Modeling of the spectral velocity tensor including buoyancy effects

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Outline

- Background
- Definitions and properties
- RDT equations
- Results
- Summary and future work

Background

- Risø has produced a model for the 3-D spectrum of turbulence, Φ_{ij} (Mann,1994 (M94))
- Widely used in wind energy industry to simulate inflow turbulence for load calculations

Assumptions & Limitations :

- Rapid distortion theory (RDT) : Non-linear term(s)$\equiv 0$
- Homogeneity, no {viscous+gravity+Coriolis} force
- Mean uniform shear
- Neutral surface layer
- Flat terrain and gently varying orography in neutral flow

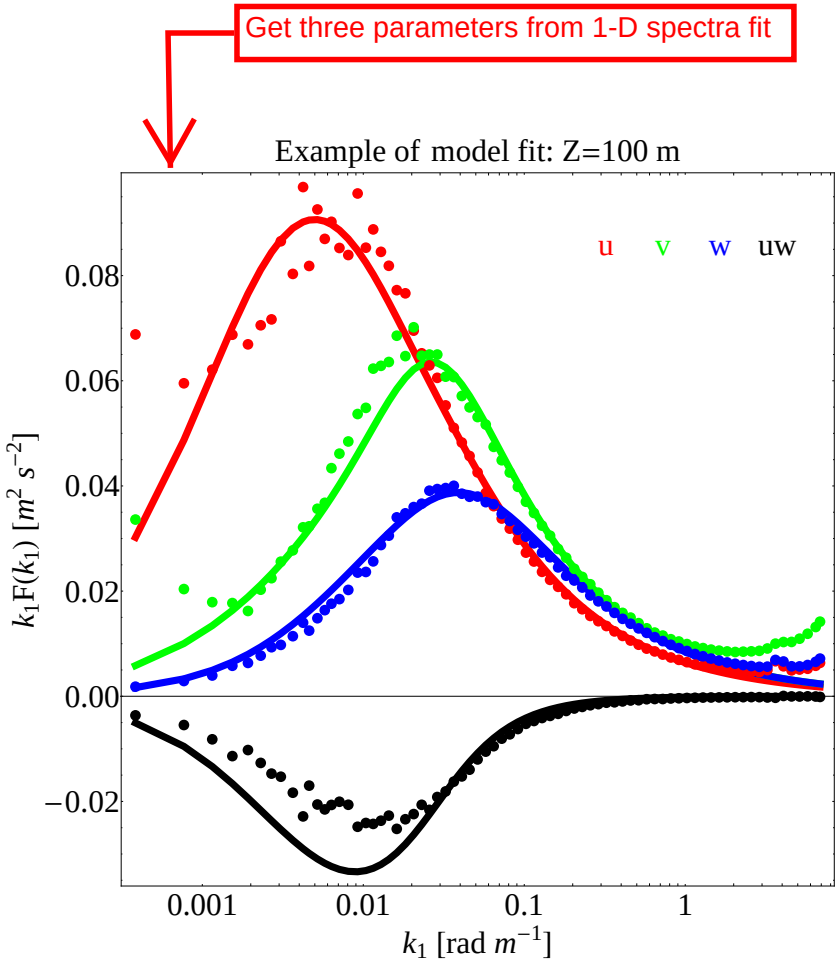
The model contains three adjustable parameters, determined from the single point measurements viz., $\alpha\varepsilon^{2/3}$, **L** and Γ , where $\alpha \approx 1.7$

Turbulence structure in rotor plane

Turbulence simulation from the Mann spectral tensor model

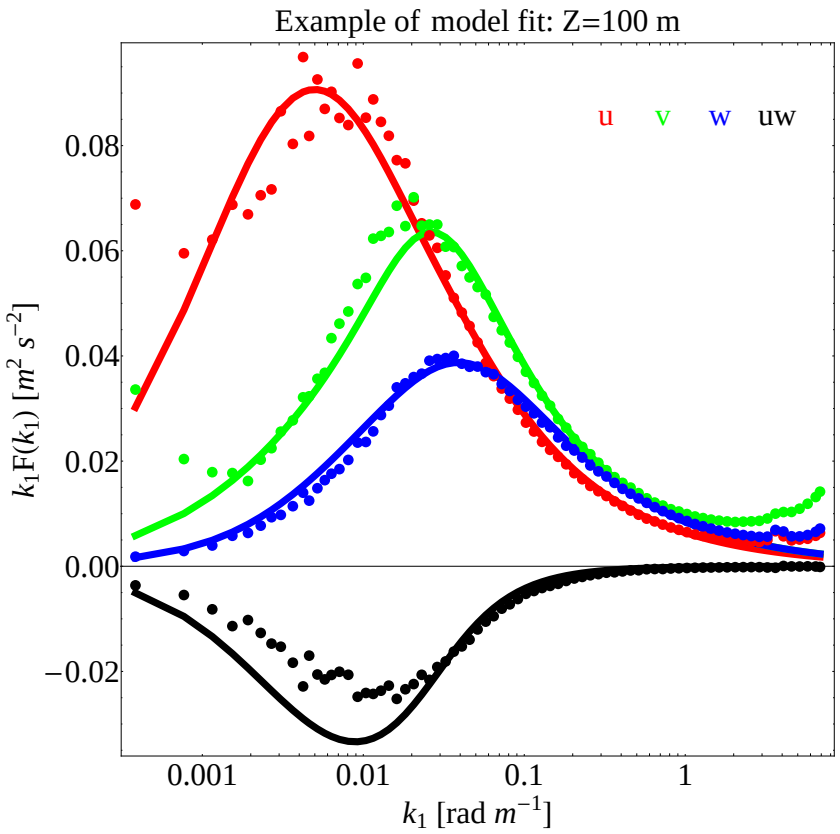
u.....v ..... w

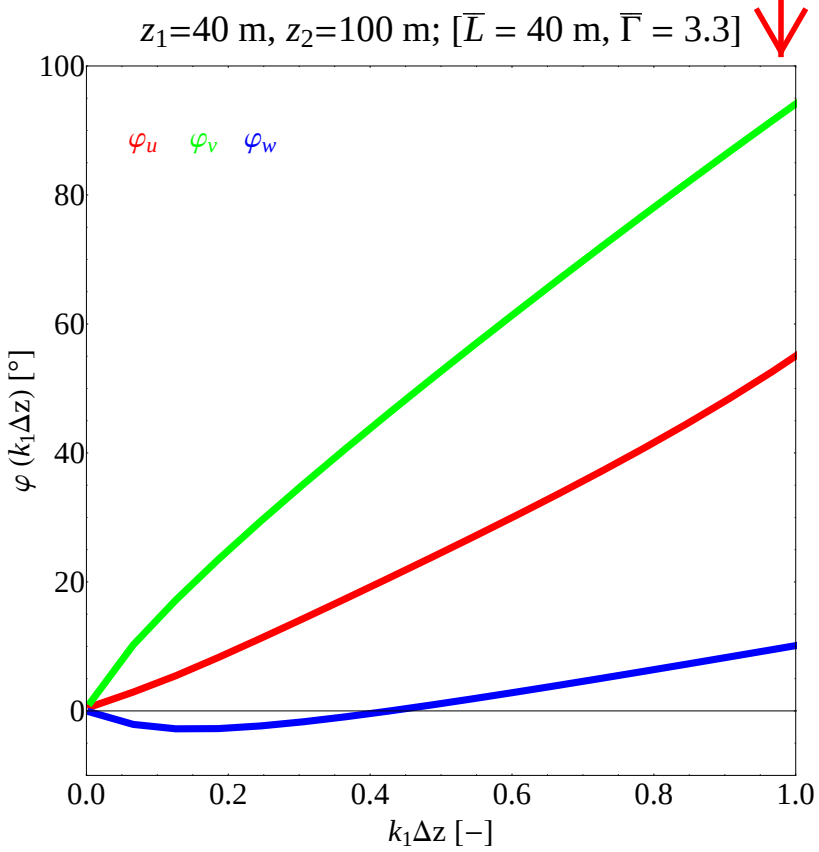
What RDT model shows



What RDT model shows

Use these parameters as an input to predict model phases





(Note : The model phases are functions of **L** and Γ parameters and are unaffected by $\alpha\varepsilon^{2/3}$)

Definitions and properties

Spectral velocity tensor

$$\frac{\langle \mathrm{d}Z_i^*(\mathbf{k})\mathrm{d}Z_j(\mathbf{k})\rangle}{\mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3} = \Phi_{ij}(\mathbf{k}) \tag{1}$$

Definitions and properties

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Cross-spectra :

$$\chi_{ij}(k_1, \Delta y, \Delta z) = \int \Phi_{ij}(\mathbf{k})\mathrm{e}^{\mathrm{i}(k_2\Delta y + k_3\Delta z)}\mathrm{d}\mathbf{k}_\perp \tag{2}$$

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Coherence :

$$\mathrm{coh}_{ij}(k_1, \Delta y, \Delta z) = \frac{|\chi_{ij}(k_1, \Delta y, \Delta z)|^2}{F_i(k_1)F_j(k_1)} \tag{3}$$

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Phase :

$$\varphi_{ij}(k_1, \Delta y, \Delta z) = \arg(\chi_{ij}(k_1, \Delta y, \Delta z)) \tag{4}$$

where $\mathbf{k}$ is the three-dimensional wavenumber vector, $\int \mathrm{d}\mathbf{k}_\perp = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}k_2\mathrm{d}k_3$, $F_i(k_1) = \chi_{ii}(k_1, 0, 0)$ (with no index summation).

Φ_{ij} from M94

The model calculates the evolution of Fourier modes under the influence of the mean shear from an initial isotropic state. In isotropic turbulence, the velocity-spectrum tensor is

$$\Phi_{ij}(\mathbf{k}_0) = \frac{E(k)}{4\pi k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right), \tag{5}$$

where $\mathbf{k}_0 = \mathbf{k}(0)$ and k is the length of the vector $\mathbf{k}$. The energy spectrum $E(k)$ given by von Kármán as

$$E(k) = \alpha \varepsilon^{2/3} L^{5/3} \frac{(kL)^4}{(1+(kL)^2)^{17/6}}, \tag{6}$$

where $\alpha \approx 1.7$ is the Kolmogorov constant, ε is the rate of viscous dissipation of specific turbulent kinetic energy (TKE), and L is a turbulence length scale.

Φ_{ij} from M94 ...continued

In order to make the model stationary, the time dependency in the model was removed by incorporating the general concept of an eddy life time, $\tau(k)$, and the parameterization of $\tau(k)$ in M94 was

$$\tau(k) = \Gamma \left(\frac{\mathrm{d}U}{\mathrm{d}z}\right)^{-1} (kL)^{-2/3} \left[{}_2F_1\left(\frac{1}{3}, \frac{17}{6}; \frac{4}{3}; -(kL)^{-2}\right)\right]^{-1/2}, \quad (7)$$

where Γ is a parameter to be determined and ${}_2F_1$ is the Gaussian or ordinary hypergeometric function. The analytical forms of $\Phi_{ij}(\mathbf{k})$ in M94 can be expressed as

$$\Phi_{ij}(\mathbf{k}) \equiv \Phi_{ij}(\mathbf{k}, \alpha\varepsilon^{2/3}, L, \Gamma). \quad (8)$$

RDT equations

$$\frac{\mathrm{D}}{\mathrm{D}t}\mathrm{d}Z_i(\mathbf{k}(t),t) = \left(2\frac{k_i k_1}{k^2} - \delta_{i1}\right)\left(\frac{\mathrm{d}U}{\mathrm{d}z}\right)\mathrm{d}Z_3(\mathbf{k}(t),t) - \frac{g}{\overline{\theta}}\left(\frac{k_i k_3}{k^2} - \delta_{i3}\right)\mathrm{d}\Theta(\mathbf{k}(t),t) \tag{9}$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\mathrm{d}\Theta(\mathbf{k}(t),t) = -\left(\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}z}\right)\mathrm{d}Z_3(\mathbf{k}(t),t) \tag{10}$$

Equations 9, and 10 constitute the governing RDT equations for homogeneous turbulent flow that is stratified $(\mathrm{d}\overline{\theta}/\mathrm{d}z)$ and sheared $(\mathrm{d}U/\mathrm{d}z)$ in vertical z direction.

RDT equations (Neutral surface-layer)

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$$- \frac{g}{\overline{\theta}}\left(\frac{k_i k_3}{k^2} - \delta_{i3}\right)\mathrm{d}\Theta(\mathbf{k}(t),t)$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\mathrm{d}\Theta(\mathbf{k}(t),t) = -\left(\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}z}\right)\mathrm{d}Z_3(\mathbf{k}(t),t)$$

Equations 9, and 10 constitute the governing RDT equations for homogeneous turbulent flow that is stratified $(\mathrm{d}\overline{\theta}/\mathrm{d}z)$ and sheared $(\mathrm{d}U/\mathrm{d}z)$ in vertical z direction.

For temperature, the isotropic three-dimensional spectrum is given as

$$\Phi_{\theta\theta}(\mathbf{k}_0, 0) = \frac{S(k)}{4\pi k^2}, \tag{11}$$

where $S(k)$ is the potential energy spectrum containing the form of the inertial subrange (Kaimal and Finnigan [1994]) as

$$S(k) = \beta_1 \varepsilon^{-1/3} \varepsilon_\theta L^{\frac{5}{3}} \frac{(kL)^2}{(1+(kL)^2)^{\frac{11}{6}}} \tag{12}$$

Here ε_θ is the dissipation rate for half the temperature variance and $\beta_1 = 0.8$ is a universal constant.

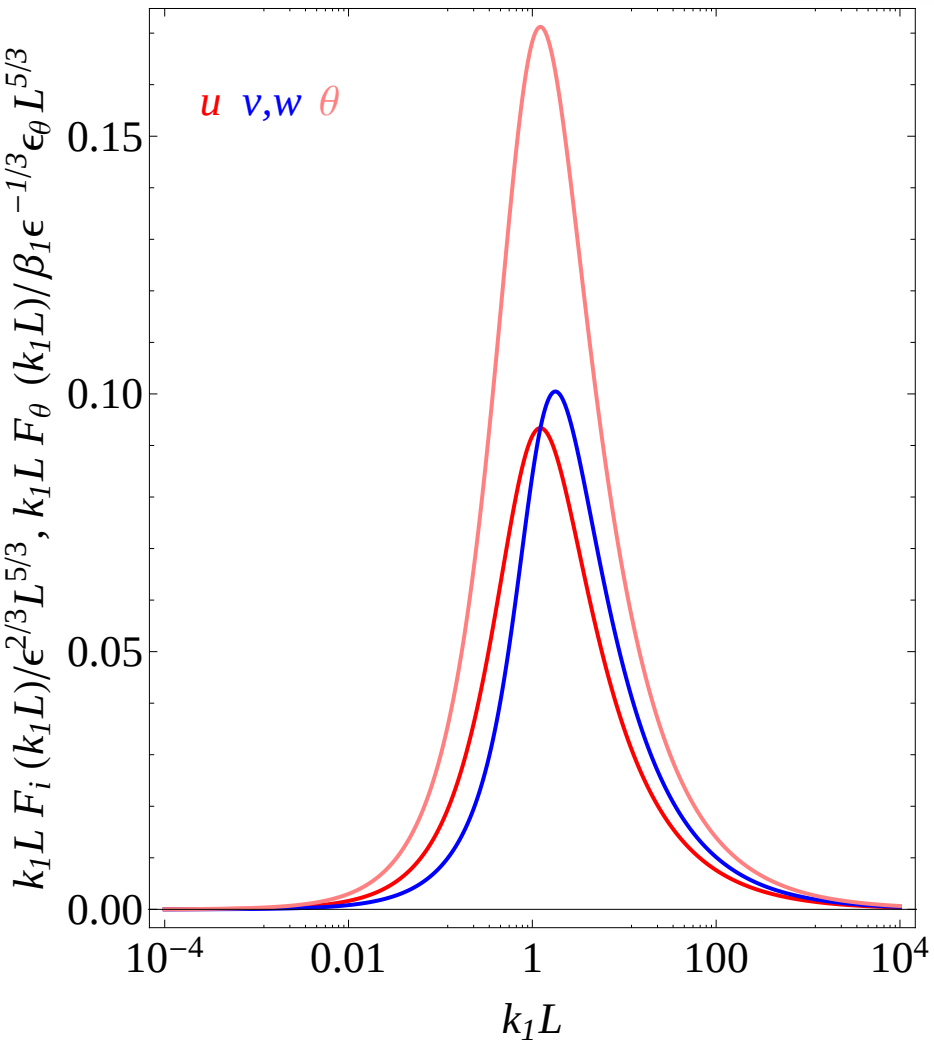


FIGURE: Normalized initial velocity and temperature spectra.

Anisotropic tensor

$$\Phi_{lm}(\mathbf{k}) \equiv \Phi_{lm}(\mathbf{k}, \alpha\varepsilon^{2/3}, L, \Gamma, Ri, \beta\eta_\theta), \tag{13}$$

where $l, m = 1, 2, 3, 4$,

$$\mathrm{d}Z_4(\mathbf{k}(t), t) = \frac{g}{\overline{\theta}} \left(\frac{\mathrm{d}U}{\mathrm{d}z} \right)^{-1} \mathrm{d}\Theta(\mathbf{k}(t), t), \tag{14}$$

the Richardson number

$$Ri = \frac{(g/\overline{\theta})\left(\mathrm{d}\overline{\theta}/\mathrm{d}z\right)}{(\mathrm{d}U/\mathrm{d}z)^2}, \tag{15}$$

and

$$\eta_\theta \equiv \frac{\varepsilon_\theta}{\varepsilon} \left[\frac{g}{\overline{\theta}} \left(\frac{\mathrm{d}U}{\mathrm{d}z} \right)^{-1} \right]^2. \tag{16}$$

Results

We estimate the velocity (auto-) spectra and co-spectrum of u and w from the measured time series as

$$F_{ij}(f,z) \equiv \langle \hat{u}_i(f)\hat{u}_j^*(f)\rangle, \tag{17}$$

and the temperature spectrum and the component-wise kinematic heat fluxes, respectively as

$$F_\theta(f,z) \equiv \langle \hat{\theta}(f)\hat{\theta}^*(f)\rangle, \tag{18}$$

$$F_{i\theta}(f,z) \equiv \langle \hat{u}_i(f)\hat{\theta}^*(f)\rangle, \tag{19}$$

where $\hat{u}_i(f)$, and $\hat{\theta}(f)$ are the complex-valued Fourier transforms of the ith velocity component, and temperature, respectively at height z.

Observations from Høvsøre test site in Denmark

116.5 m tall Met-mast: 56°26'26'' N, 08°09'03''E. Wind turbines

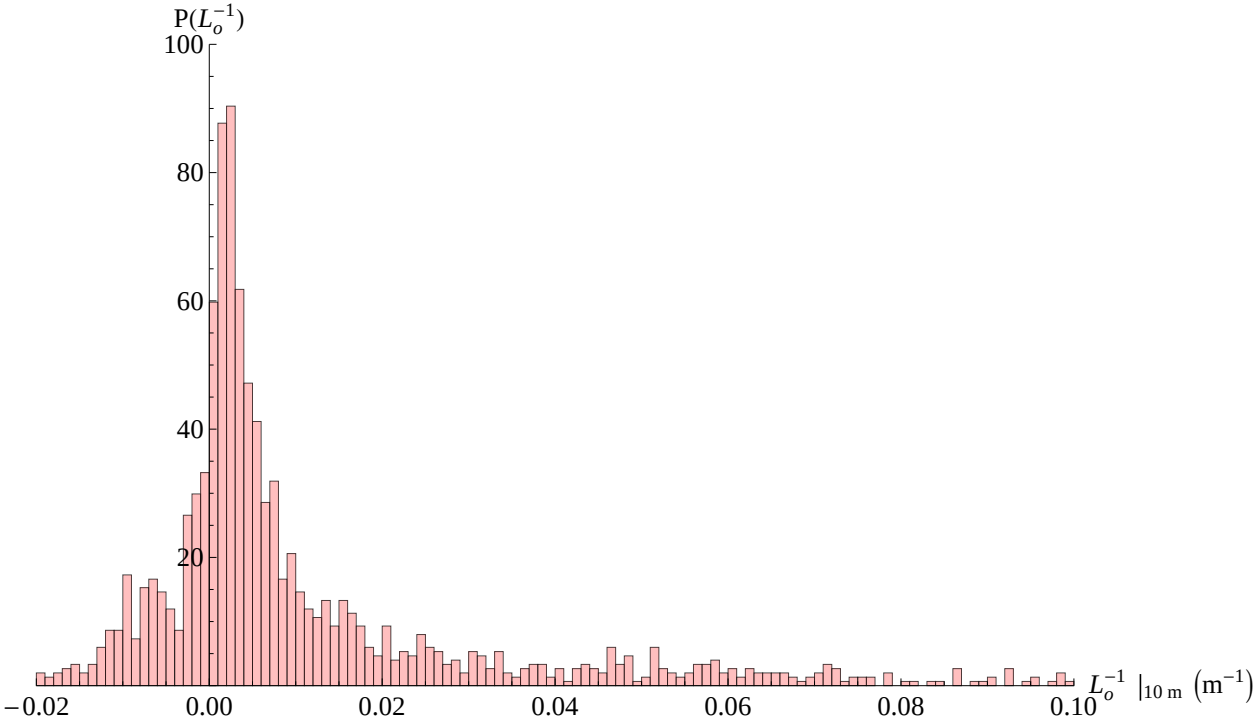
TABLE: Classification of atmospheric stability according to inverse Obukhov length intervals (in m^{-1}).

Stable (S)	$0.005 \leq L_o^{-1} \leq 0.02$
Near-neutral stable (NNS)	$0.002 \leq L_o^{-1} \leq 0.005$
Near-neutral unstable (NNU)	$-0.005 \leq L_o^{-1} \leq -0.002$
Unstable (U)	$-0.01 \leq L_o^{-1} \leq -0.005$

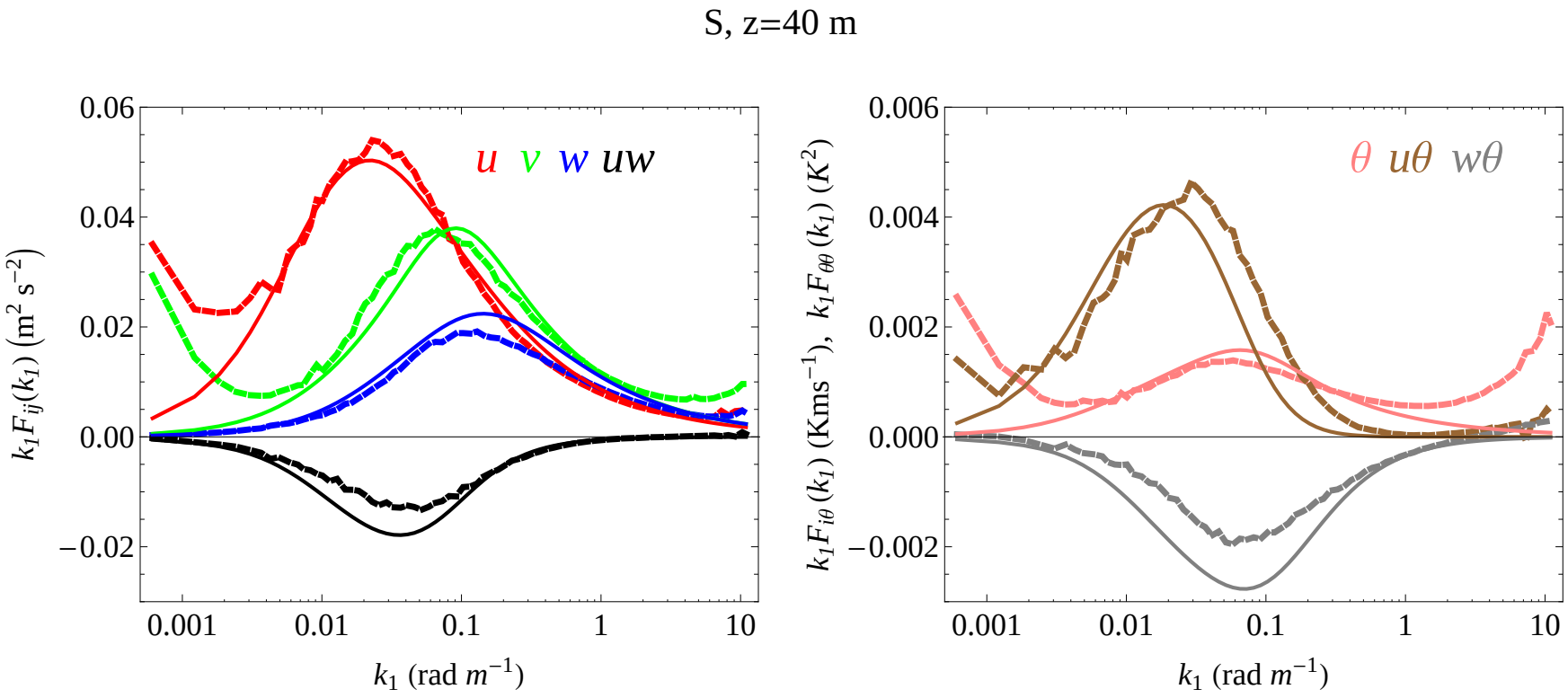
$$L_0 = \frac{-{u_*}^3}{\kappa(g/\overline{\theta})\overline{w'\theta'}_0} \tag{20}$$

Data :

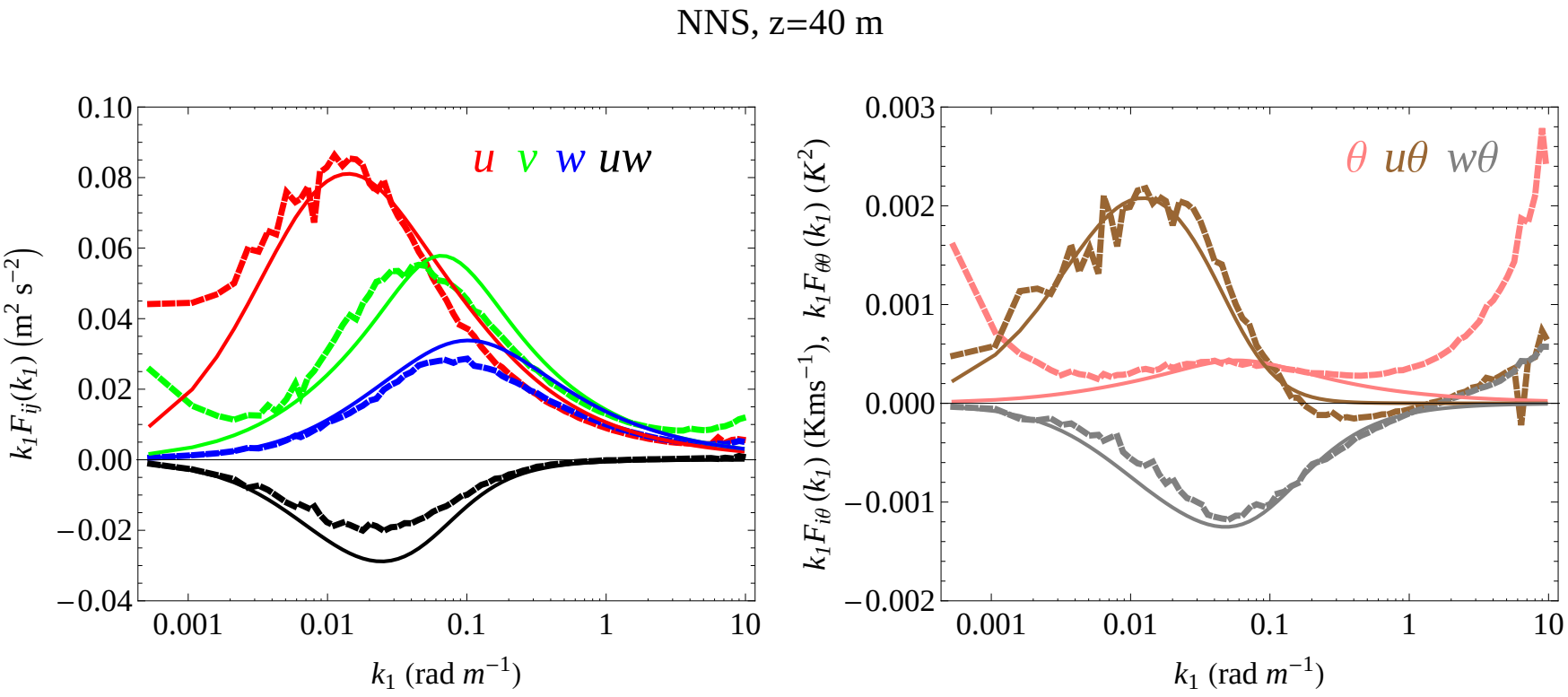
- Wind speeds are selected between 8-9 ms^{-1} at 80 m
- Wind speed direction between 60° and 120°
- 20 Hz sonics at 10, 20, 40, 60, 80 and 100 m
- Seven years of data from 2004 to 2010
- 30 min. averaged wind speeds



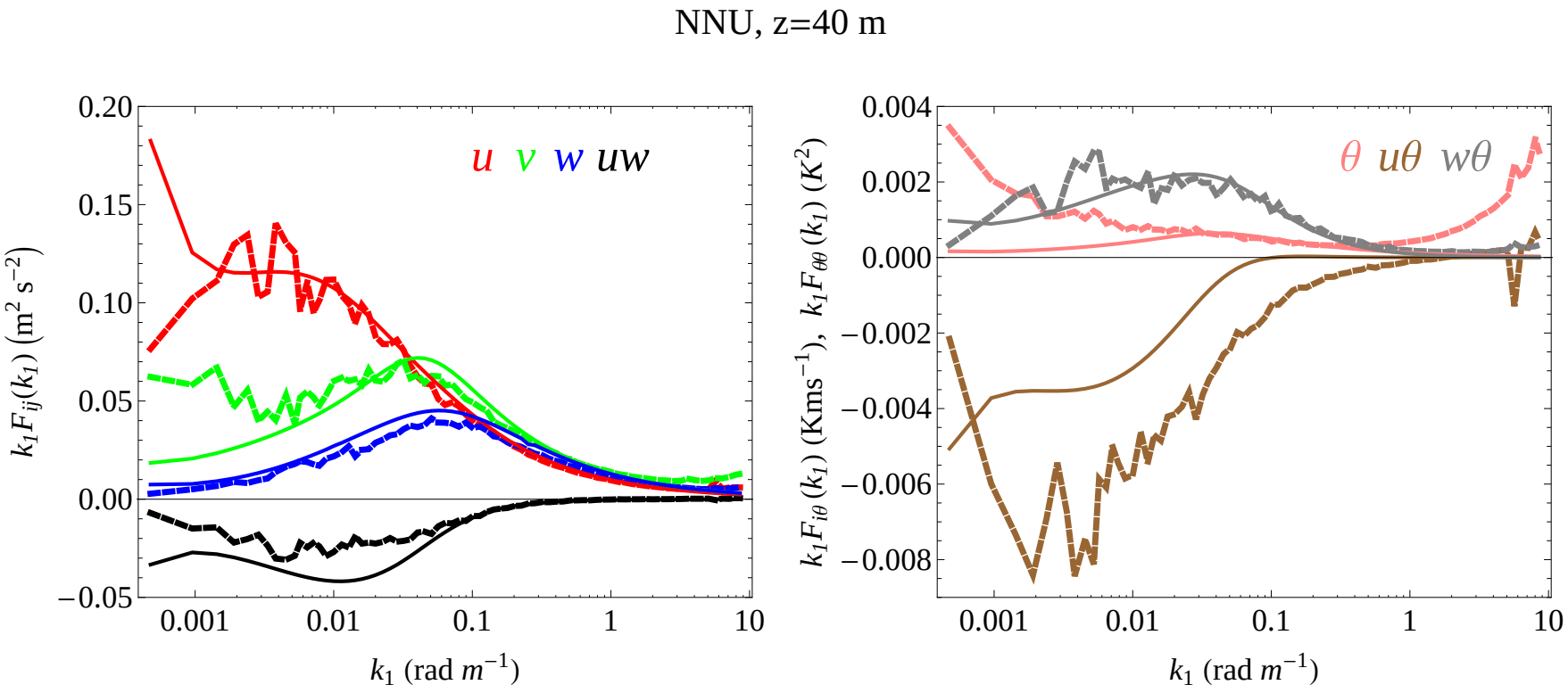
Spectra and co-spectra : S



Spectra and co-spectra : NNS



Spectra and co-spectra : NNU



Spectra and co-spectra : U

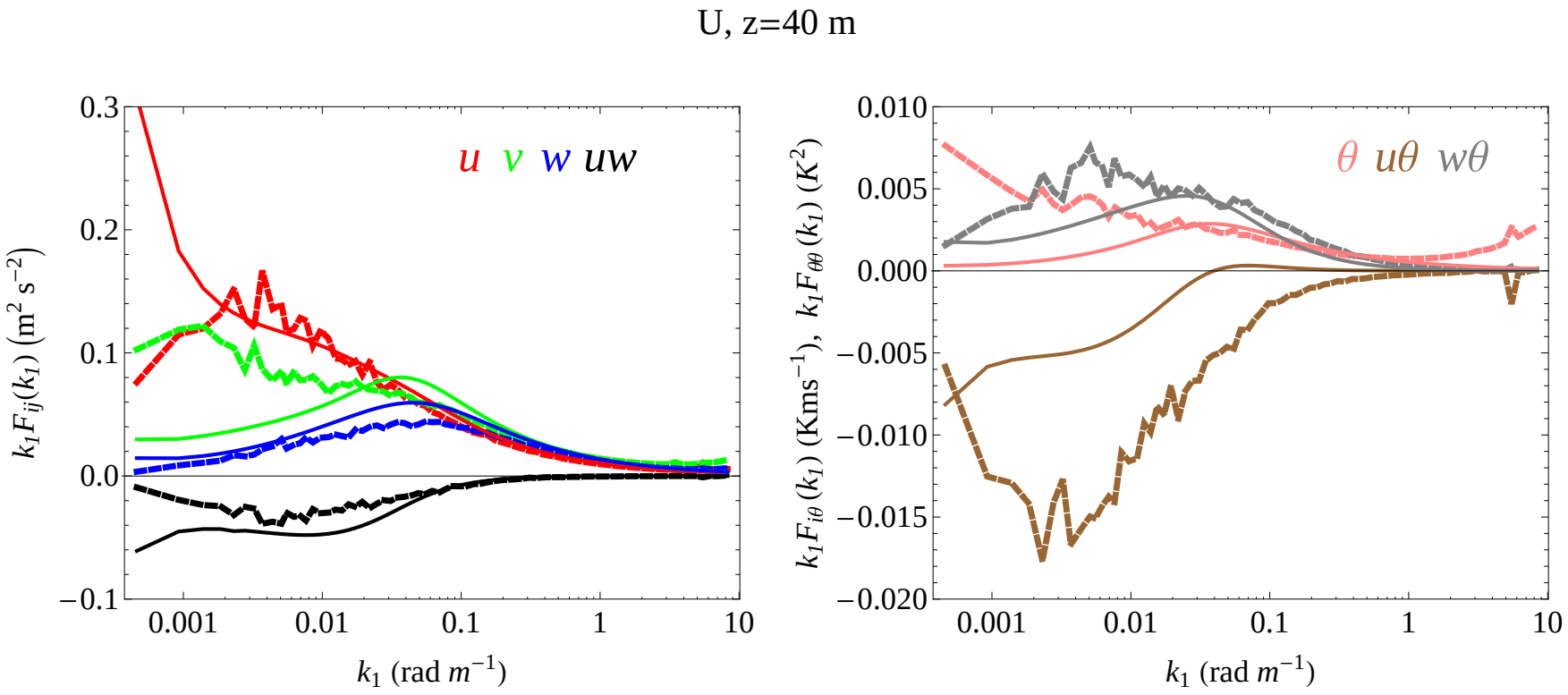


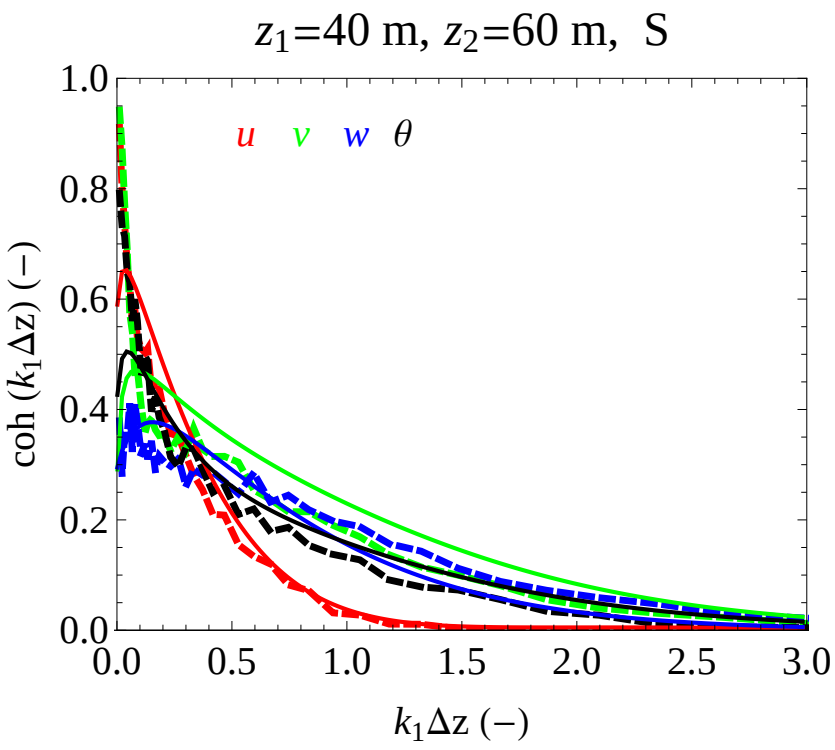
TABLE: The spectral tensor parameters from the fits for S, NNS, NNU, and U stability cases for the velocity bin 8–9 m s^{-1} at $z = 40$ m.

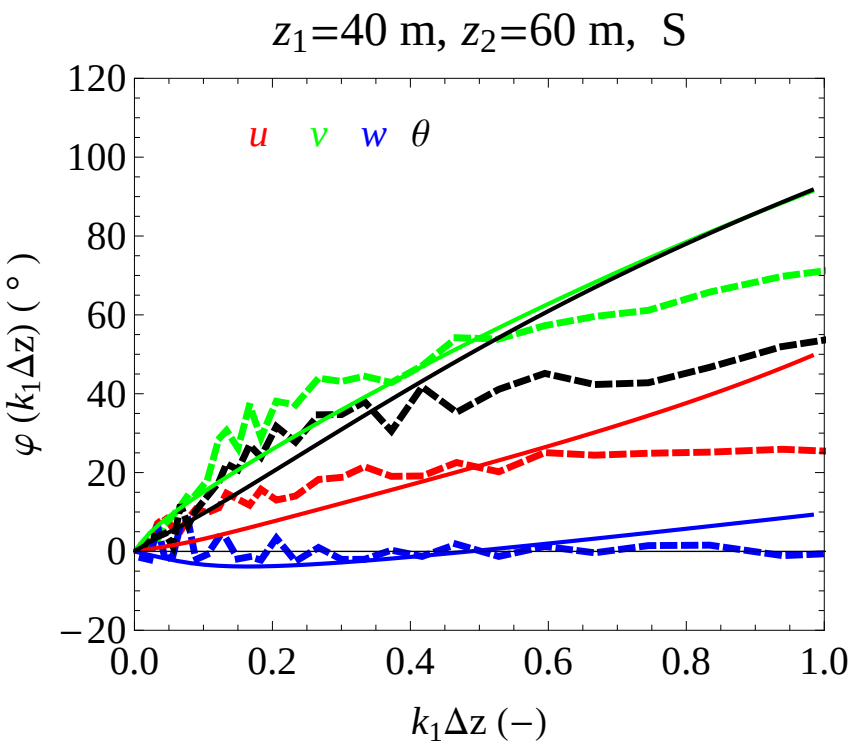
Stability	n	$\alpha\varepsilon^{2/3}$ (m$^{4/3}$ s^{-2})	L (m)	Γ	Ri	η_θ
S	359	0.053	15	3.0	0.06	0.02
NNS	298	0.065	20	3.1	0.03	0.012
NNU	71	0.06	30	3.01	-0.032	0.025
U	106	0.0635	35	2.5	-0.033	0.15

TABLE: The M94 model parameters from the fits at $z = 40$ m.

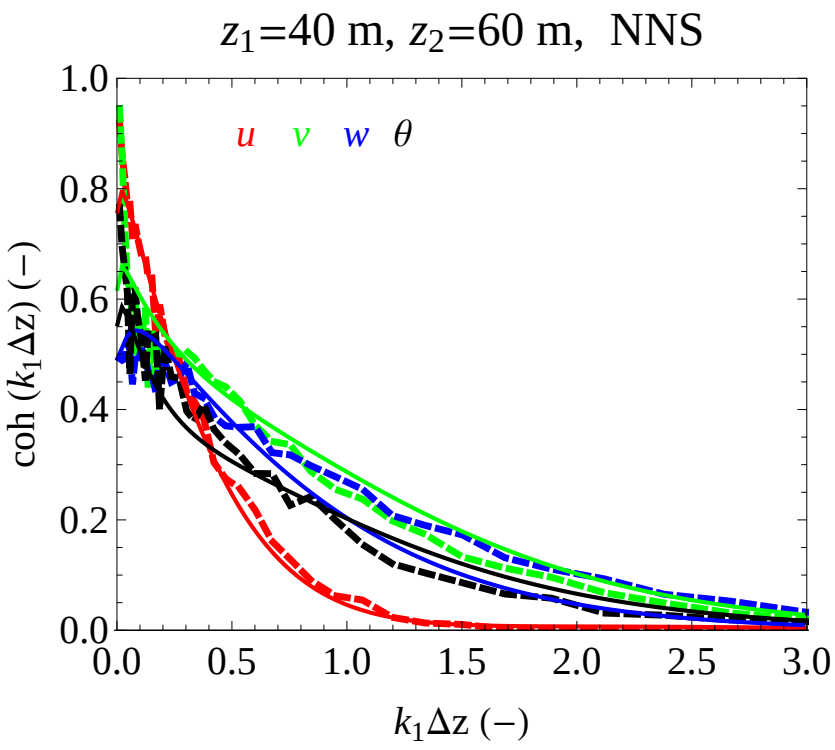
Stability	$\alpha\varepsilon^{2/3}$ (m$^{4/3}$ s^{-2})	L (m)	Γ
S	0.05	12	3.1
NNS	0.053	21	3.4
NNU	0.04	45	3.7
U	0.04	75	3.5

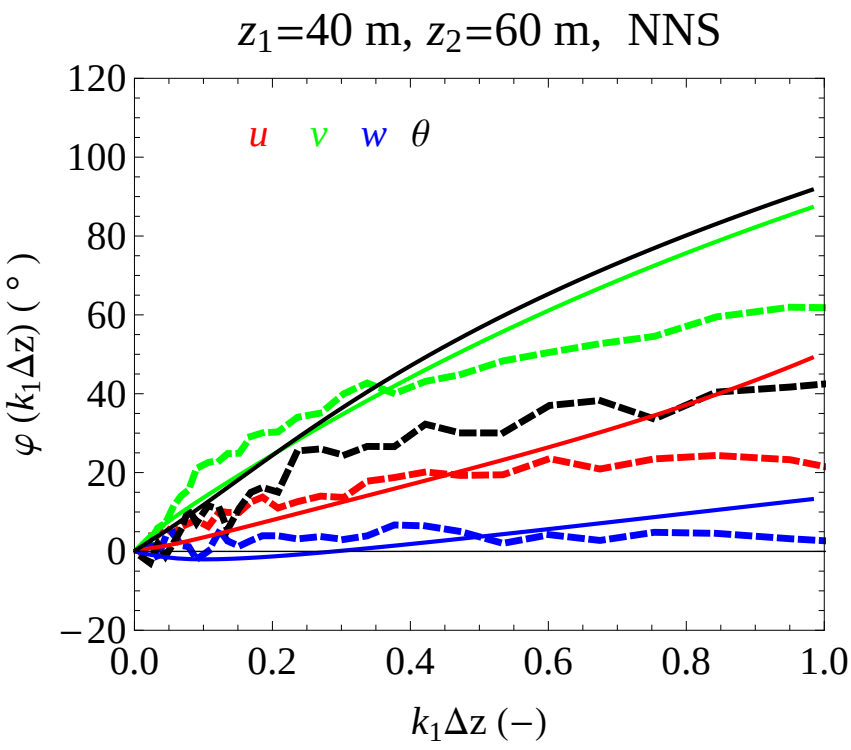
Cross-spectra : S



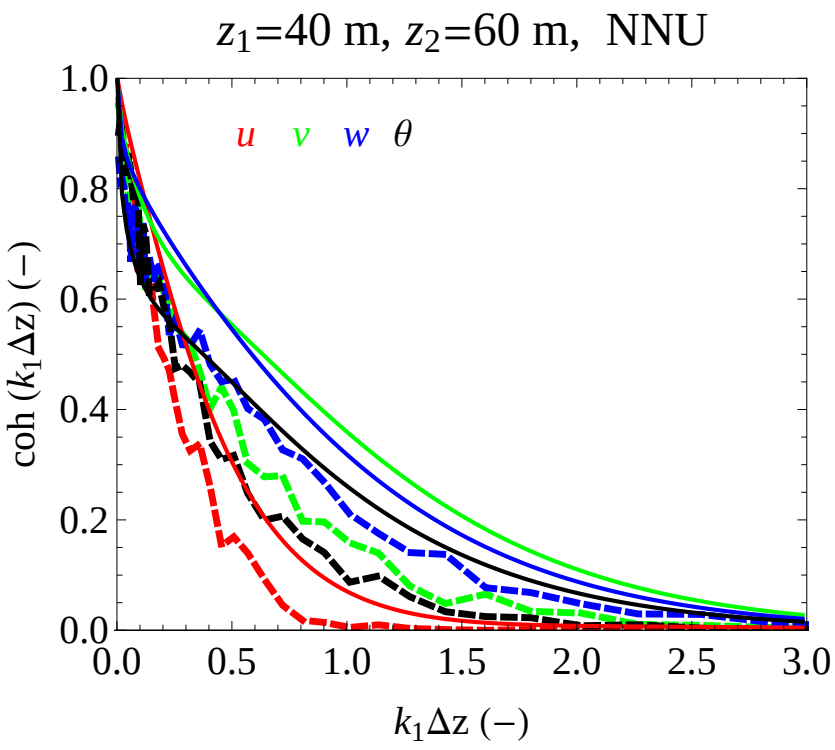


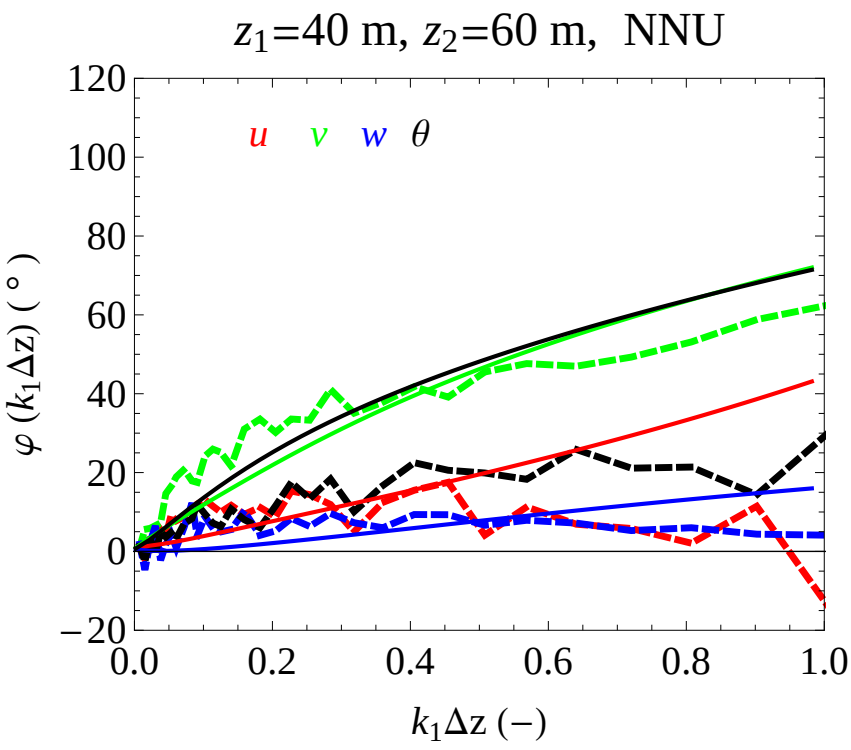
Cross-spectra : NNS





Cross-spectra : NNU





Summary and future work

- The new RDT-based spectral tensor model is proposed, which, in addition to the velocity spectra and co-spectra, also gives the temperature spectrum and the temperature fluxes, as a result of including buoyancy effects via a mean uniform temperature gradient
- The preliminary results show that the model seems to work better for stable than unstable conditions
- The length scale of $u\theta$ is roughly equal to that of u-spectra, and the length scales of θ and $w\theta$ are roughly equal to that of w-spectra, in both stable and unstable ABLs
- In the intertial subrange, $w\theta$ co-spectrum is proportional to $k_1^{-7/3}$, and
- The model can be tested against alternative forms of eddy life time

Summary and future work cont...

- The two extra parameters Ri and η_θ are obtained from spectral fits, while the model will be tested against the Ri values determined from the measurements
- The temperature spectra from the Høvsøre measurements are noisy, and therefore, in the future we could use other measurements
- There is possibility of improving the spectral fits by making the look-up table and making the interpolations function, which will be done by many (co-)spectra calculations on a computer cluster of DTU's Wind Energy department
- We have not tested the model using the empirical relationships from the MOST, where the number of parameters is reduced to four instead of five, and that it could be useful to test RDT model against MOST within the surface layer

Thank you