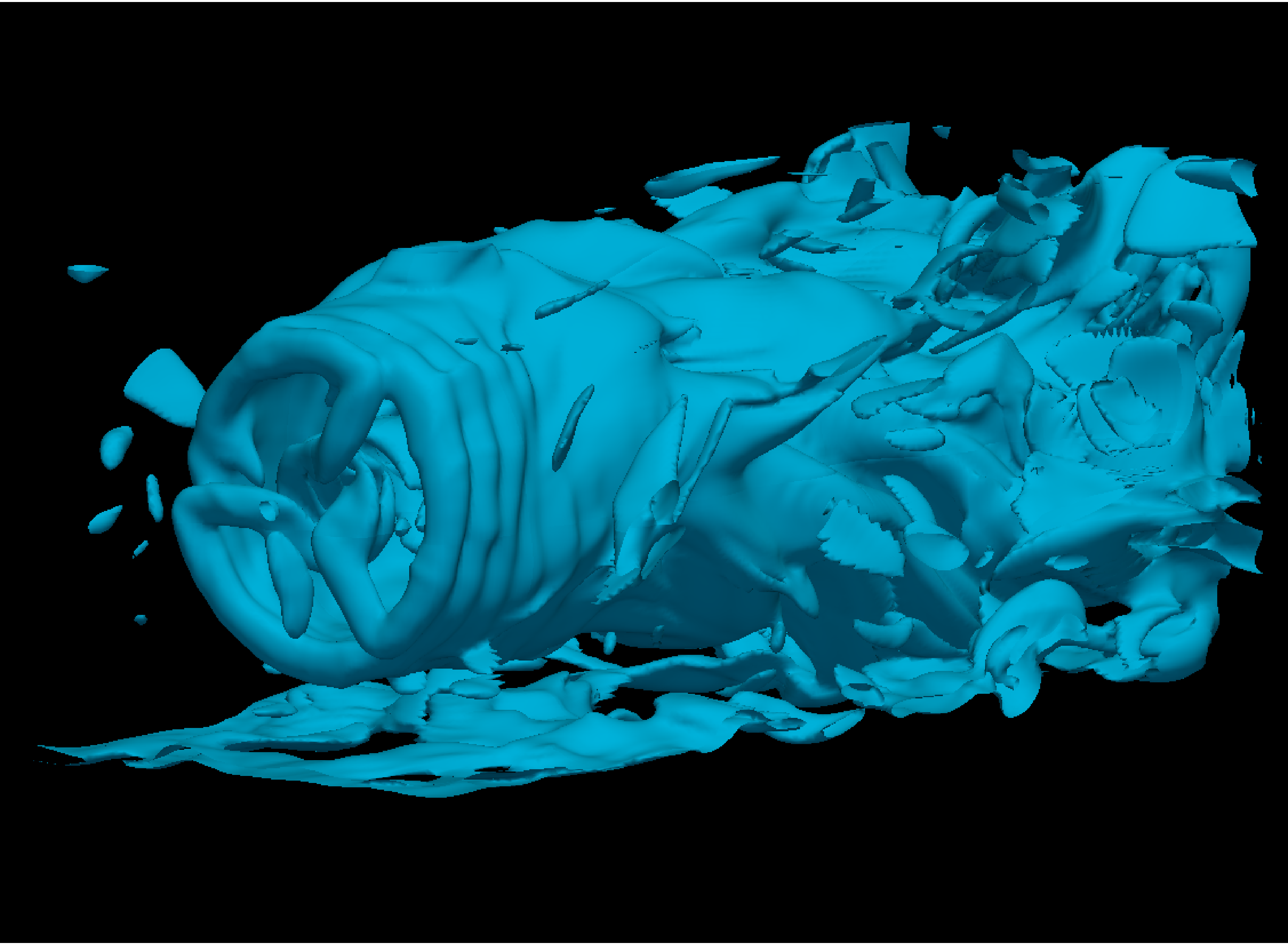
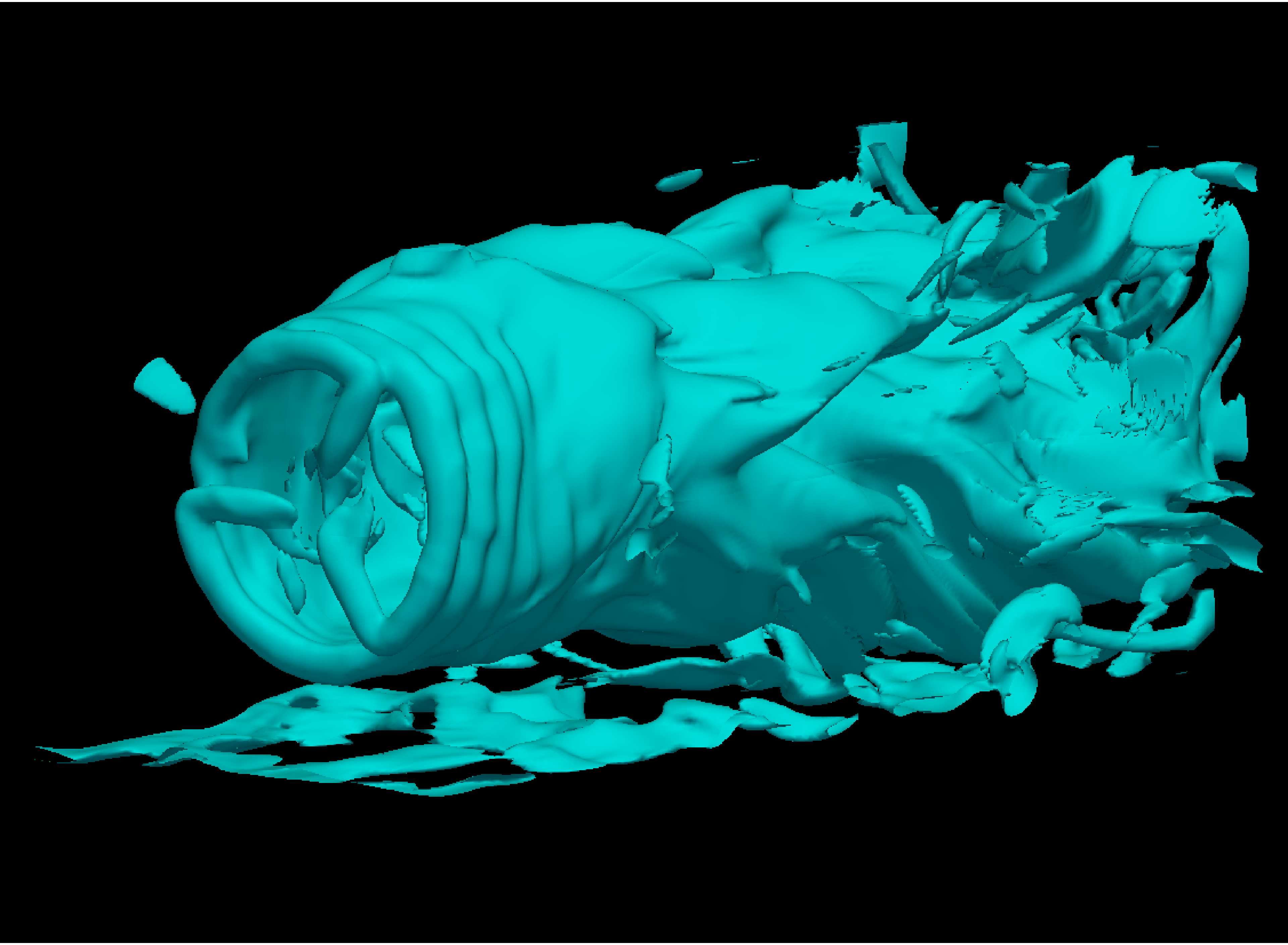
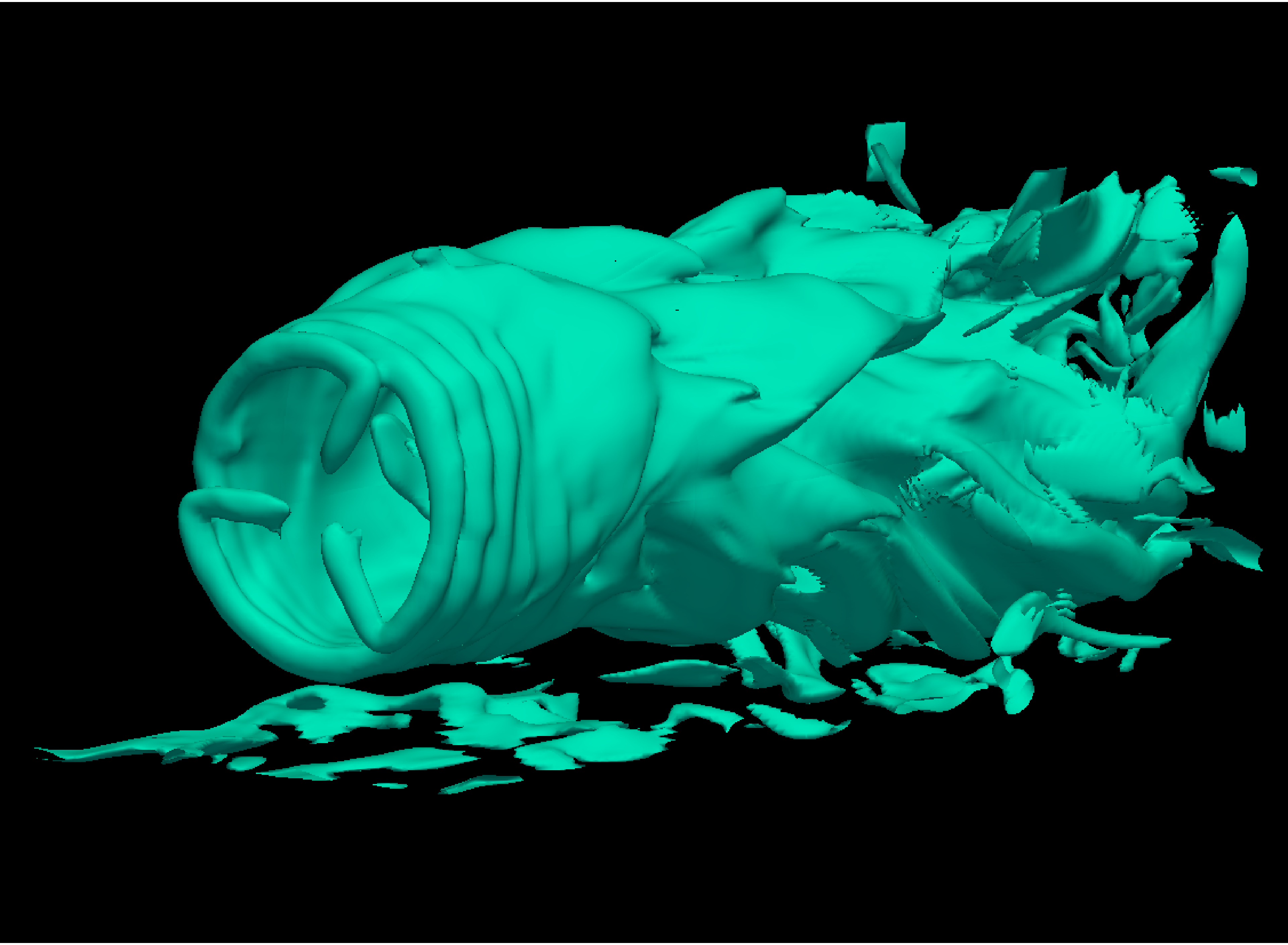


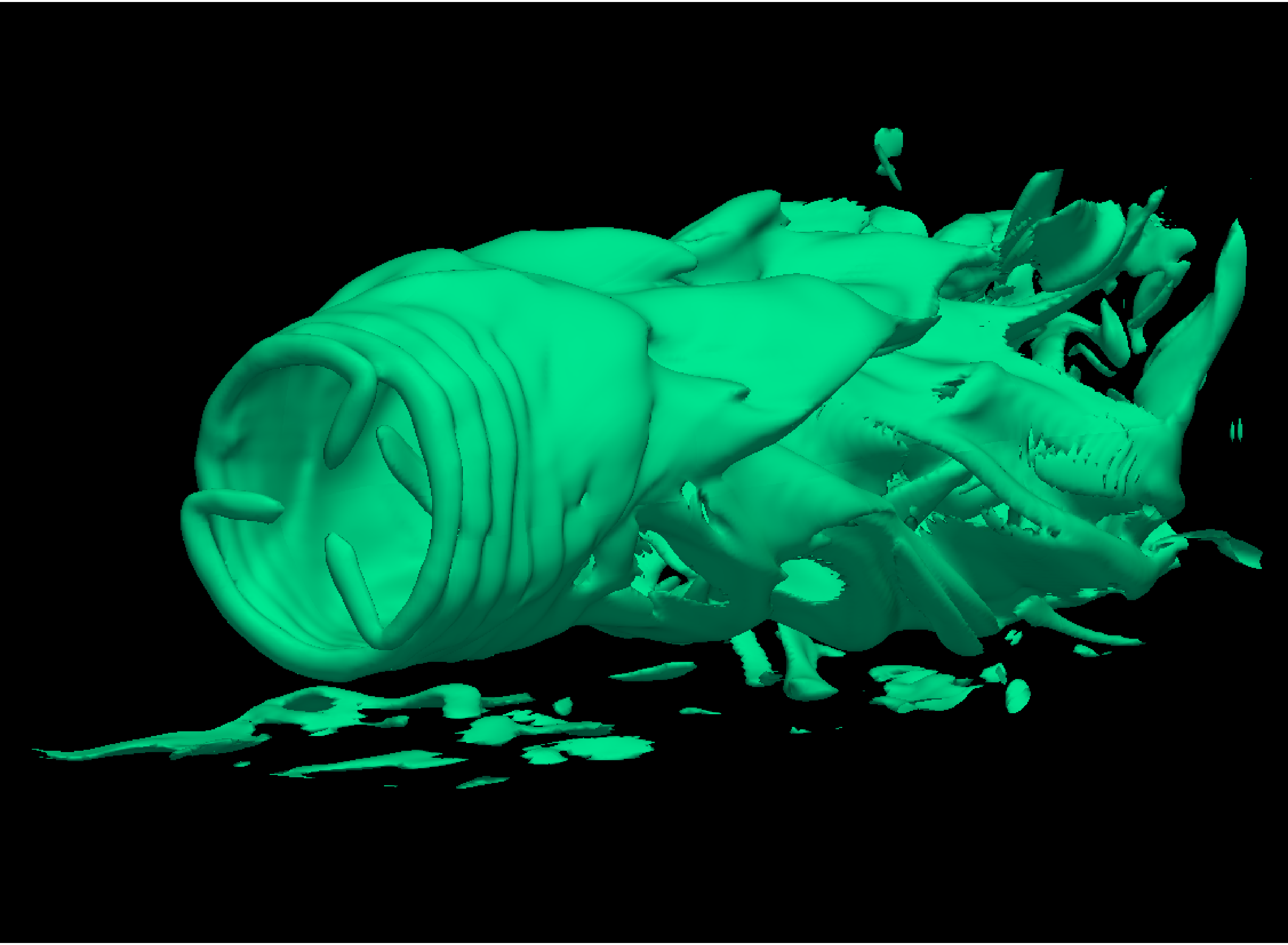


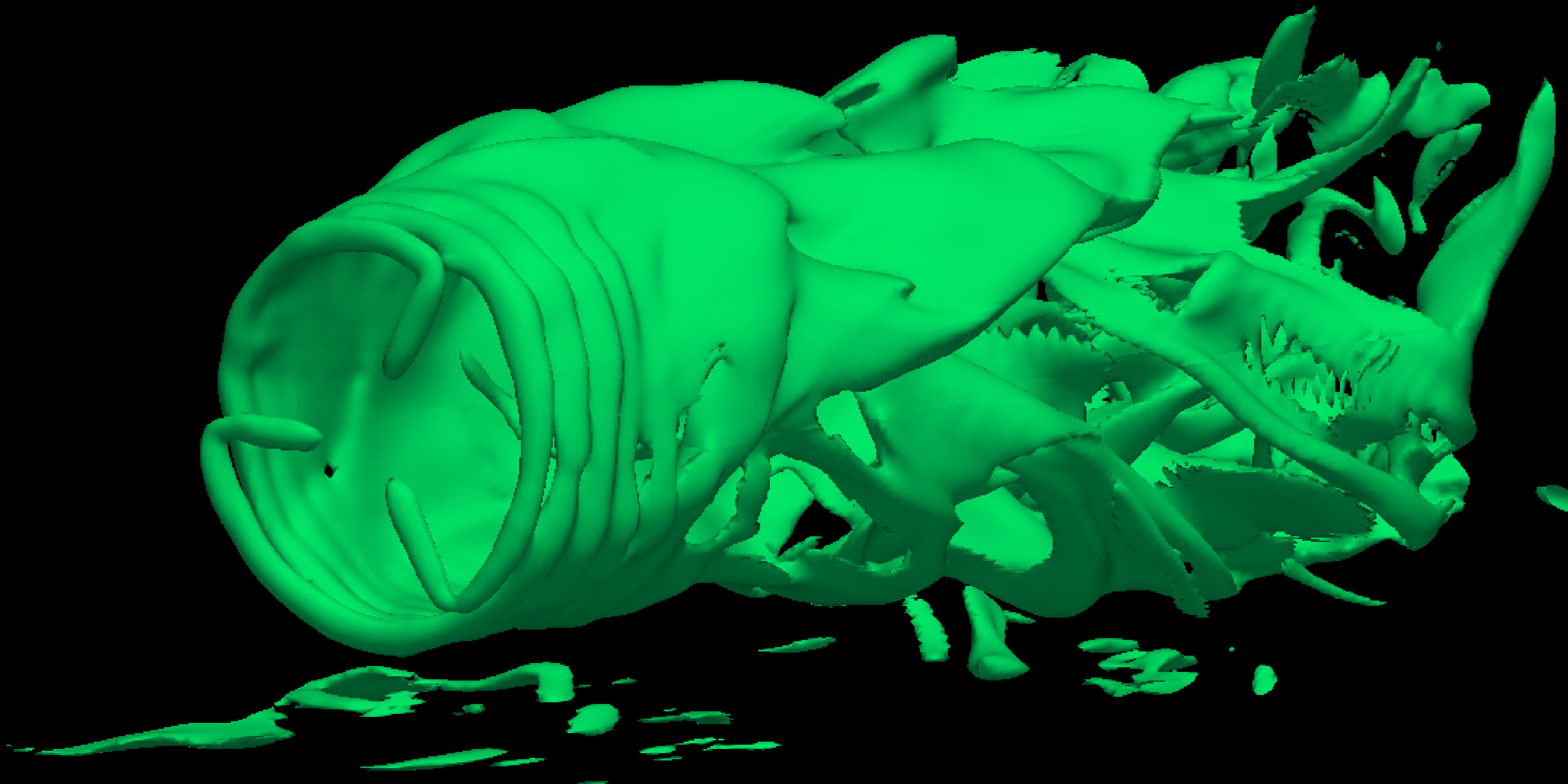


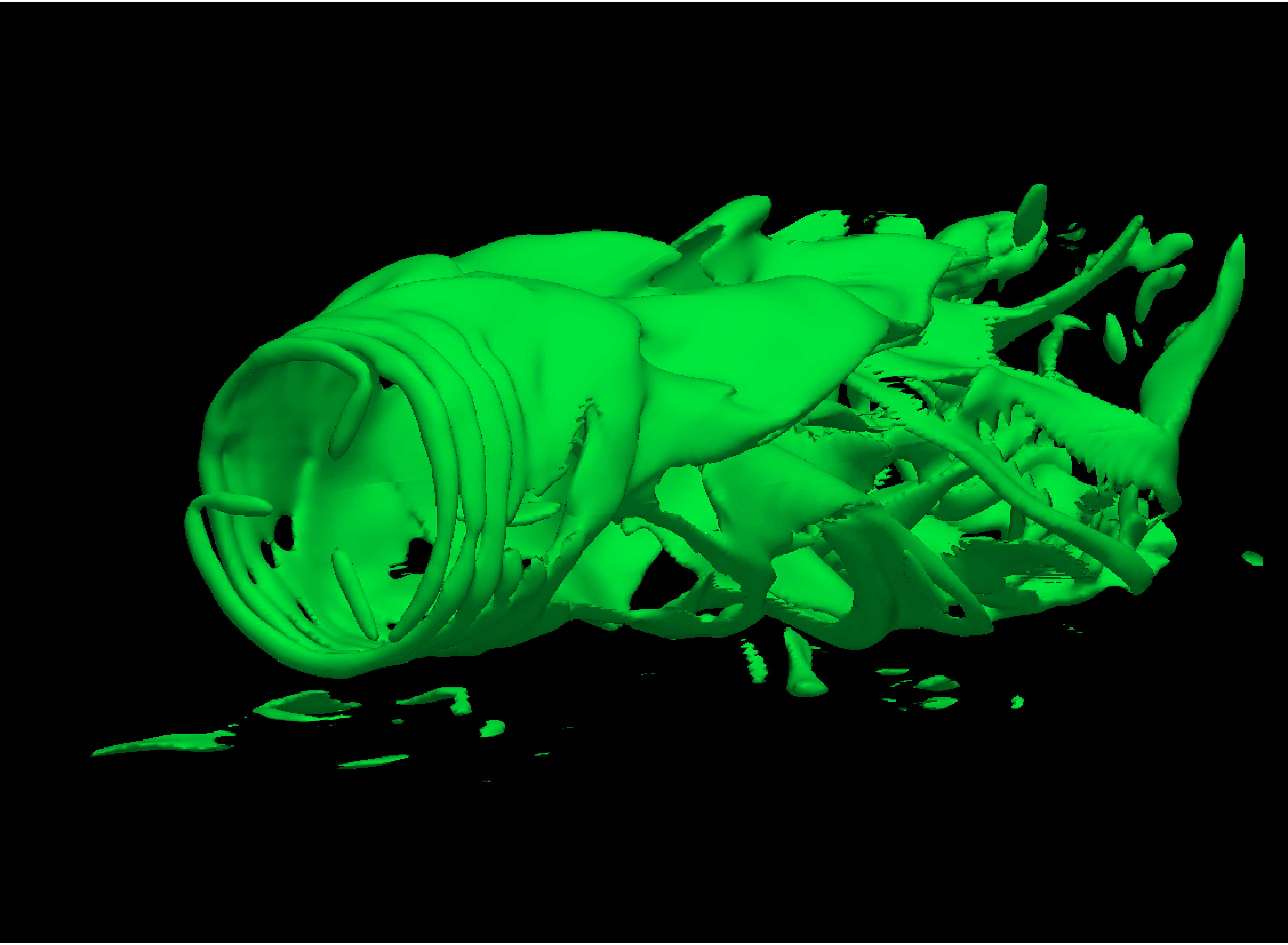


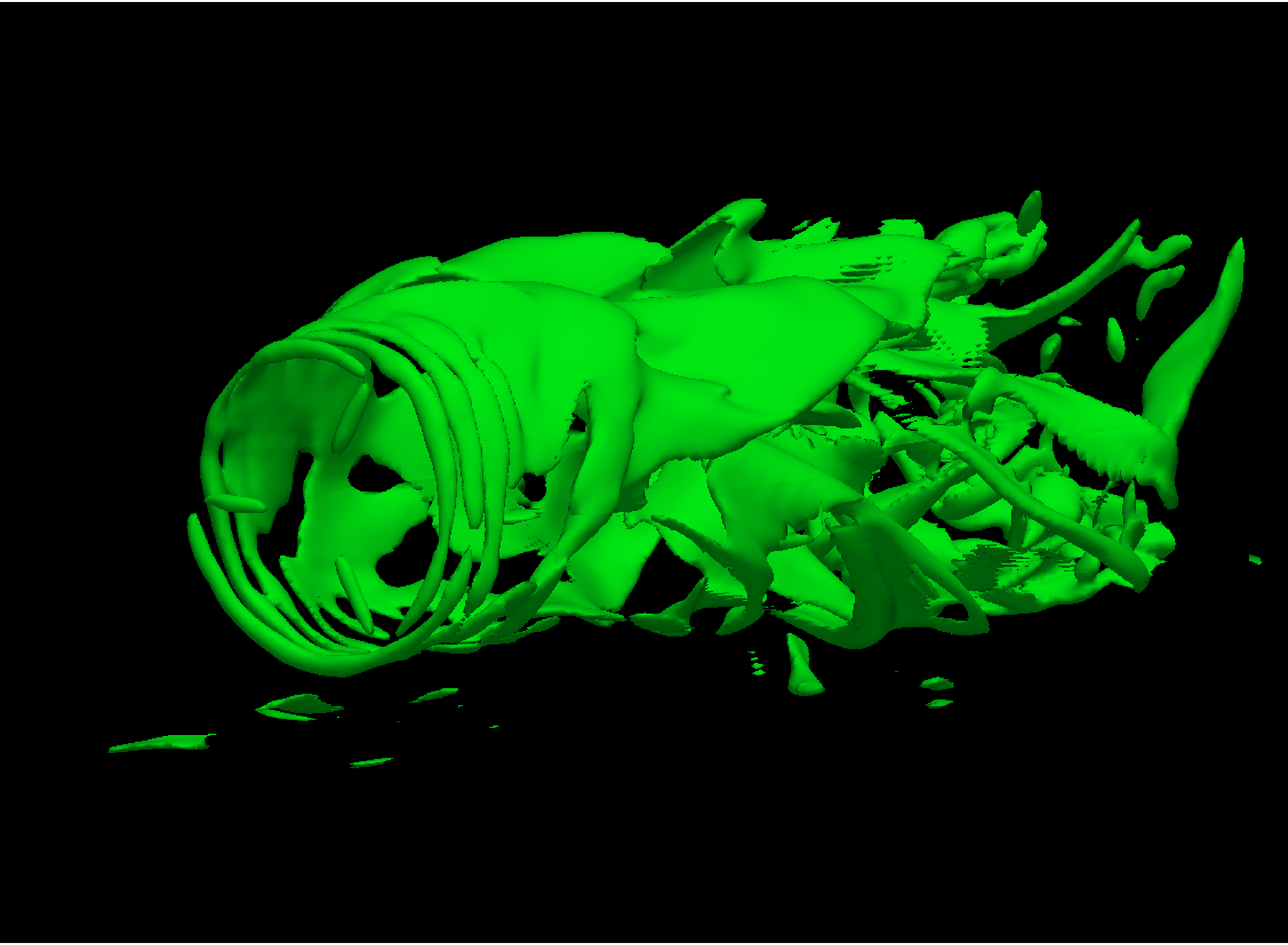


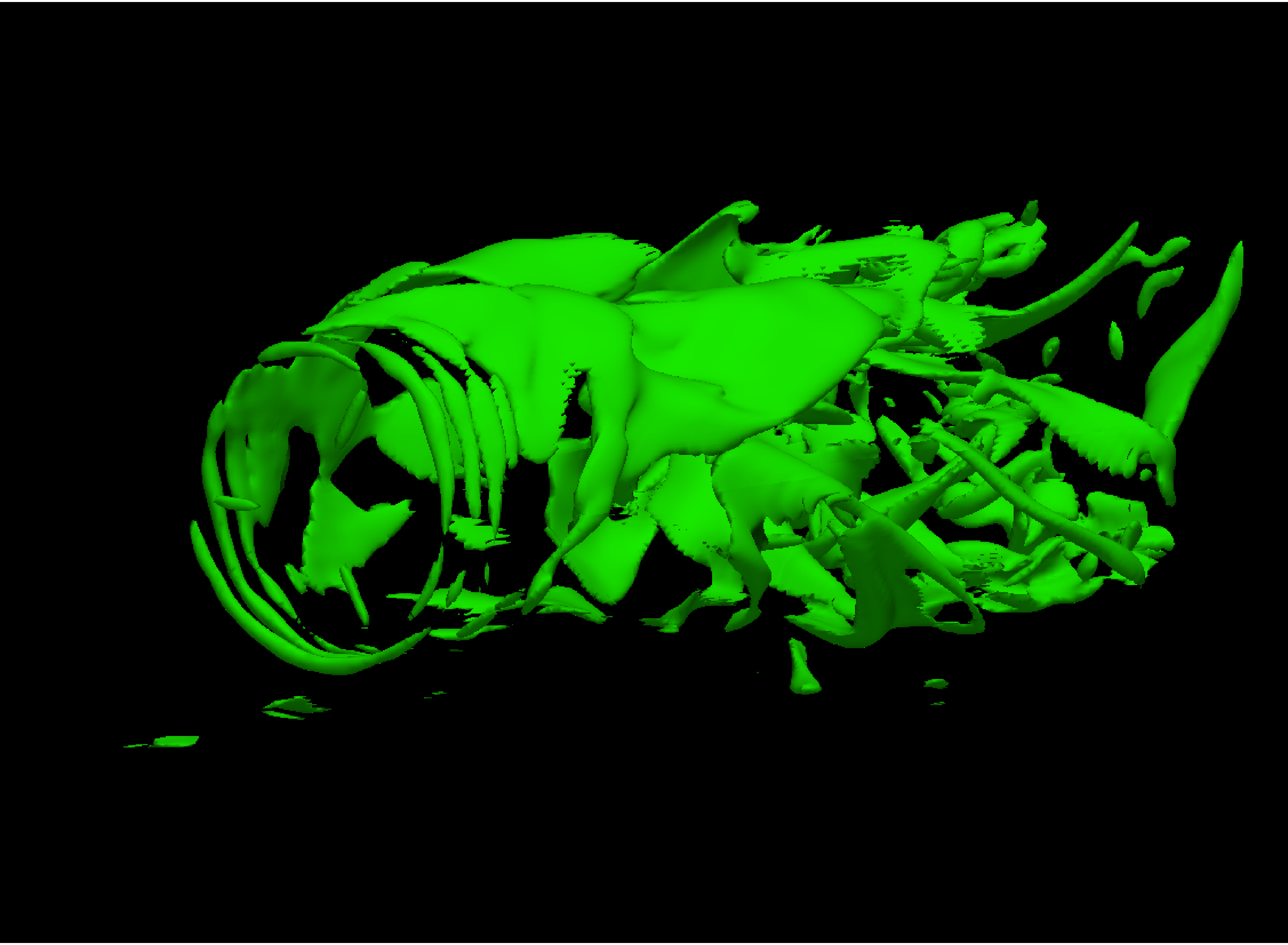


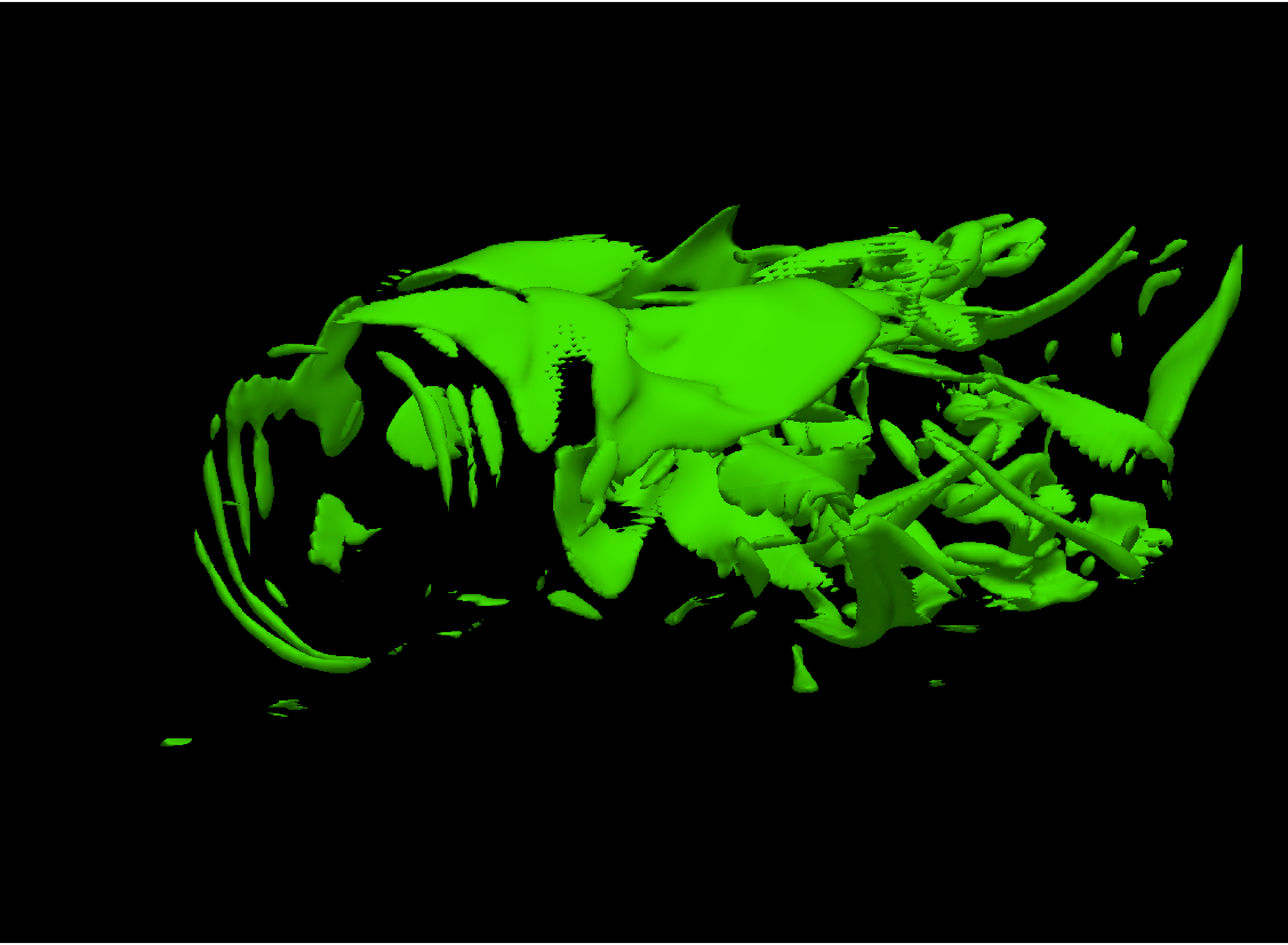


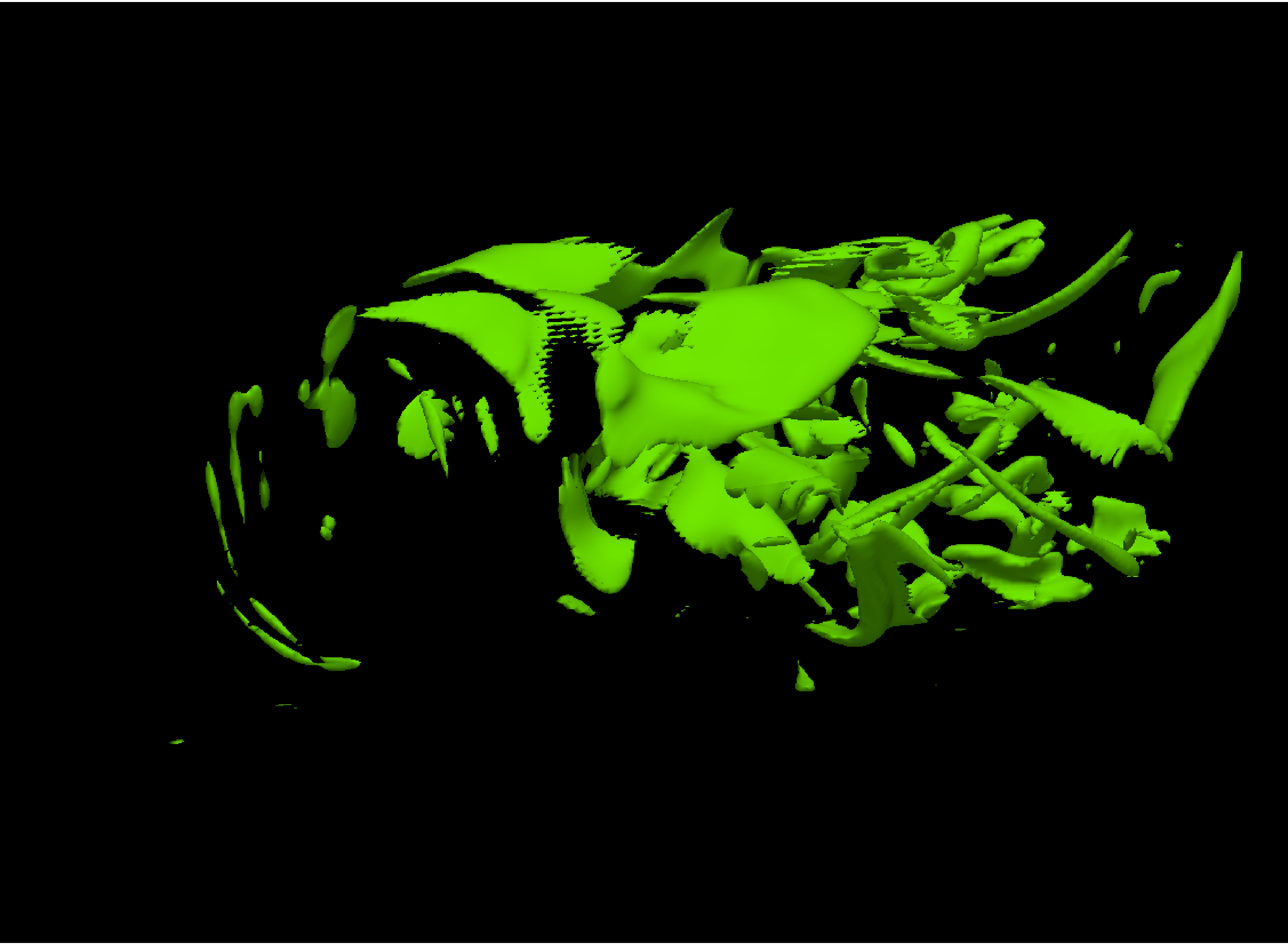


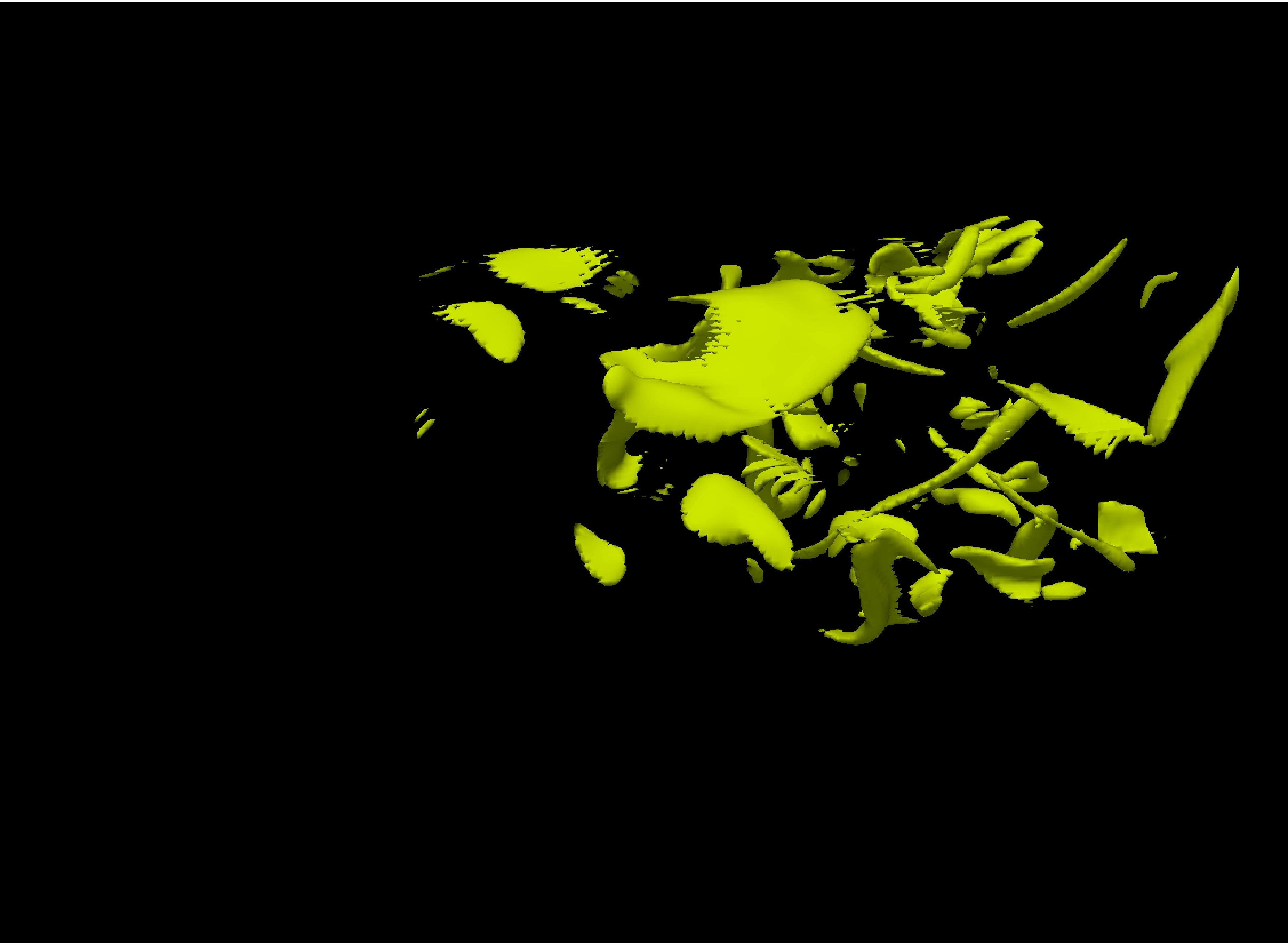


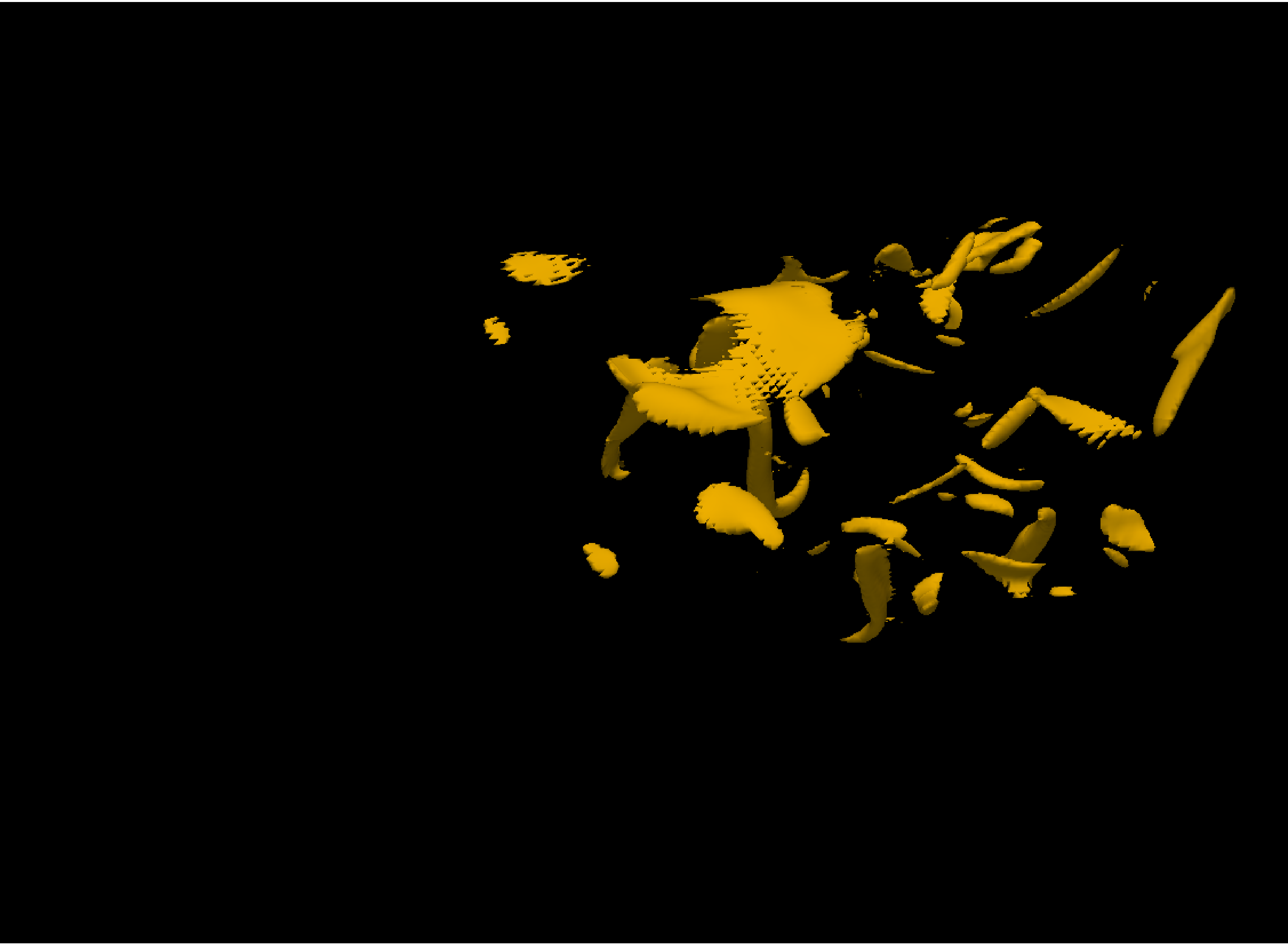


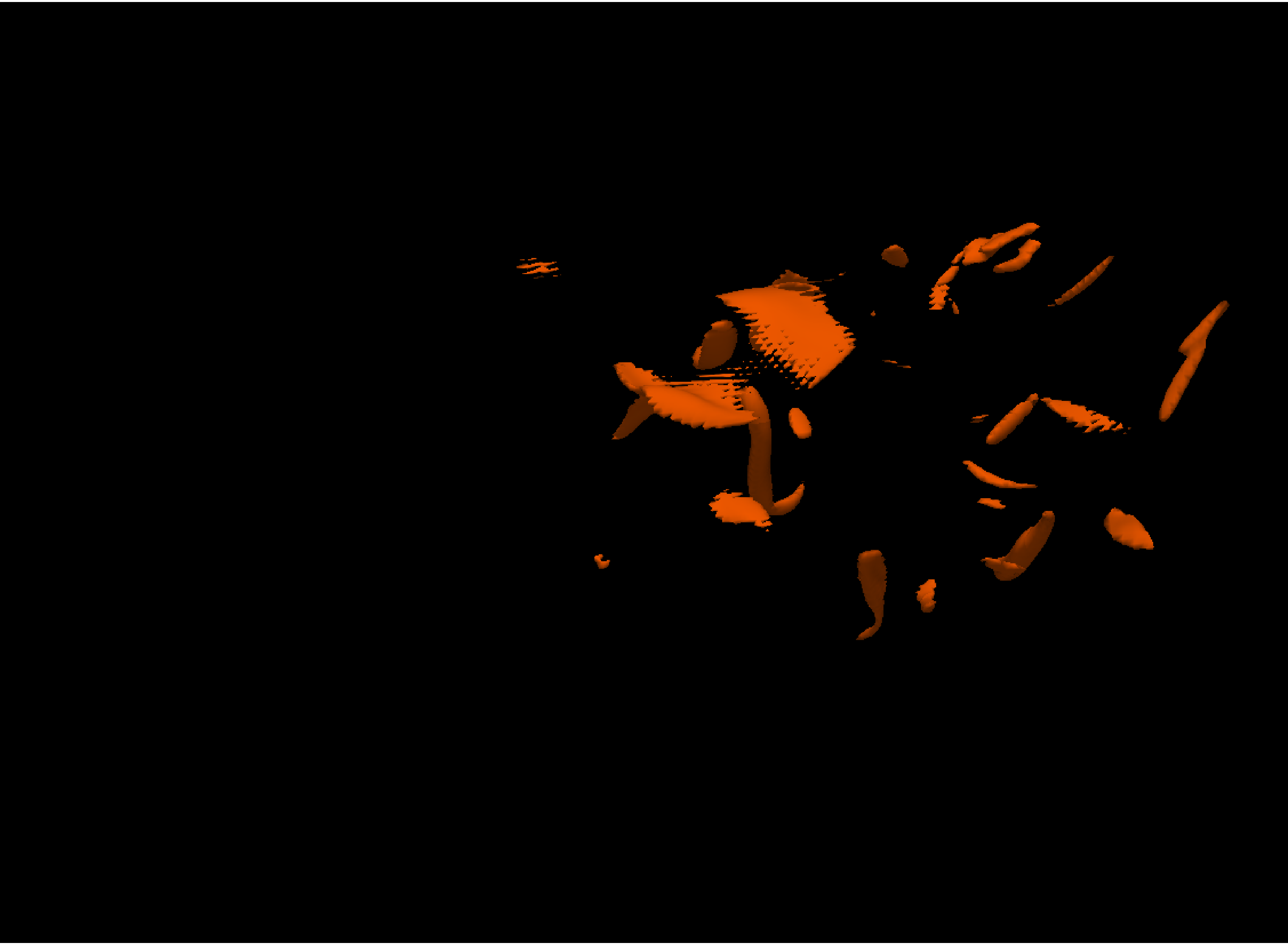








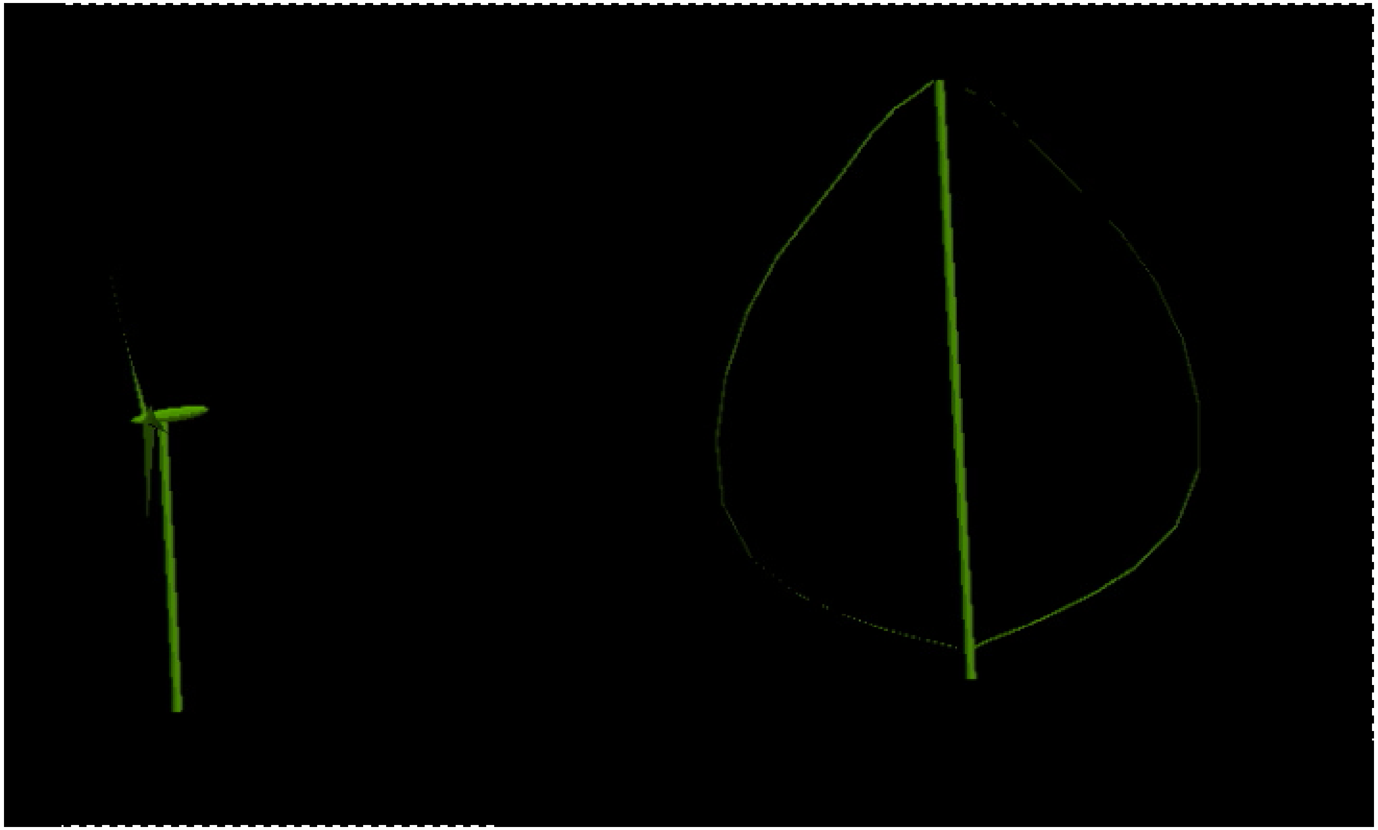




Part II:

Implementation of a vortex code for wind turbine aerodynamics and aeroelasticity

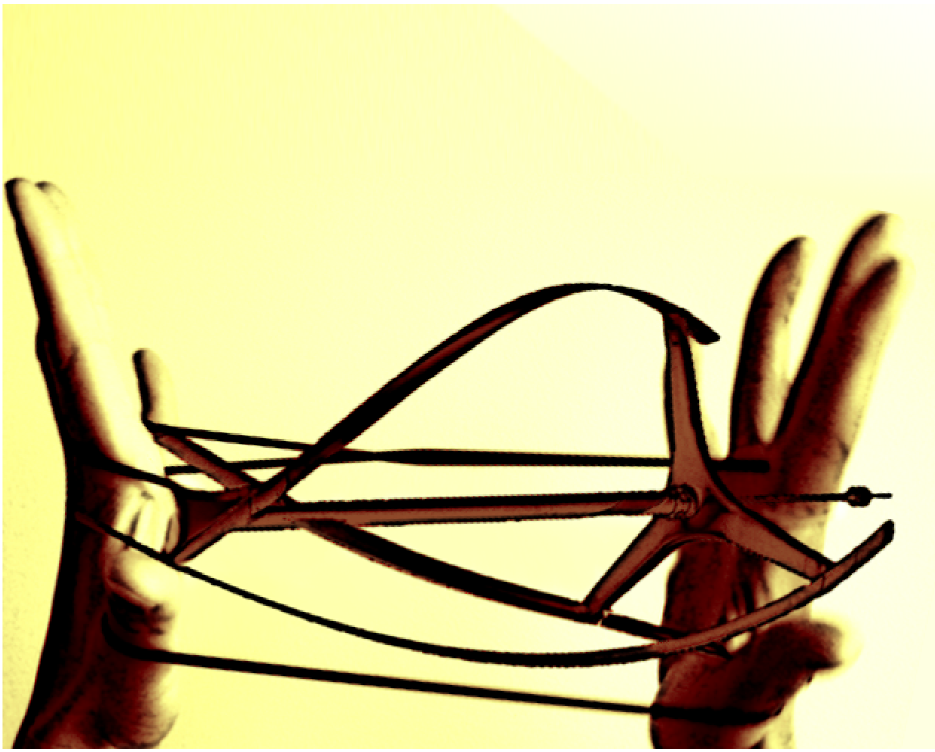
An illustration



Usage

```
begin aero;
    nblades 3;
    hub_vec shaft -3 ; rotor rotation vector (normally
    link 1 mbdy_c2_def blade1;^M
    link 2 mbdy_c2_def blade2;^M
    link 3 mbdy_c2_def blade3;
    ae_filename ./input/blade82.ae ;
    pc_filename ./input/blade.pc ;
    aerocalc_method 1 ; 0=no aerodynamic, 1=with aerod
    aerosections       30 ;
    ae_sets 1 1 1;
    tiploss_method 1; 0=none, 1=prandtl
    dynstall_method 0; 0=none, 1=stig øye method,2=mhh
    induction_method 4 ; 0=none, 1=normal, 2.. 3... 4
    begin freewake_params ;
        input_file ./vortexwake_ntk500_elast.oin ;
    end freewake_params ;
end aero ;
```

Aeroelasticity with vortex methods - Challenges



- Vortex-structure interaction
- Turbulence
- Shear
- Dynamic-stall models
- Computational time

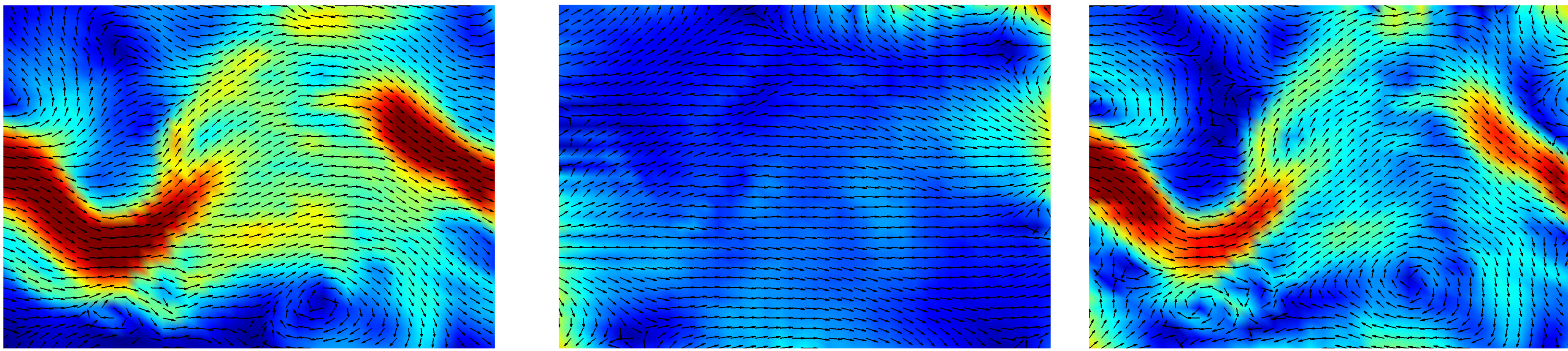
Part I:

Introduction to vortex methods

1 – Kinematics

Helmholtz decomposition and Poisson's equation

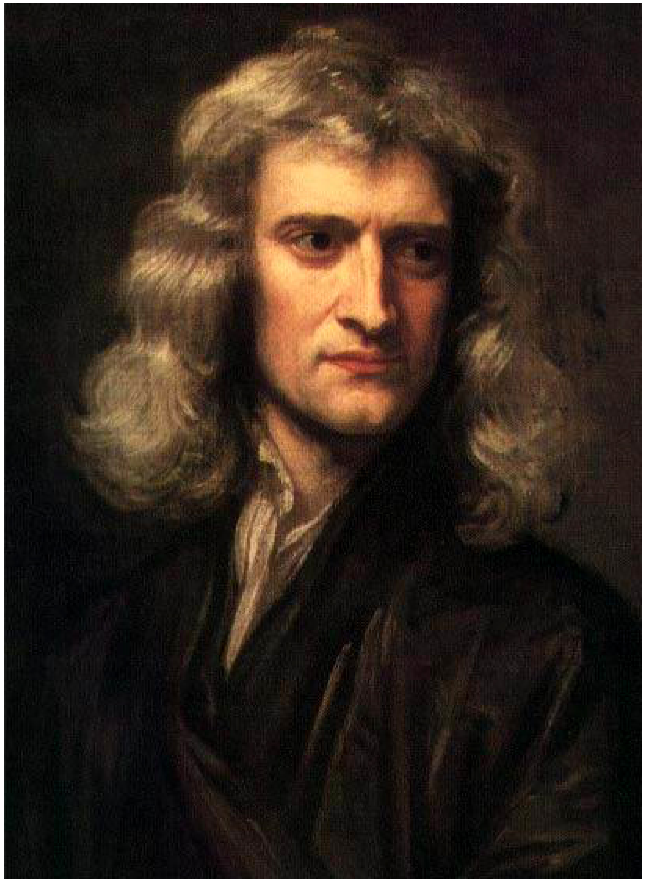
$$\boldsymbol{U} = \qquad \operatorname{grad}\phi \qquad + \qquad \operatorname{curl}\psi$$



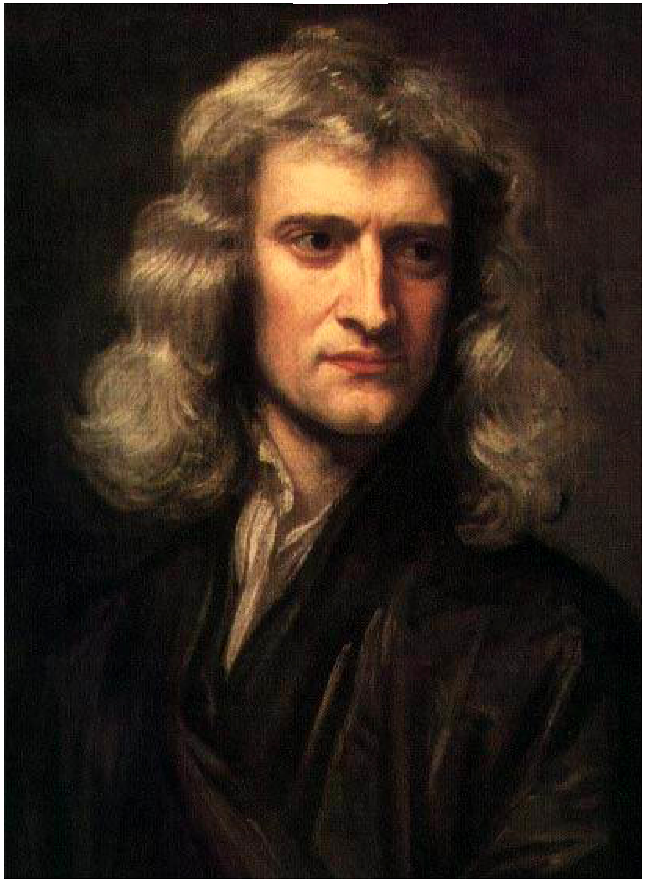
(vorticity definition) $$\Delta\psi = -\omega$$

(incompressible) $$\Delta\phi = 0$$

2 – Dynamics



2 – Dynamics



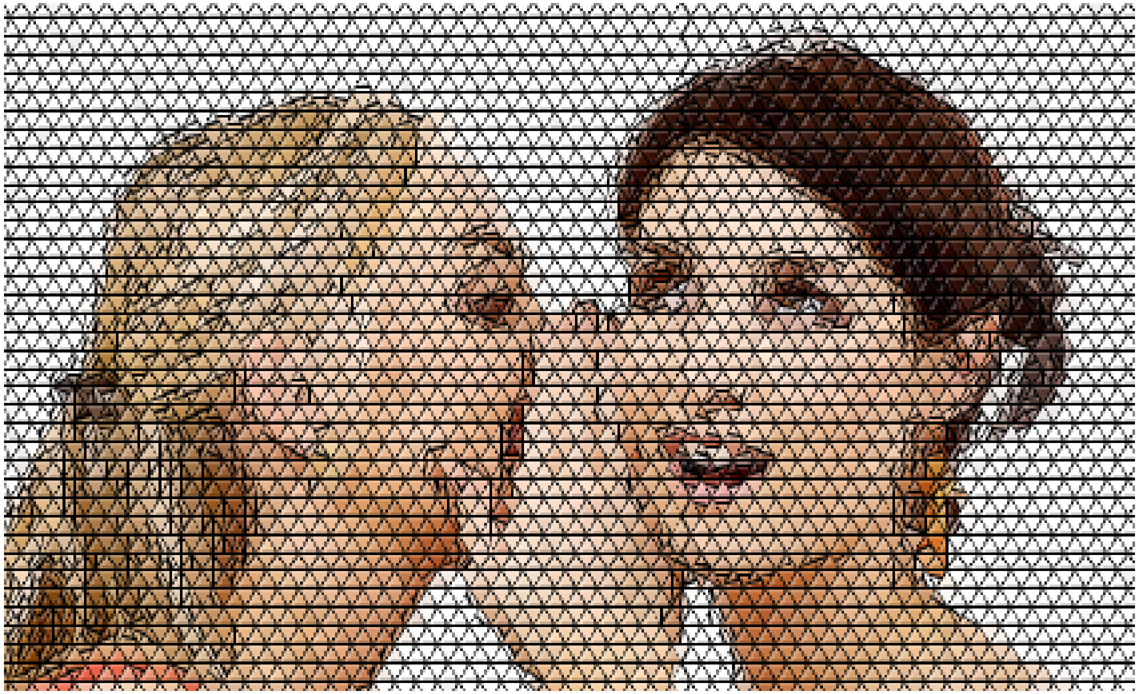
Vorticity formulation of NS equations

- $$\frac{d\boldsymbol{\omega}}{dt} = \frac{\partial \boldsymbol{\omega}}{\partial t} + \underbrace{(\boldsymbol{U} \cdot \nabla)}_{\text{convection}} \boldsymbol{\omega} = \underbrace{(\boldsymbol{\omega} \cdot \nabla)\boldsymbol{U}}_{\text{strain}} + \underbrace{\nu \Delta \boldsymbol{\omega}}_{\text{diffusion}}$$

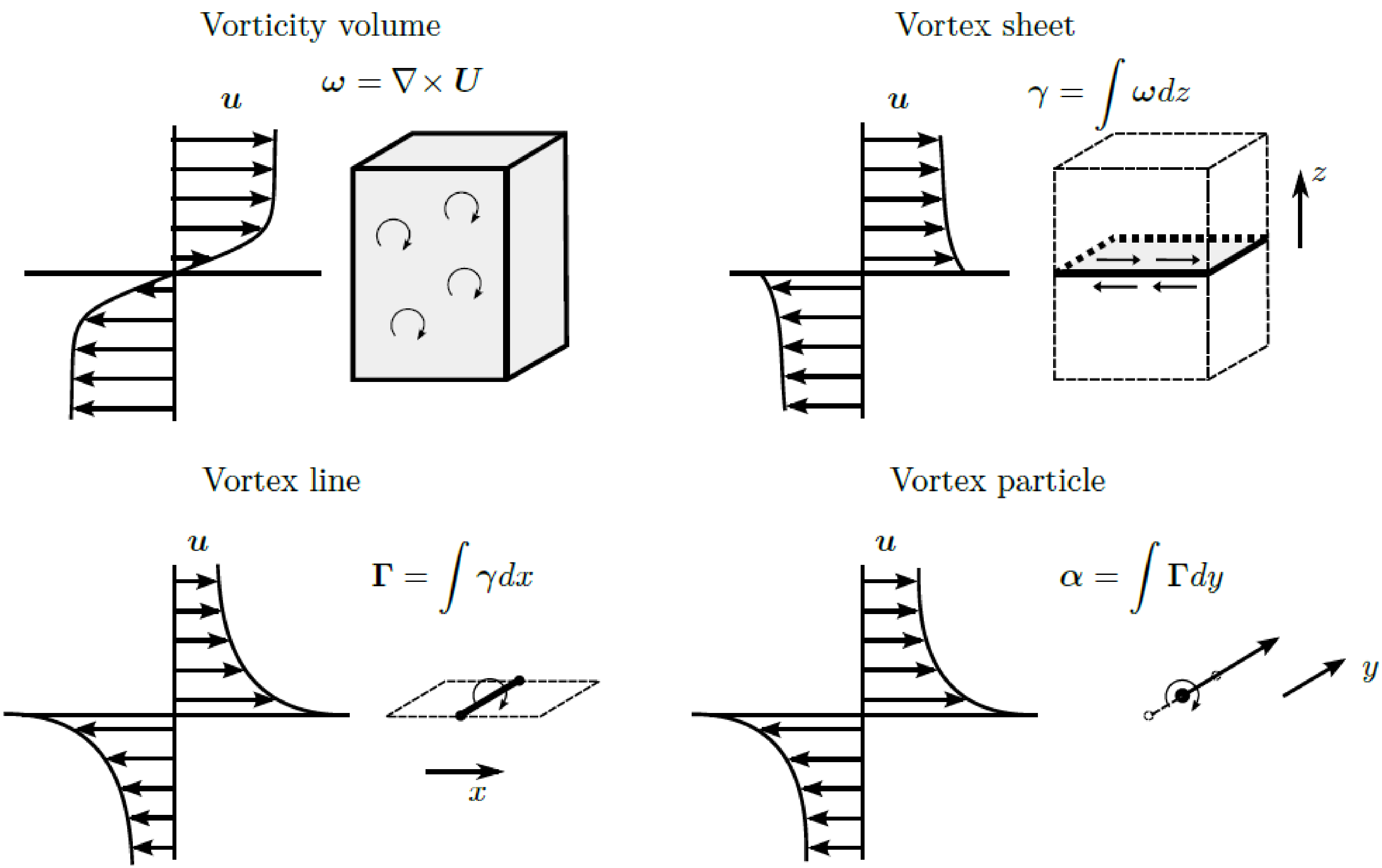
- $$\frac{\nabla^2 p_\omega}{\rho} = -\nabla \cdot (\boldsymbol{\omega} \times \boldsymbol{u})$$

Helmholtz's (2nd) theorem (consequence of Kelvin's)

3 – Discretization

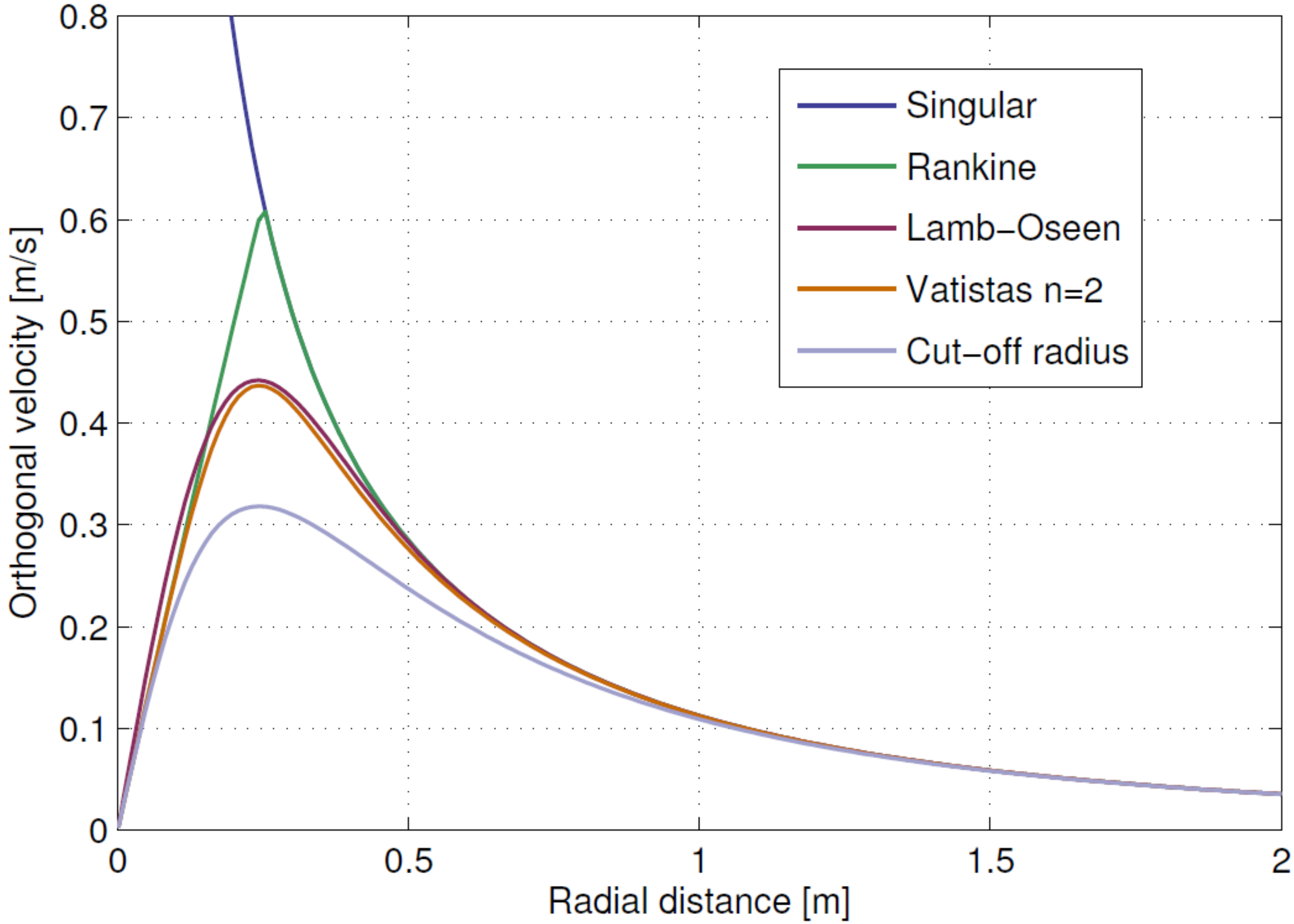


Generalized vorticity



4 – Smoothing and viscosity

Smoothing



Singular
Rankine
Lamb−Oseen
Vatistas n=2
Cut−off radius
Orthogonal velocity [m/s]
Radial distance [m]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.5
1
1.5
2

Viscous diffusion (thanks to viscous splitting)

- Viscous splitting: $$\frac{d\boldsymbol{\omega}}{dt} = \underbrace{(\boldsymbol{\omega} \cdot \nabla)\boldsymbol{U}}_{\text{strain}} + \underbrace{\nu \Delta \boldsymbol{\omega}}_{\text{diffusion}}$$

Convection/Strain

$$\frac{d\boldsymbol{x}_v(t)}{dt} = \boldsymbol{u}(\boldsymbol{x}_v, t)$$

$$\frac{d\boldsymbol{\omega}}{dt} = \underbrace{(\boldsymbol{\omega} \cdot \nabla)\boldsymbol{U}}_{\text{strain}}$$

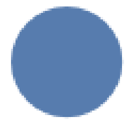
Diffusion

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nu \Delta \omega$$

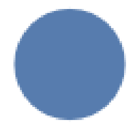
$$G(r) = \frac{1}{(4\pi\nu\Delta t)^{3/2}} \exp\left[-\frac{r^2}{4\nu\Delta t}\right]$$

Viscous diffusion
(thanks to viscous splitting)

- Initial state:

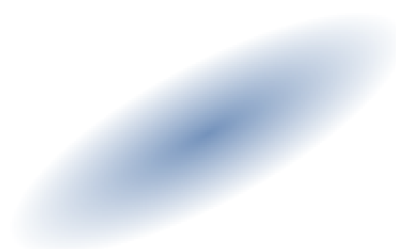


- Convection

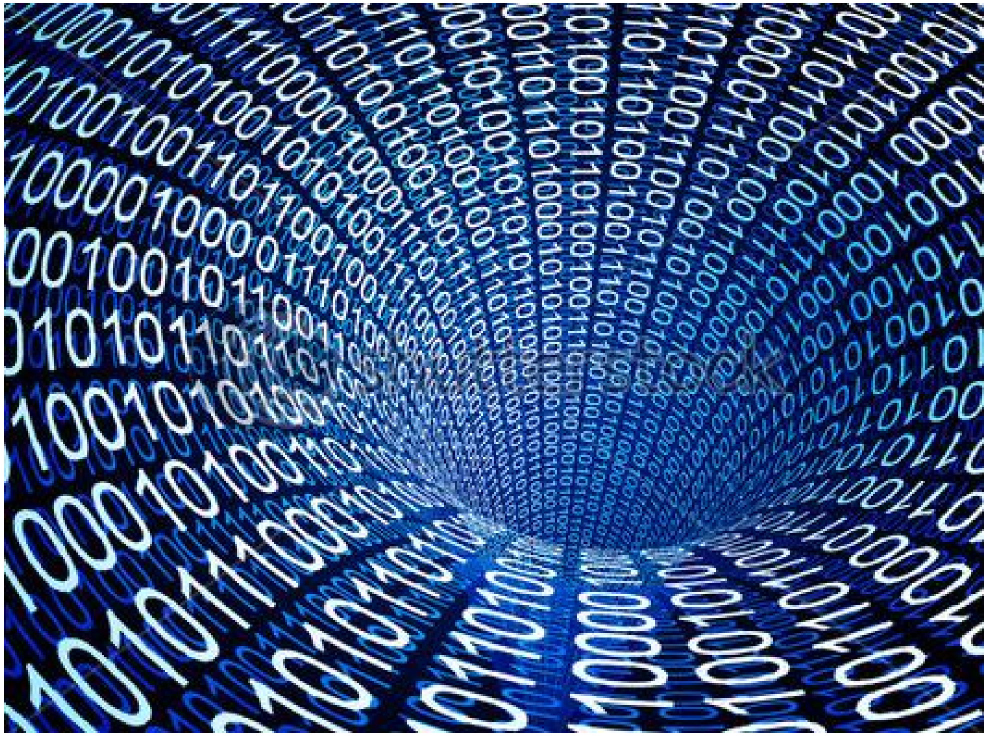


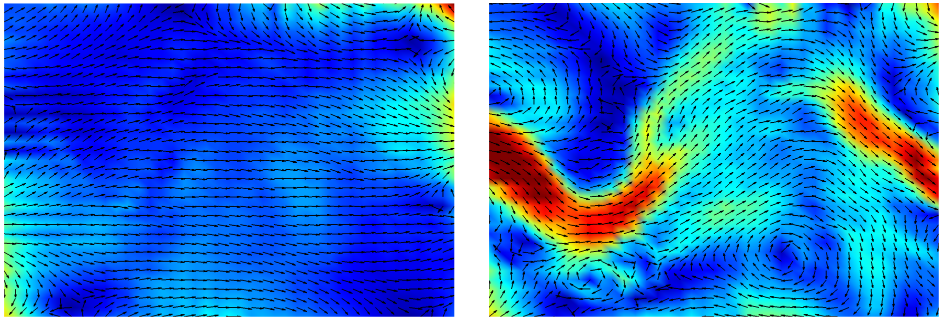
- Strain

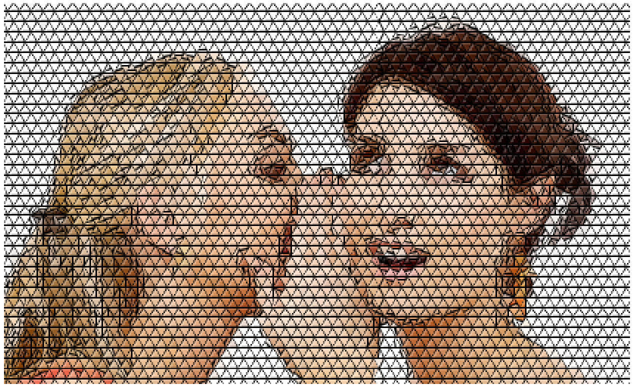
- Diffusion



5– Vortex methods (summary)





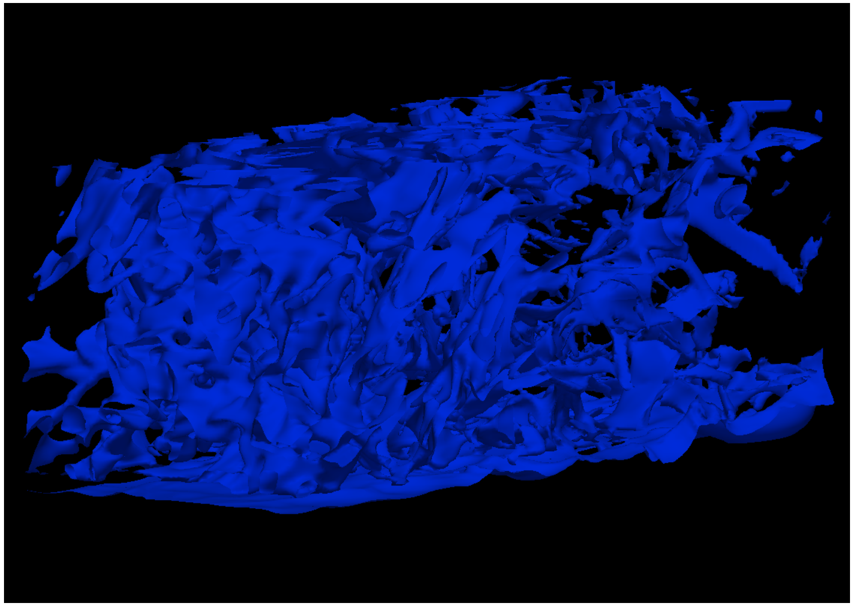


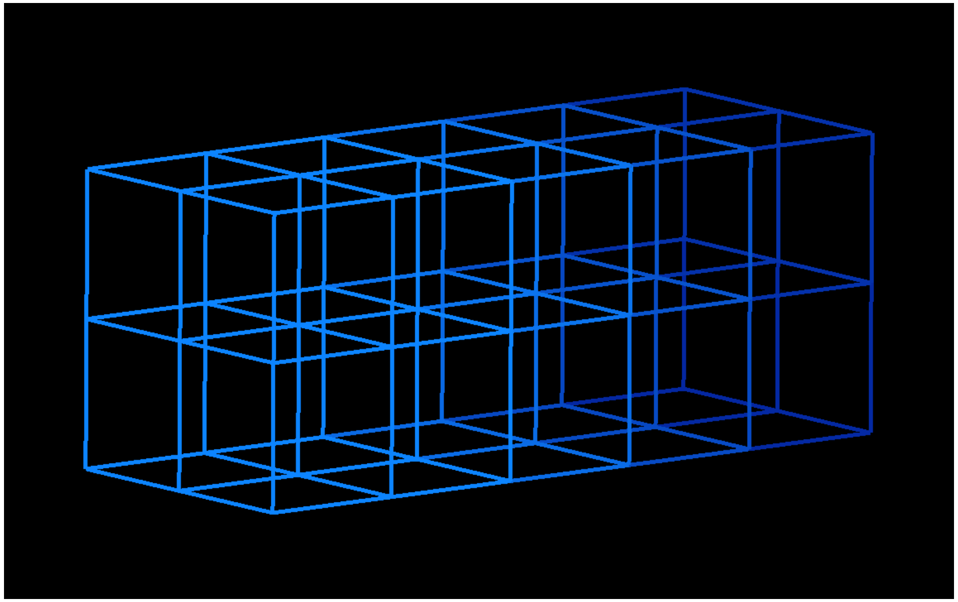
Conclusions

- Preliminary aero-elastic vortex code released
- More applications and users needed
- The challenges presented in the previous section should be faced
- Further improvements of the code to be developed

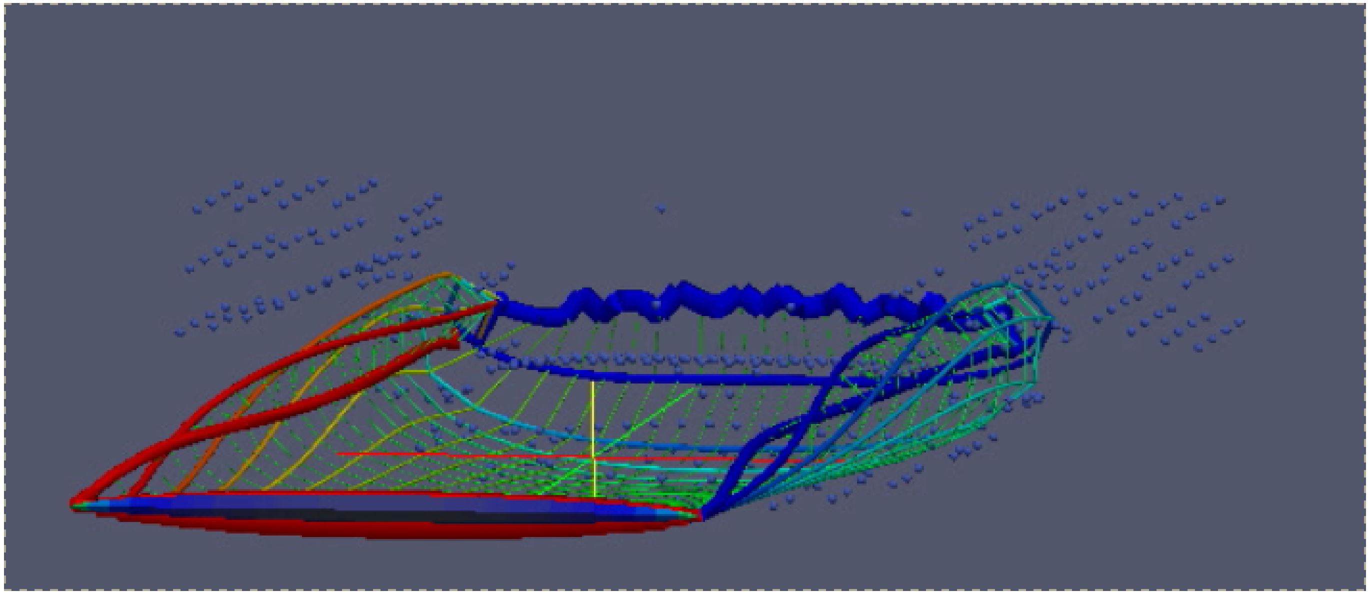
Thank you for your attention

Vortex method as projection





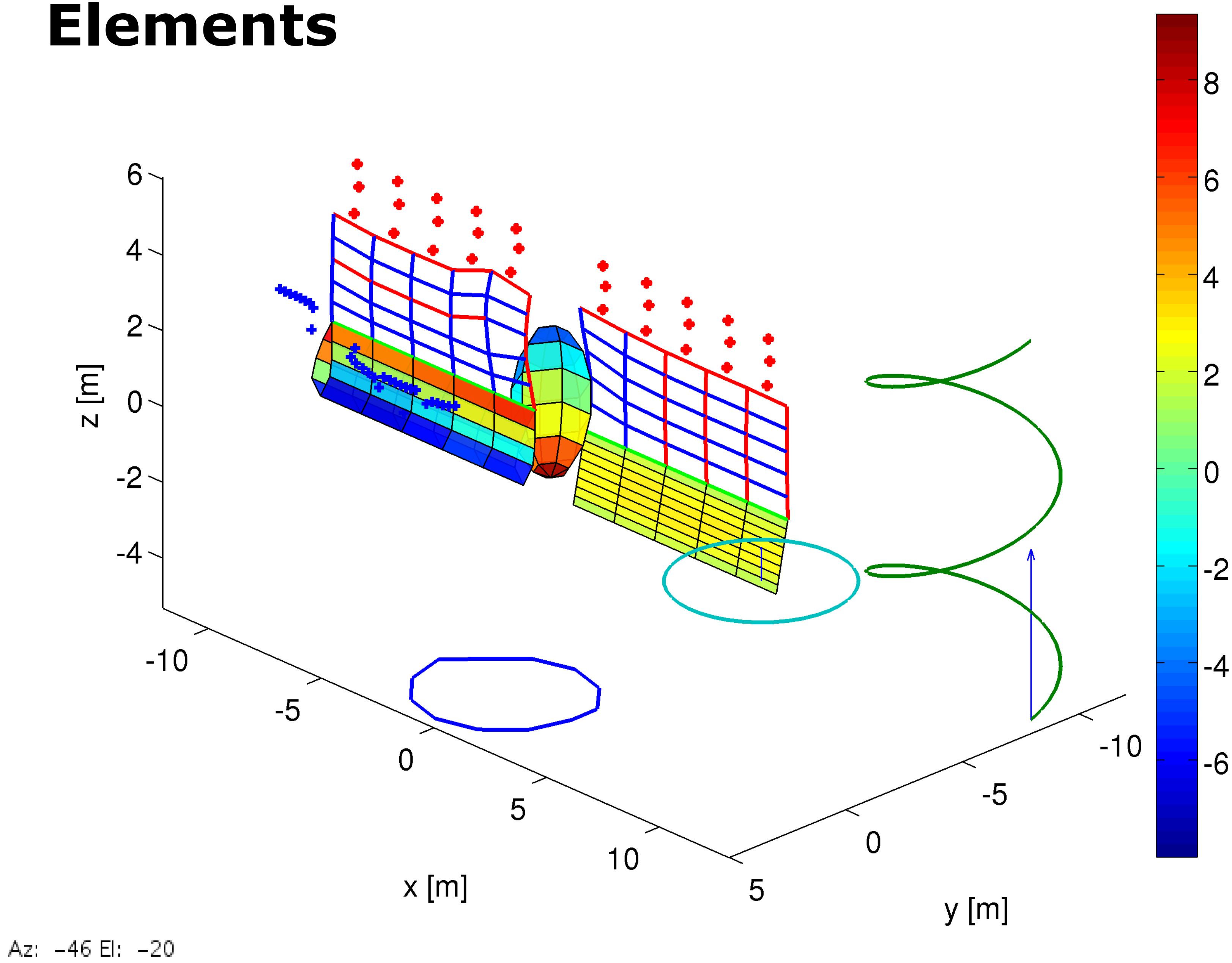
Vortex method as an aerodynamic tool



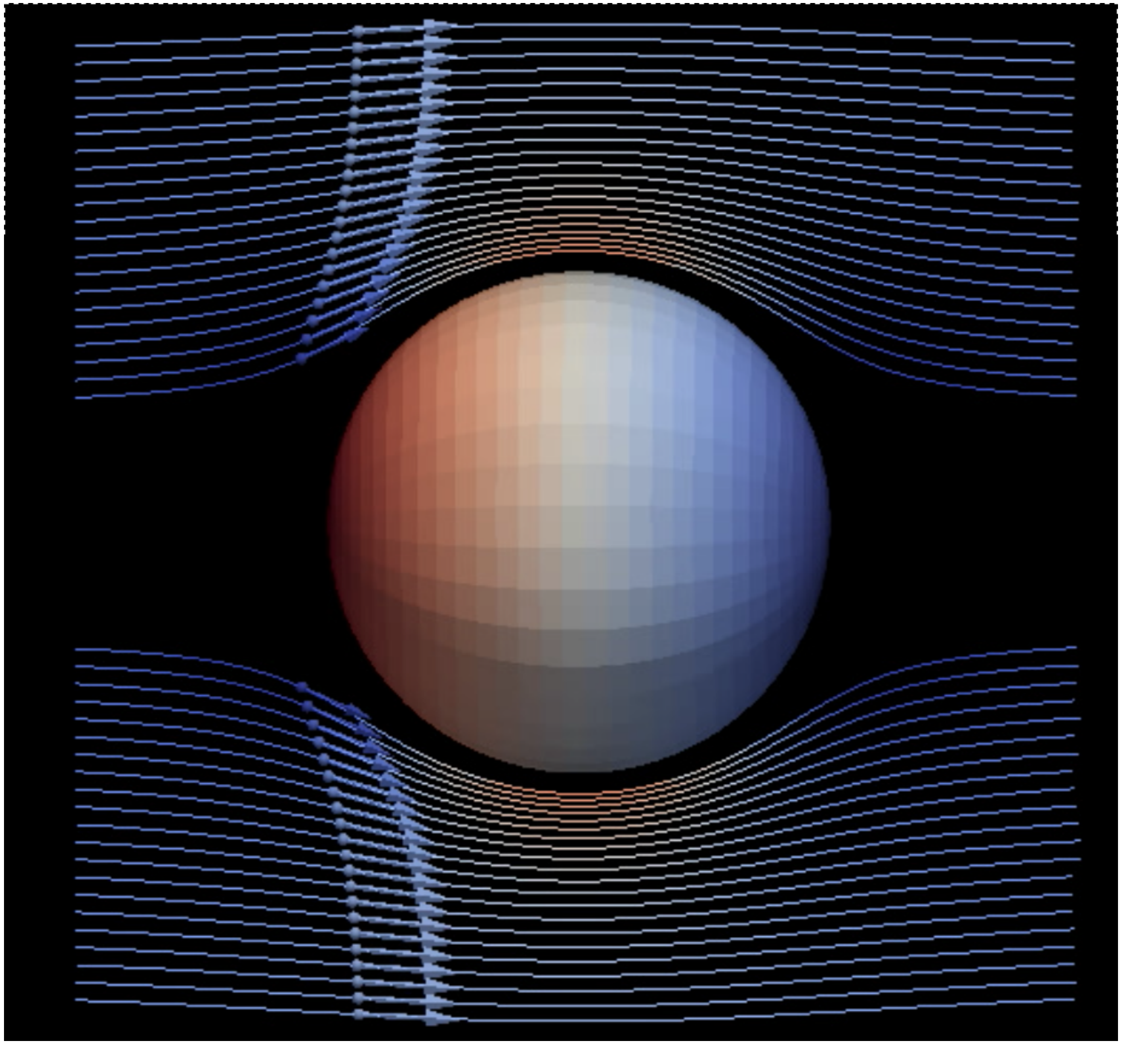
Movie

4 – Implemented vortex code: Omnivor

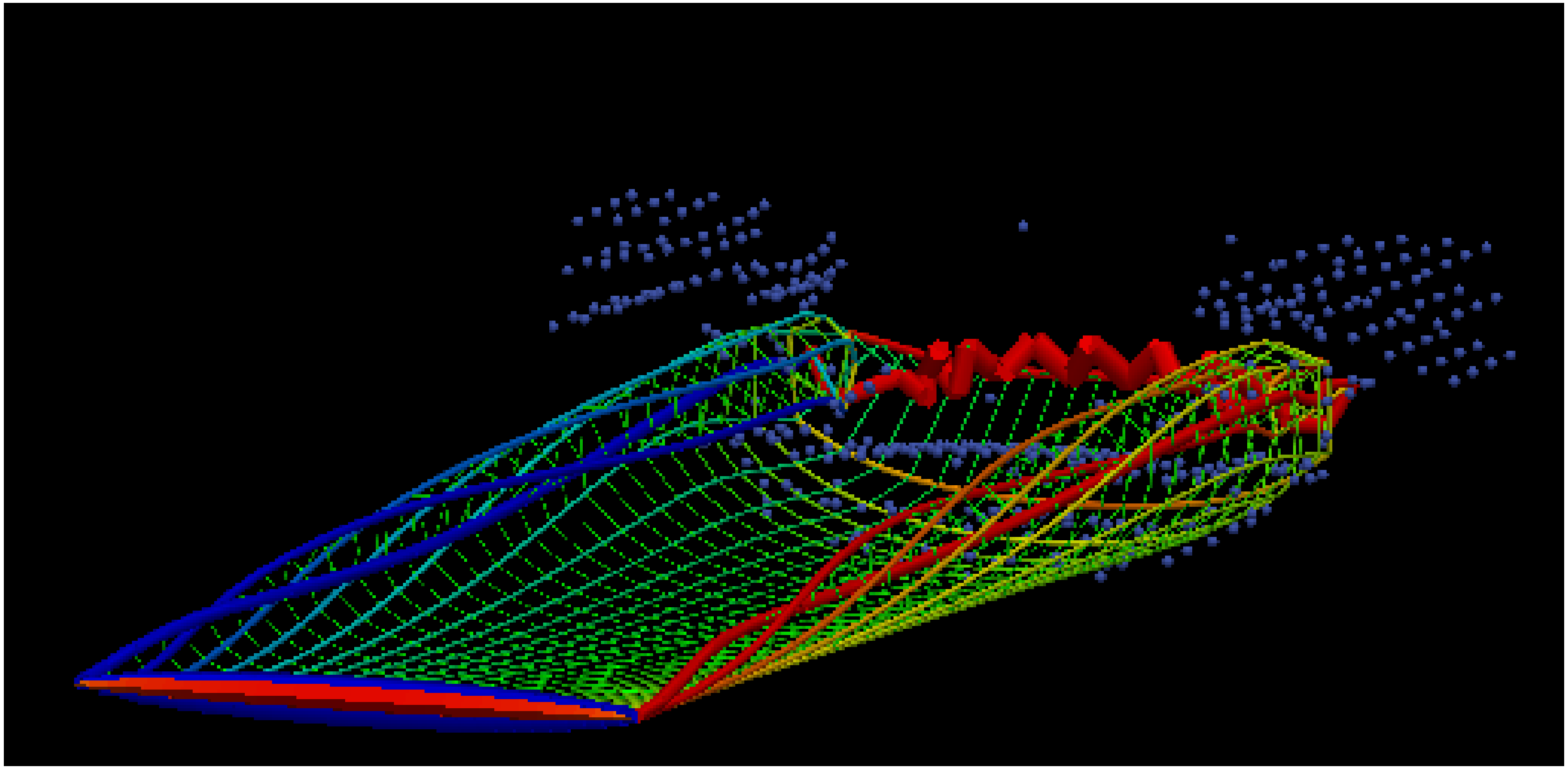
Elements



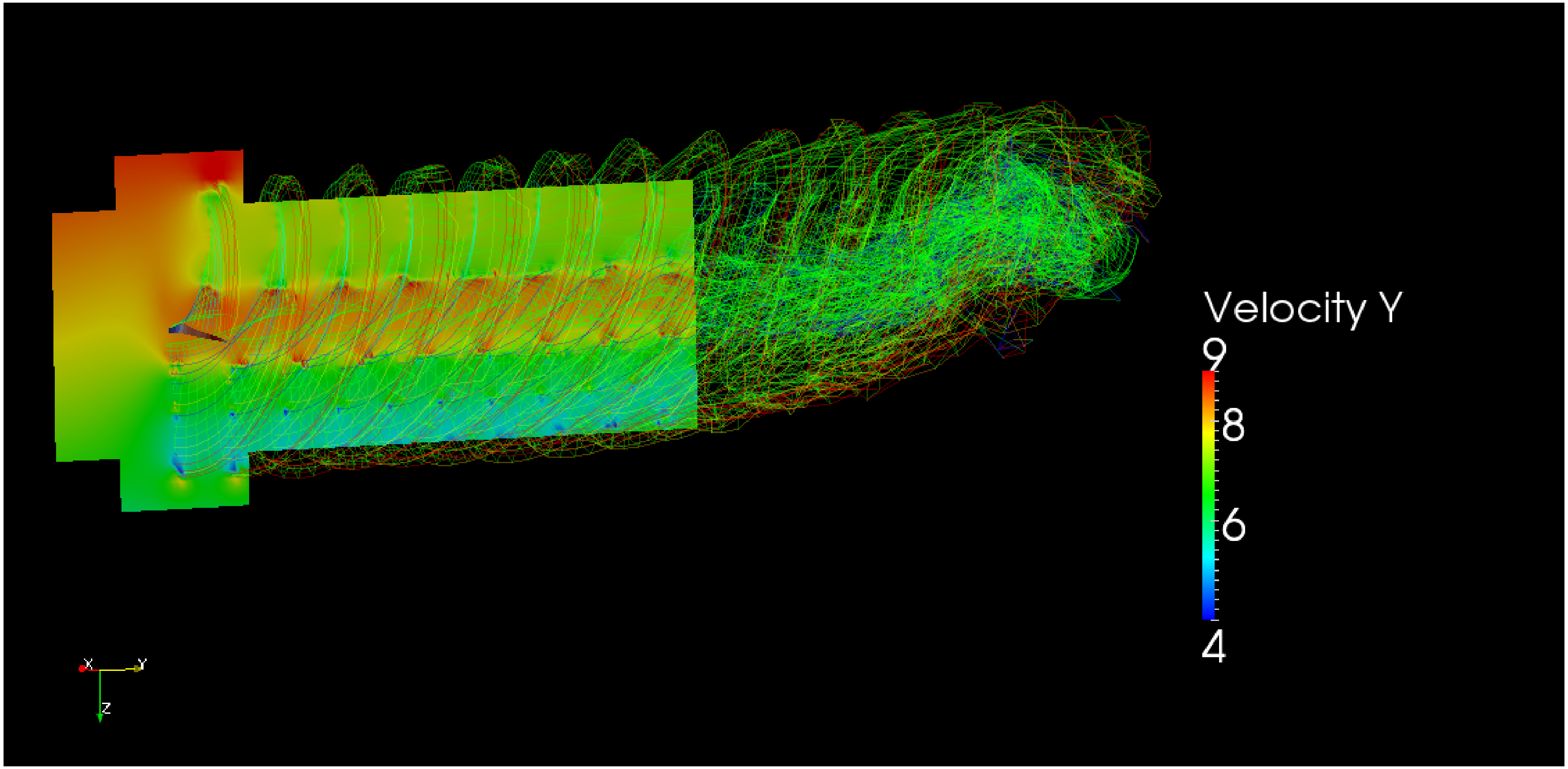
Applications



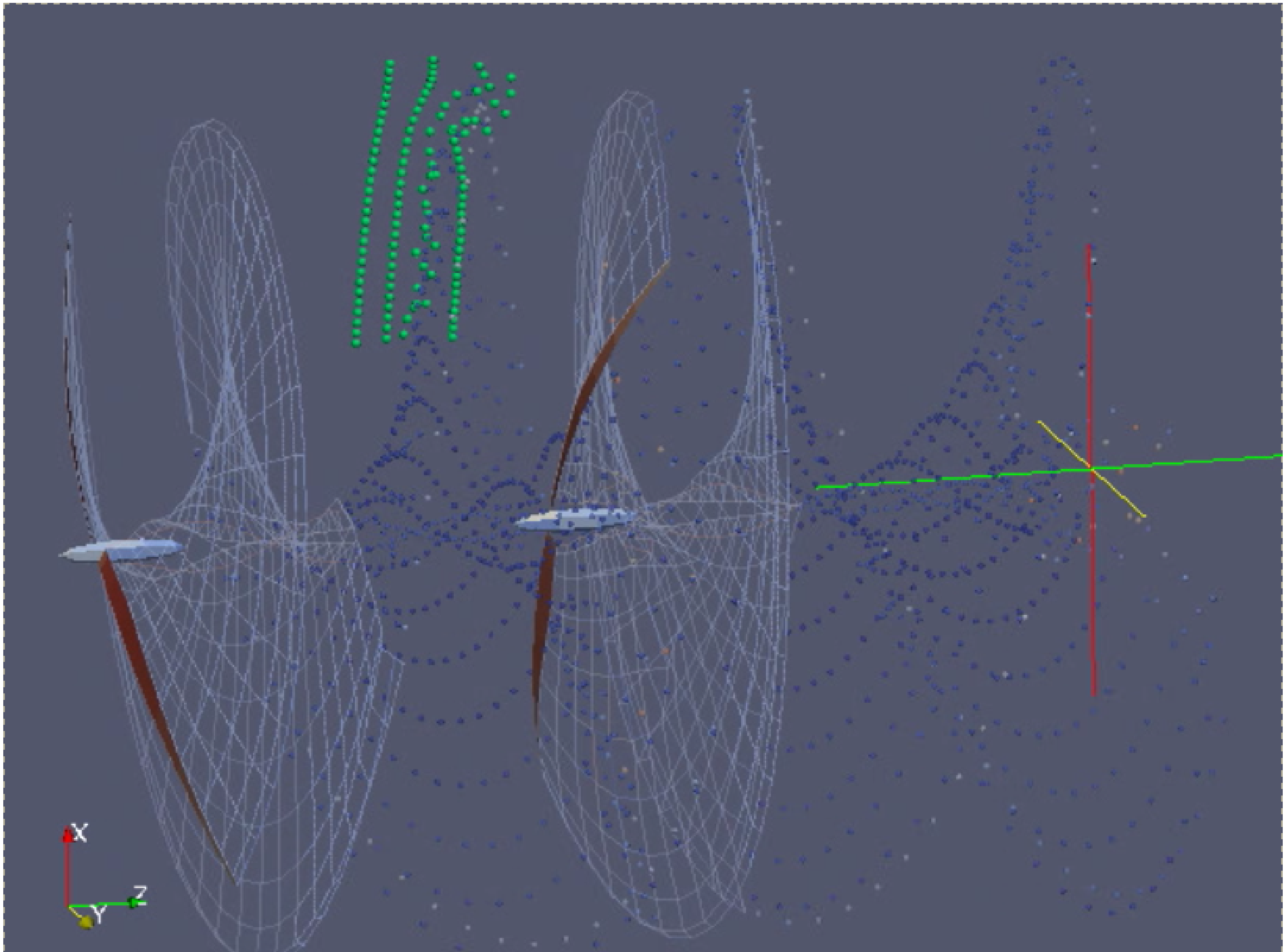
Applications



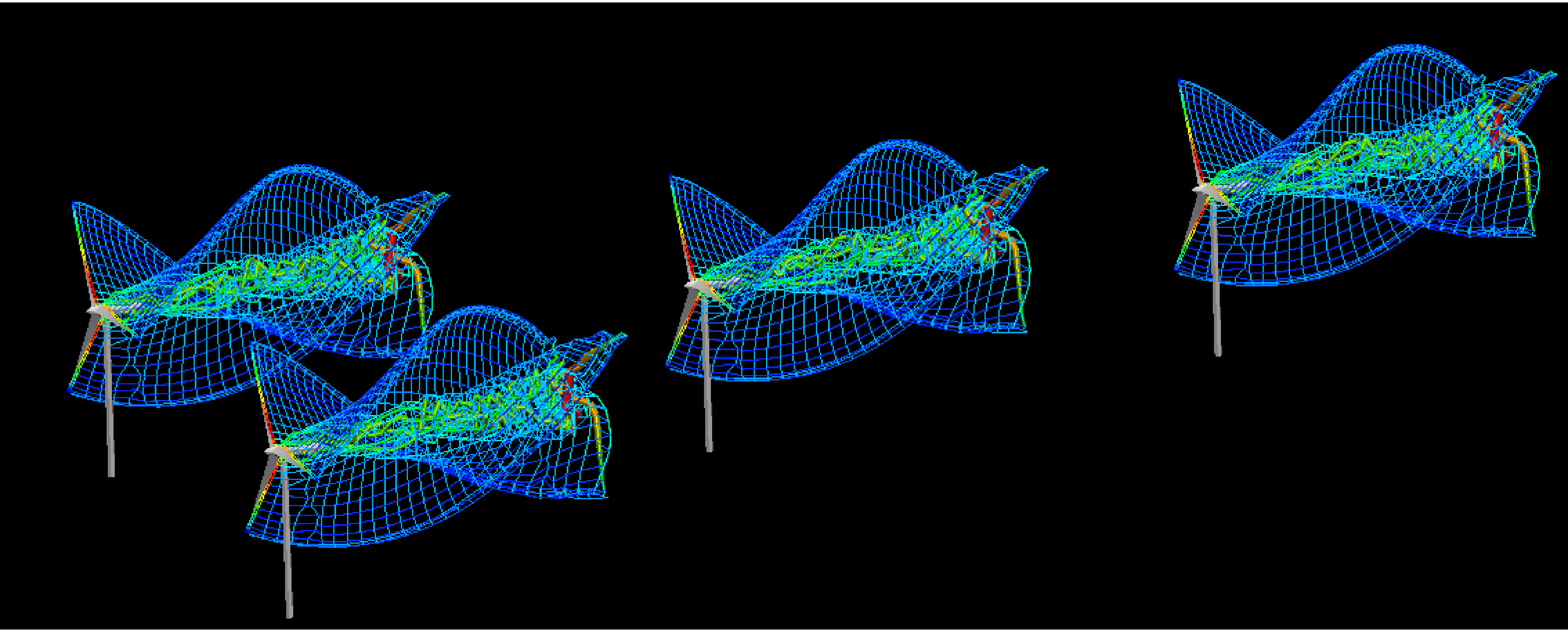
Applications



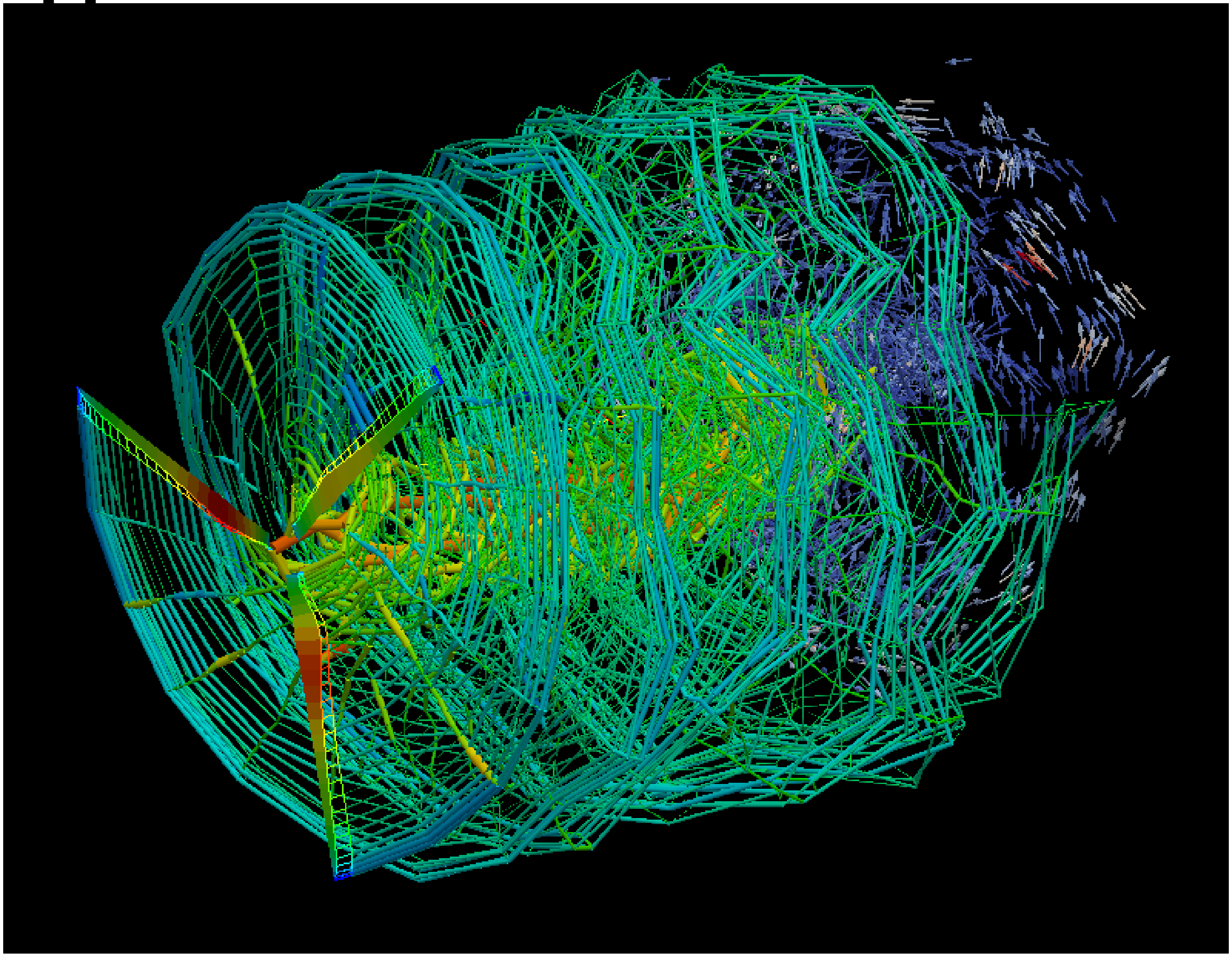
Applications



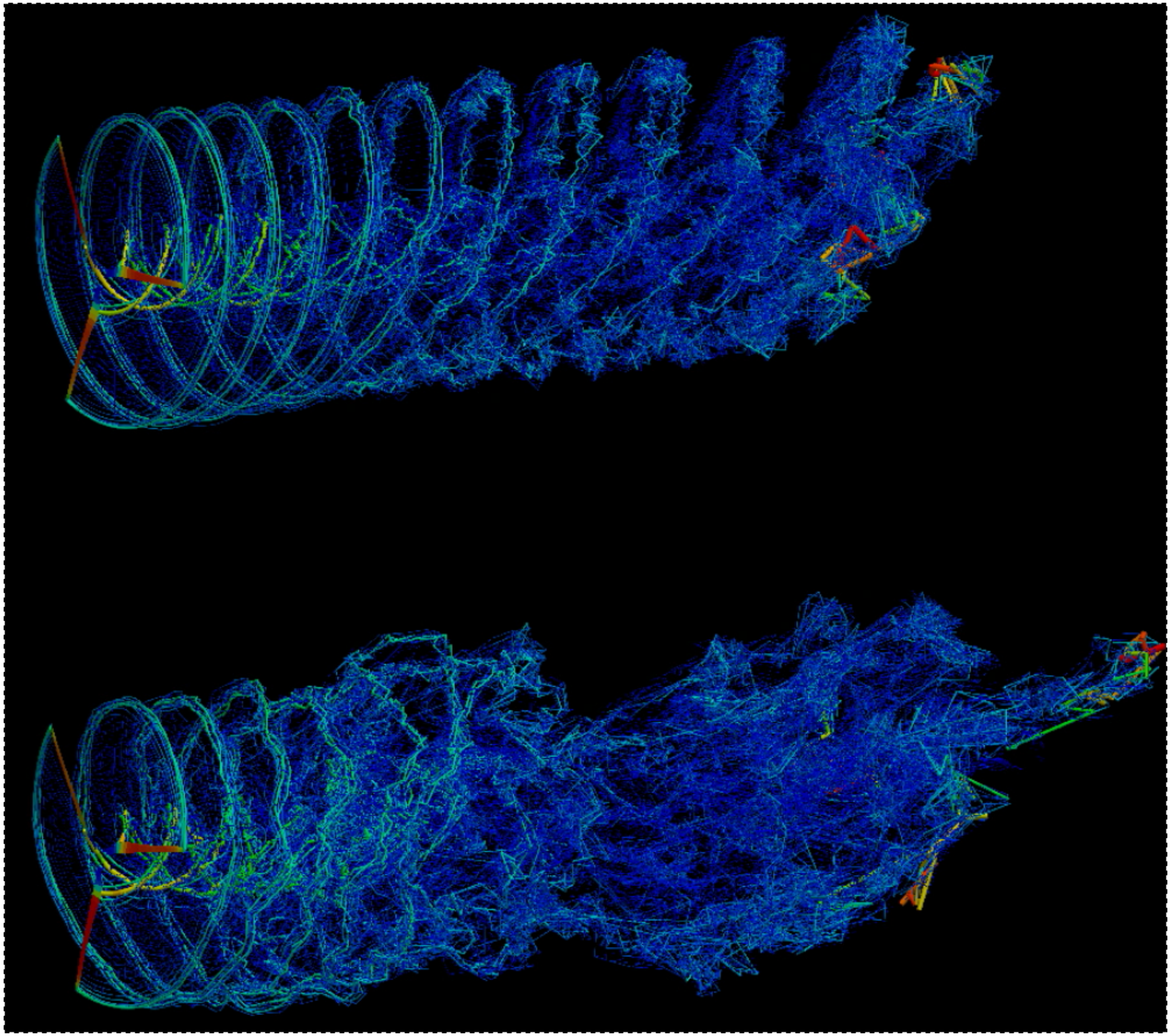
Applications



Applications



Applications



Movie

Solution: Biot-Savart law and Green's theorem

$$\boldsymbol{U}_{\psi}(\boldsymbol{x}) = \frac{1}{4\pi} \int_{\Omega(t)} \frac{\omega(\boldsymbol{x'}) \times (\boldsymbol{x} - \boldsymbol{x'})}{|\boldsymbol{x} - \boldsymbol{x'}|^3} \, dv(\boldsymbol{x'})$$

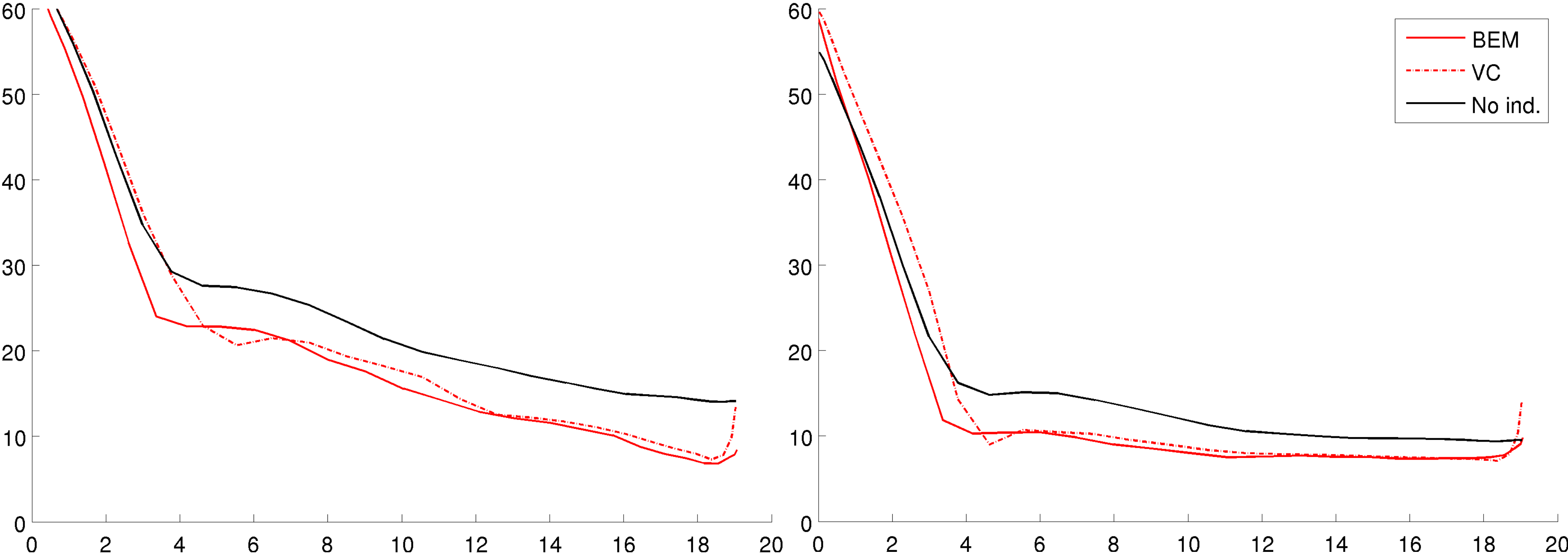
$\boldsymbol{U}_{\phi}$ More complex

Vorticity and sources are the keys.

It's easy to find particular, elementary solutions

The boundary conditions are the tricky part.

Results



Potential flow elements

Potential flow elements used

- Vortex segments
- Vortex particles (blobs)
- Plan quadrilateral source panel
- Constant strength double panel

Potential flow elements not-used but available

- Point sources
- Point vortex
- Point doublet

Theoretical elements

- Vortex Rings
- Semi-helical vortices

Other

- Lagrangian markers
- Eulerian markers

Helmholtz decomposition

$$\boldsymbol{U} = \mathrm{grad}\,\phi + \mathrm{curl}\,\boldsymbol{A}$$

!!!Decomposition not unique, involves some things on the BC already, bounded/unbounded domain tricky

$$\begin{aligned}\boldsymbol{U} &= \underbrace{\mathrm{grad}\,\phi}_{\text{curl free}} + \underbrace{\mathrm{curl}\,\psi}_{\text{divergence free}} \\ &= \boldsymbol{U}_\phi + \boldsymbol{U}_\psi\end{aligned}$$

$$\begin{aligned}\boldsymbol{U} &= \underbrace{\mathrm{grad}\,\phi_0}_{\text{curl and div free}} + \underbrace{\mathrm{grad}\,\tilde{\phi}}_{\text{curl free}} + \underbrace{\mathrm{curl}\,\psi}_{\text{divergence free}} \\ &= \boldsymbol{U}_0 + \boldsymbol{U}_{\tilde{\phi}} + \boldsymbol{U}_\psi\end{aligned}$$

(incompressible) $\mathrm{div}\,\boldsymbol{U} = 0 \quad \text{and} \quad \mathrm{div}\,\boldsymbol{U}_\psi = 0 \quad \Rightarrow \quad \boxed{\Delta\phi = 0}$

(definition) $\mathrm{curl}\,\boldsymbol{U} = \omega \quad \text{and} \quad \mathrm{curl}\,\boldsymbol{U}_\phi = 0 \quad \Rightarrow \quad \boxed{\Delta\psi = -\omega}$

Vorticity equation

Curl of Newton's law:

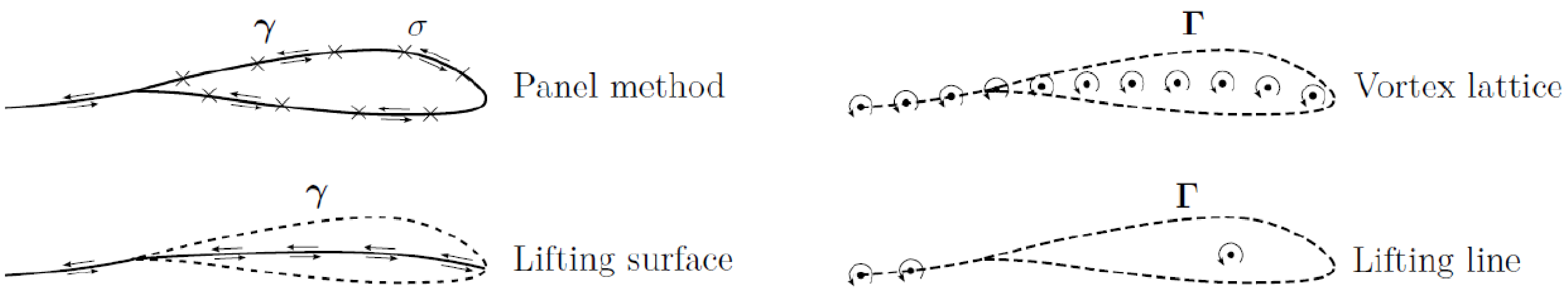
$$\frac{d\omega}{dt} = \frac{\partial \omega}{\partial t} + \underbrace{(\boldsymbol{U} \cdot \nabla)\omega}_{\text{advection}} = \underbrace{(\omega \cdot \nabla)\boldsymbol{U}}_{\text{stretching}} - \underbrace{\omega \nabla \cdot \boldsymbol{U}}_{\text{dilatation}} + \underbrace{\nabla \times \boldsymbol{F}}_{\text{n.-c. forces}} + \underbrace{\frac{1}{\rho^2}\nabla\rho \times \nabla p}_{\text{baroclin}} + \underbrace{\nabla \times \left(\frac{1}{\rho}\nabla \cdot \underline{\underline{\boldsymbol{\tau}}}\right)}_{\text{v. diffusion}}$$

Incompressible, homogeneous, Newtonian fluid with conservative forces:

$$\frac{\partial \omega}{\partial t} + \underbrace{(\boldsymbol{U} \cdot \nabla)}_{\text{convection}}\ \omega = \underbrace{(\omega \cdot \nabla)\boldsymbol{U}}_{\text{strain}} + \underbrace{\nu \Delta \omega}_{\text{diffusion}}$$

Presssure obtained by integration

Different vortex codes



The entire flow field is known by summation of the velocity induced by all the singularities

Pros and cons

Pros

- Fast and higly parallel (GPU)
- Grid free
- Small diffusion compared to CFD
- Good amount of theoretical results available for validation
- Easy to study the effect of different elements
- Fun

Cons

- Computational time increases with time.
- Everything should fit in the potential flow context (turbulence, shear)
- Viscosity at the boundary layer needs more work
- Compressibility can be added, but out of initial assumptions.